A CENSUS OF 3-(12,6,4) AND 2-(11,5,4) DESIGNS

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CONTENTS

PART I

THE DEVELOPMENT OF THE 3-(12,6,4)

AND 2-(11,5,4) DESIGNS

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ABSTRACT

This thesis documents the development of all possible non-isomorphic 3-(12,6,4) and 2-(11,5,4) designs. A representative copy of each of the 545 non-isomorphic 3-(12,6,4) designs along with all its non-trivial automorphisms is given. The point orbits of these designs can then be used to produce a copy of any of the 4393 non-isomorphic 2-(11,5,4) designs found to occur.

INTRODUCTION

In combinatorial theory the concept of t-(v,k,λ) 'block designs' is well known. This is the collective name for designs with a fixed number of 'blocks' of size k chosen from v different 'points' or 'symbols' so that the resulting structure has some degree of balance; i.e. each singleton, unordered pair, unordered triple, etc. will occur a fixed number of times. Given a set of design parameters a design possessing them may or may not exist. Where designs do exist, a common combinatorial problem is that of determining all possible different (non-isomorphic) structures. Thus it is the aim of this thesis to produce all possible non-isomorphic 3-(12,6,4) and 2-(11,5,4) designs.

The material for this thesis has been arranged into two main sections labelled Part I and Part II. The latter of these contains a key and data for obtaining a representative copy of each of the non-isomorphic designs produced in the former. Thus Part II is primarily a 'reference' section to provide a permanent record of and easy access to any particular non-isomorphic design.

Part I documents the development of the designs. In chapter 1 some general concepts and formulae are introduced. These are then applied to the 3-(12,6,4) and 2-(11,5,4) designs to give possible structures for the blocks of these designs, i.e. their 'block types'. The relationship between the block types of the 3-designs and the 2-designs is then discussed and this and first principles are used to discount the existence of some of the block types.

Further basic material is added in chapter 2 where a summary of the work done by Thompson [6] on the decomposable 3-(12,6,4) and 2-(11,5,4) designs is presented. The last two sections of this chapter contain the first new results beyond those of [6]: those of the enumeration of the decomposable 3-(12,6,4) and 2-(11,5,4) designs with repeated blocks.

All the non-isomorphic designs created in this chapter have been listed in Part II, Section I.

Chapters 3 and 4 cover the development of all non-isomorphic designs without, and with repeated blocks respectively. In chapter 3 the first block of a 3-(12,6,4) design was required to have one of the two legitimate block types. Under this assumption a 'skeleton' for this block was laid down and then completed for the six points of the block in all possible non-equivalent ways. With the assistance of a computer and subject to the non-occurrence of repeated blocks, these structures were balanced for the remaining points to produce a pool of 3-designs from which isomorphs must be sieved. Once the isomorphs had been eliminated all the non-isomorphic 3-(12,6,4) designs' automorphisms were developed so that their point orbits are available to provide easy access to the non-isomorphic 2-(11,5,4) designs they contain as point restrictions. This whole procedure was repeated with the first block being required to be of the other block type. Now as well as repeated blocks, blocks of the previous block type are also disallowed, i.e. the designs must consist entirely of blocks of this second type. A complete list of the non-isomorphic designs created by these two processes appears in Part II, Sections II and III.

A similar approach is adopted in chapter 4 to develop all the designs with repeated blocks. Only here the smaller 2-(11,5,4) designs are produced first and the 3-designs obtained from these by complementation. The resulting non-isomorphic designs are recorded in Part II, Section IV.

The final chapter in Part I lists the eight transitive 3-(12,6,4) designs and the three transitive 2-(11,5,4) designs and discusses their automorphism groups.

At the conclusion of the fourth section of Part II there appear three appendices which contain the input data associated with the various programs used to develop the designs.

CHAPTER 1

INTRODUCTORY CONCEPTS AND RESULTS

As this thesis provides an extension of the work done by Thompson in [6], much of the introductory analysis is already available from this source. However the reproduction of this material here will provide a useful starting point and help to keep this volume reasonably self-contained. Some minor modifications have been made to allow for repeated blocks which were not covered in [6]. Should the reader be familiar with this earlier work then undesired repetition can be avoided by beginning at Chapter 2, Section IV.

Section I: Standard Formulae and Concepts

A t-(v,k,λ) design on v symbols (or points) has b distinct blocks each containing k distinct points, with every unordered t-tuple of points appearing in exactly λ blocks.

Let $\lambda_s \leqslant t$ be the number of times a particular s-tuple appears in the design, and count incidences of t-tuples containing s-tuples. Then $(t-s)$ points of the t-tuple do not occur in the s-tuple and these can be chosen from the remaining $(k-s)$ points of the block in $\binom{k-s}{t-s}$ ways. Therefore as the s-tuples each occur λ_s times, the number of such incidences is

$$\lambda_s \binom{k-s}{t-s} .$$

Counting these a different way, we have v points in all. If a particular s-tuple is chosen then there are $(v-s)$ other points available to extend the s-tuple to a t-tuple. These points can be taken $(t-s)$ at a time in $\binom{v-s}{t-s}$ ways. As each t-tuple occurs λ times in the design the number of incidences is

$$\lambda \binom{v-s}{t-s} .$$

Equating the two expressions gives the standard formula

$$\lambda_s \binom{k-s}{t-s} = \lambda \binom{v-s}{t-s}$$

$$\lambda_s = \frac{(v-s)(v-s-1)(v-s-2)\dots(v-t+1)\lambda}{(k-s)(k-s-1)(k-s-2)\dots(k-t+1)}, \quad 0 \leqslant s \leqslant t.$$

Note that λ_0 is the number of blocks, usually denoted by b, and λ_1 is the replication number for the design, usually denoted by r.

Associated with every design is an incidence matrix $A = [a_{ij}]$ with $a_{ij} = 1$ if the i^{th} symbol is on the j^{th} block and $a_{ij} = 0$ otherwise. If two designs have incidence matrices A and B such that PAQ = B where P and Q are permutation matrices then A is said to be isomorphic to B. If A is the incidence matrix of a design and there are permutation matrices P and Q such that PAQ = A, then the design is said to have an automorphism. Any automorphism of a design D can be represented as a permutation π on the points of D. The set of all such π under successive applications forms the automorphism group, Aut D, of D.

If the automorphism group of the design is such that it allows any symbol to be transformed to any other symbol then the group, and hence the design, is said to be transitive. If any ordered pair can be sent to any other ordered pair the group and the design are said to be 2-transitive, and so on.

Let D be a t-(v,k,λ) design and B_0 be a block of D. Let n_i be the number of blocks intersecting B_0 in exactly i points. Now count blocks; so

$$n_0 + n_1 + n_2 + \dots + n_k = b = \lambda_0 .$$

Then count flags $(B;x)$ where $x \in B$ and $x \in B_0$, to get

$$n_1 + 2n_2 + 3n_3 + \dots + kn_k = k\lambda_1 .$$

Also count double flags $(B;x,y)$, where $x \in B$, $y \in B$, $x \in B_0$, $y \in B_0$ and $x \neq y$, to get

$$\binom{2}{2}n_2 + \binom{3}{2}n_3 + \binom{4}{2}n_4 + \dots + \binom{k}{2}n_k = \binom{k}{2}\lambda_2 ,$$

and so on. In general

$$\sum_{i=s}^{k} \binom{i}{s} n_i = \binom{k}{s} \lambda_s, \quad 0 \leqslant s \leqslant t \; .$$

This gives a set of (t+1) diophantine equations for the block intersection numbers n_i and as the system is usually under-determined this can give two or more block types (the type being given by $(n_0, n_1, n_2, \ldots, n_k)$). Thus for a given set of design parameters, a knowledge of the number of blocks of each type helps to determine and catalogue the possible non-isomorphic designs.

For a given t-(v,k,λ) design it may be possible to form a $(t+1)$-$(v+1,k+1,\lambda)$ design by adding a new point to each of the given blocks; and then by adding new blocks of (k+1) points not containing the new point. The resulting (t+1)-design is known as an extension of the t-design. A design produced by deleting a particular point from the blocks and discarding all blocks not containing that point is called a restriction of the given design.

The extension of a t-$(2n+1,n,\lambda)$ design to a $(t+1)$-$(2n+2,n+1,\lambda)$ design is always possible by complementation [5]. This involves adding a new point to the given blocks and then taking complements with respect to the new point set to get further new blocks. For the particular case of $\lambda = \frac{n-1}{2}$ and n odd, we get the family of Hadamard designs. It has been shown by Dembowski [4] that these 2-designs can be extended to 3-$(2n+2,n+1, \frac{n-1}{2})$ designs in only one way, and that is by complementation.

If the blocks of two t-(v,k,λ) designs on the same set of points with the same parameters are taken together then a t-$(v,k,2\lambda)$ design is formed. However it does not follow that any t-$(v,k,2\lambda)$ design can be decomposed into two t-(v,k,λ) designs. If a t-(v,k,μ) design can be decomposed into t-(v,k,λ) designs, with $\lambda < \mu$, then it is said to be reducible; otherwise it is irreducible. The terms decomposable and non-decomposable are also used.

Section II: The Designs Block Types

The current study concerns the 2-(11,5,2) and 2-(11,5,4) designs and their respective extensions to 3-(12,6,2) and 3-(12,6,4) designs. If the block intersection equations are applied to these designs the following parameters and block intersection numbers result:

Design	Parameters				Possible n_i's							Type
	λ_0	λ_1	λ_2	λ_3	n_0	n_1	n_2	n_3	n_4	n_5	n_6	
2-(11,5,2)	11	5	2	-	0	0	10	0	0	1	-	-
3-(12,6,2)	22	11	5	2	1	0	0	20	0	0	1	-
2-(11,5,4)	22	10	4	-	1	0	15	5	0	1	-	A
					0	3	12	6	0	1	-	B
					0	2	15	3	1	1	-	C
					0	1	18	0	2	1	-	D
					0	0	20	0	0	2	-	R
3-(12,6,4)	44	22	10	4	1	2	1	36	1	2	1	E
					0	4	3	28	8	0	1	F
					1	1	5	30	5	1	1	AC
					1	0	9	24	9	0	1	B
					2	0	0	40	0	0	2	R

It is important to note that a solution of the block intersection equations gives only a necessary condition for the existence of a block of that type. We therefore proceed by showing that some of the above block types cannot exist. The non-occurrence of blocks of type D in any of the known 2-(11,5,4) designs suggested that blocks of this type are impossible.

<u>Lemma</u>: A block of type D(0 1 18 0 2 1) is impossible in a 2-(11,5,4) design.

<u>Proof</u>: Let H be a 2-(11,5,4) design. Let N_4 be the number of times a particular set of four points occur together and let N_i be the number of blocks containing any i of these points, allowing for multiplicities. By

the principle of inclusion and exclusion the number of blocks N_0 not containing any of the four points is

$$\begin{aligned} N_0 &= b - N_1 + N_2 - N_3 + N_4 \\ &= 22 - \binom{4}{1}.10 + \binom{4}{2}.4 - N_3 + N_4 \\ &= 6 - N_3 + N_4 \ . \end{aligned}$$

Suppose H contains a block of type D and call this block X. Then X intersects two other blocks Y,Z (say) in four points as $n_4 = 2$. Taking the four intersecting points of X and Y as reference points then either three or four of these must occur in Z. Define $\alpha \geqslant 0$ to be the number of three-point intersections not already counted in X,Y and Z.

If Z contains four reference points then $N_4 = 3$, $N_3 = 12 + \alpha$ and therefore

$$N_0 = 6 - (12+\alpha) + 3 = -3 - \alpha \ .$$

If Z contains three reference points then

$$N_0 = -1 - \alpha \ .$$

But α, $N_0 \geqslant 0$, so both possibilities cannot occur. □

This completes the analysis for the 2-(11,5,4) design as designs containing blocks of the remaining four types are already known. The number of blocks of each of the four types will help to determine and catalogue the non-isomorphic designs.

Section III: The Relationship Between the Block Types of the 2-(11,5,4) and 3-(12,6,4) Designs

A 2-(11,5,4) design can be extended to a unique 3-(12,6,4) design by complementation, while taking a restriction on a point of the 3-design will produce a 2-design. Thus, there are useful connections between the block types of the 2-(11,5,4) designs and those of the 3-(12,6,4) designs. Suppose a 2-(11,5,4) design is to be extended by complementation to a 3-(12,6,4) design. Denote the block intersection numbers for the 3-design by $\bar{n}_i$. Adding a point to the 22 existing blocks results in one more point of intersection for each of these blocks in the new design. This gives a contribution to the $\bar{n}_i$'s of

$$\bar{p}_{i+1} = n_i , \quad i = 0,1,2,3,4,5 \ .$$

The remaining 22 blocks of the design obtained by taking the complement of these blocks with respect to the new point set, gives a further contribution to the $\bar{n}_i$'s of

$$\bar{q}_i = n_{5-i} \quad , \quad i = 0,1,2,3,4,5 \ .$$

Thus the block intersection numbers for the 3-design are obtained from those of the 2-design using these two expressions, or equivalently

$$\bar{p}_i + \bar{q}_i = \bar{n}_i = \begin{cases} n_5 & i = 0,6 \ . \\ n_{5-i} + n_{i-1} & i = 1,2,3,4,5 \ . \end{cases}$$

i.e. the sets of the $\bar{n}_i$ for a self-complementary 3-(12,6,4) design are palindromic.

This result shows that the A(1 0 15 5 0 1) and the C(0 2 15 3 1 1) type blocks of the 2-(11,5,4) design extend to AC(1 1 5 30 5 1 1) type blocks of the 3-(12,6,4) design. The B(1 0 9 24 9 0 1) and R(2 0 0 20 0 0 2) type blocks of the 3-design have been labelled to demonstrate a similar correspondence to the B and R type blocks of the 2-design. The process is seen to work in reverse in that if a restriction is taken on a point of an AC type block, then the correspondin block in the 2-design is either of type A or type C. A similar result holds for the B and R type blocks, i.e.

2-design		3-design
B(0 3 12 6 0 1)	$\xrightarrow{\text{extension by complementation}}$ $\xleftarrow[\text{of the block}]{\text{restriction on any point}}$	B(1 0 9 24 9 0 1)
R(0 0 20 0 0 2)	$\rightleftarrows$	R(2 0 0 40 0 0 2)

If the D(0 1 18 0 2 1) type block of a 2-(11,5,4) design were possible, the above argument shows it must extend to an E(1 2 1 36 1 2 1) type block of the 3-design. Thus the restriction on at least one point of an E type block will produce a smaller design containing a D type block. As the D type block is impossible an E type block can never occur.

The F(0 4 3 28 8 0 1) type block of the 3-design cannot be produced from the blocks of the 2-design by complementation because $\bar{n}_0 = 0$. Thus, either this block is impossible or another method of extension will produce it.

<u>Lemma:</u> For a 3-(12,6,4) design a block of type F(0 4 3 28 8 0 1) can never exist.

<u>Proof:</u> Suppose a 3-(12,6,4) design does have a block [1 2 3 4 5 6] of type F(0 4 3 28 8 0 1). Since [1 2 3 4 5 6] does not have five or six points in common with any other block, a restriction on any of its points can only yield blocks of types A(1 0 15 5 0 1) or B(0 3 12 6 0 1) in the resulting 2-(11,5,4) design.

If a block intersects [1 2 3 4 5 6] in just one point x then a restriction on x leads to an A type block, with $n_0 = 1$, in the 2-(11,5,4) design. Therefore the four blocks intersecting [1 2 3 4 5 6] in just one point, each do so in a different point. As a restriction on each of the four points chosen for the single-point intersection blocks will produce an A type block in the 2-(11,5,4) design, none of these points can occur in a block that intersects [1 2 3 4 5 6] in just two points. Therefore the three two-point intersection blocks all contain the same pair. These parts of the skeleton for an F type block [1 2 3 4 5 6] would therefore have a structure equivalent to:

[1 • • • • •]		[5 6 • • • •]	
[2 • • • • •]	$(\bar{n}_1 = 4)$	[5 6 • • • •]	$(\bar{n}_2 = 3)$
[3 • • • • •]		[5 6 • • • •]	
[4 • • • • •]			

where $\cdot \not\in \{1,2,3,4,5,6\}$.

Now consider adding a new point x to the skeleton of an F type block and let a_i be the number of times x occurs in blocks containing exactly i of $\{1,2,3,4,5,6\}$. Obviously for the F type block $a_i \geqslant 0$, $a_0 = a_5 = a_6 = 0$, $a_1 \leqslant 4$, $a_2 \leqslant 3$, $a_3 \leqslant 28$ and $a_4 \leqslant 8$.

For single-point, pair and triple balance for x we have respectively that:

$$a_1 + a_2 + a_3 + a_4 = \lambda_1 = 22 \quad ,$$

$$a_1 + 2a_2 + 3a_3 + 4a_4 = \lambda_2 \binom{6}{1} = 60 \quad ,$$

and

$$a_2 + 3a_3 + 6a_4 = \lambda_3 \binom{6}{2} = 60 \quad .$$

The general solution of these equations in terms of a_1 is

$$(a_1,\ a_2,\ a_3,\ a_4)^T = (a_1,\ 12-3a_1,\ 4+3a_1,\ 6-a_1)^T.$$

The previously stated conditions on the a_i's reduce this infinite set to just the two legitimate possibilities,

$$(a_1,\ a_2,\ a_3,\ a_4)^T = (3,\ 3,\ 13,\ 3) \quad ,$$

$$\text{or} \quad = (4,\ 0,\ 16,\ 3) \quad .$$

From these, either x occurs in all three two-point intersection (a_2) blocks or it fails to occur in any of them. As it was shown earlier that these three blocks contain the same pair, any attempt to balance the skeleton results in the illegal occurrence of <u>three</u> identical blocks. □

The three remaining block types, AC, B and R, which can occur in the 3-(12,6,4) design all have $n_0 = n_6$, i.e. each block has its complement in the design and hence the designs must be self-complementary. That is any 2-(11,5,4) design can be extended to a 3-(12,6,4) design in just one way, by complementation: while it has been shown by Breach [1] that there are 2-(9,4,3) designs which may be extended to 3-(10,5,3) designs other than by complementation.

CHAPTER 2

THE REDUCIBLE 2-(11,5,4) AND 3-(12,6,4) DESIGNS

To provide some background, this chapter summarizes the research undertaken by the author on the *reducible* 3-(12,6,4) and 2-(11,5,4) designs. A more detailed account can be obtained from [6]. For completeness, a section detailing the development of the reducible designs containing repeated blocks has been added. By allowing repeated blocks, the number of non-isomorphic reducible 2-(11,5,4) designs was increased from 58 to 95. The corresponding increase in the number of non-isomorphic reducible 3-(12,6,4) designs was 14, giving 26 reducible 3-designs in all.

In each case, the approach used was to develop all non-isomorphic reducible 2-(11,5,4) designs and then apply the result that;

every reducible 2-(11,5,4) design can be extended to a unique self-complementary reducible 3-(12,6,4) design;

to obtain the non-isomorphic 3-designs. It is also important to remember that in the following three sections, the repeated block, type R(0 0 20 0 0 2) has been disallowed.

Section I: The Reducible 2-(11,5,4) Designs With Two or More B Type Blocks.

The general method used to develop the reducible 2-(11,5,4) designs was to append a variable 2-(11,5,2) design labelled $\underset{\sim}{D}^*$, to a fixed 2-(11,5,2) design labelled $\underset{\sim}{D}$. The well-known 2-(11,5,2) design which is unique, is symmetric, i.e. $b = v$, and is also a Hadamard design. As it is unique up to isomorphism, let $\underset{\sim}{D}$ be represented by the design in its usual cyclic form, i.e. the blocks developed from [1 3 4 5 9] under the point transformation $x \to x + 1 \pmod{11}$. $\underset{\sim}{D}^*$ is thus some isomorphic copy of this design.

Now although Aut $\underset{\sim}{D}$ and Aut $\underset{\sim}{D}^*$ as abstract groups are isomorphic, their representations as permutation groups may or may not have elements in common, i.e. a permutation of 0,1,..., 10 fixing $\underset{\sim}{D}$ need not fix $\underset{\sim}{D}^*$. Such a permutation may however fix patches of $\underset{\sim}{D}^*$. Thus the elements of Aut $\underset{\sim}{D}$ will be used to keep the number of equivalence classes of partially completed skeletons for $\underset{\sim}{D}^*$ to a minimum.

Suppose that the first block, [1 3 4 5 9], of $\underset{\sim}{D}$ in its standard form, is a B(0 3 12 6 0 1) type block of a 2-(11,5,4) design. As this block intersects all other blocks of D in exactly two points, it must intersect three blocks of $\underset{\sim}{D}^*$ in one point, two blocks in two points, and six blocks in three points. The blocks of $\underset{\sim}{D}^*$ must intersect each other in exactly two points. Under these requirements the automorphisms of $\underset{\sim}{D}$ force the different point positionings into one equivalence class. With a dot denoting a member of the set {0,2,6,7,8,10}, the point positioning chosen to represent this equivalence class is

		$\underset{\sim}{D}$					$\underset{\sim}{D}^*$			
1	3	4	5	9	1	•	•	•	•	(i)
2	4	5	6	10	4	•	•	•	•	(ii)
3	5	6	7	0	5	•	•	•	•	(iii)
4	6	7	8	1	3	9	•	•	•	(iv)
5	7	8	9	2	3	9	•	•	•	(v)
6	8	9	10	3	1	4	3	•	•	
7	9	10	0	4	4	5	3	•	•	
8	10	0	1	5	5	1	3	•	•	
9	0	1	2	6	1	4	9	•	•	
10	1	2	3	7	4	5	9	•	•	
0	2	3	4	8	5	1	9	•	•	.

The elements of Aut $\underset{\sim}{D}$ which leave this skeleton fixed are the identity and

$$\alpha = (1)(2)(8)(3\ 9)(4\ 5)(6\ 10)(0\ 7),$$

$$\beta = (0)(4)(6)(3\ 9)(1\ 5)(2\ 7)(8\ 10),$$

$$\gamma = (3)(9)(1\ 5\ 4)(6\ 8\ 10)(2\ 7\ 0),$$

$$\delta = (3)(9)(1\ 4\ 5)(6\ 10\ 8)(2\ 0\ 7),$$

$$\varepsilon = (7)(10)(5)(3\ 9)(1\ 4)(6\ 8)(0\ 2).$$

The two blocks (iv) and (v) already intersect in two points so each of the remaining six points 0,2,6,7,8,10 must occur exactly once in these two blocks. But $\underset{\sim}{D}$ contains the block [3 9 6 8 10], so to avoid repeated blocks, block (iv) must contain one symbol of the triple (6,8,10) and (v) must contain the other two. Then (v) must intersect [3 9 6 8 10] in four points and so must be a C type block. Therefore:

every reducible 2-(11,5,4) design containing B type blocks must also contain C type blocks.

The three blocks (i), (ii), (iii) mutually intersect in exactly two points so each of 0,2,6,7,8,10 must occur exactly twice in these three blocks. If $a,b,c \in \{0,2,7\}$ and $d,e,f \in \{6,8,10\}$ then blocks (i),..., (v) of $\underset{\sim}{D}^*$ have one of the two structures,

$$\begin{array}{ccccc} 1 & a & b & d & e \\ 4 & b & c & e & f \\ 5 & c & a & f & d \\ 3 & 9 & a & f & e \\ 3 & 9 & b & c & d \end{array} \qquad\qquad \begin{array}{ccccc} 1 & b & d & e & f \\ 4 & a & c & d & f \\ 5 & a & b & c & e \\ 3 & 9 & a & f & e \\ 3 & 9 & b & c & d \end{array}$$

<u>Rectangular Top</u>   <u>Triangular Top</u>

For the rectangular tops the 108 possible positionings of the numbers are reduced to 20 non-equivalent ways by the $\alpha,\beta,\gamma,\delta,\varepsilon$ elements of Aut $\underset{\sim}{D}$. The remaining six blocks of $\underset{\sim}{D}^*$ can then be completed in two ways, giving 40 reducible 2-(11,5,4) designs. A similar approach for the triangular tops produces a further 72 designs to be examined for isomorphs.

The assignation of the blocks of each design to the types A,B or C, was the first step in the elimination of isomorphs. A coarse sorting according to the number of blocks of each type puts the 112 designs into 16 equivalence classes. No two designs from different classes can be isomorphic. Within each class either a permutation mapping one design to another was found or it was shown that no such permutation exists. The 112 designs with B type blocks reduce to 53 non-isomorphic designs.

Section II: Reducible 2-(11,5,4) Designs With C Type Blocks But With No B Type Blocks.

The general approach used is similar to that of the preceding section. There is however an added advantage in that if any partially complete design contains a B type block, it may be discarded, as these have already been accounted for in Section I.

The first block [1 3 4 5 9] of $\underset{\sim}{D}$ is now required to be a block of type C(0 2 15 3 1 1) of a 2-(11,5,4) design. Fulfilling this condition and utilising the elements of Aut $\underset{\sim}{D}$, the two resulting equivalence classes for the skeletons of $\underset{\sim}{D}^*$ were represented by

Case 1:						Case 2:				
1	3	4	5	a		1	3	4	5	a
1	3	9	d	f		1	3	9	b	e
1	4	9	b	e		1	4	9	d	f
3	5	9	b	c		4	5	9	b	c
1	5	c	d	e		1	5	c	d	e
3	4	c	e	f		3	4	c	e	f
4	5	b	d	f		3	5	b	d	f
4	9	a	c	d		3	9	a	c	d
5	9	a	e	f		5	9	a	e	f
1	a	b	c	f		1	a	b	c	f
3	a	b	d	e		4	a	b	d	e.

The systematic selection of a,b,c,d,e,f from {0,2,6,7,8,10} for cases 1 and 2, subject to the non-occurrence of repeated or B type blocks, yields 28 acceptable designs. These belong to two equivalence classes in accordance with the number of A and C type blocks in each design.

Permutation techniques then gave a further reduction in numbers so that only 4 new non-isomorphic designs were created.

Section III: Reducible 2-(11,5,4) Designs With A Type Blocks Only.

If a reducible 2-(11,5,4) design is to contain A(1 0 15 5 0 1) type blocks only then the blocks must occur in disjoint pairs. There is only one such design. It is generated cyclically under the action $x \to x + 1$ (mod 11) from the two starter blocks [1 3 4 5 9] and [2 6 7 8 10] containing the quadratic and non-quadratic residues, mod 11, respectively.

Finally, the 58 non-isomorphic 2-(11,5,4) designs created were packaged into 3-(12,6,4) designs. As expected the resulting 12 non-isomorphic reducible 3-(12,6,4) designs neatly contained the 58 2-designs, with no new designs being discovered. A copy of each of these 3-(12,6,4) designs appears in Part II, Section I.

This completes the review of the work done by Thompson.

Section IV: Reducible 2-(11,5,4) Designs With Repeated Blocks.

This section will complete the census of reducible 2-(11,5,4) and 3-(12,6,4) designs, by developing all such designs containing repeated blocks. An approach analogous to that of Sections I and II will be used to create all the required 2-designs. Once isomorphs are removed from these, the 3-designs are readily obtained through the extension process.

For computational purposes it is convenient to alter the form of the standard 2-(11,5,2) design $\underset{\sim}{D}$, by interchanging the symbols 2 and 9, and substituting the symbol 11 for the symbol 0. The first block [1 2 3 4 5] of $\underset{\sim}{D}$ is now required to be a block of type R(0 0 20 0 0 2) of a 2-(11,5,4) design. The appended variable 2-(11,5,2) design $\underset{\sim}{D}^*$ must therefore contain the block [1 2 3 4 5] and its remaining blocks must intersect this in exactly two points. The properties of the 2-(11,5,2) design dictate that $\underset{\sim}{D}^*$ and hence the skeleton, can have only the one structure,

1	2	3	4	5		1	2	3	4	5	
1	2	6	9	11		1	2	•	•	•	(i)
1	3	7	9	10		1	3	•	•	•	
1	4	6	7	8		1	4	•	•	•	
1	5	8	10	11		1	5	•	•	•	
2	3	6	8	10		2	3	•	•	•	
2	4	7	10	11		2	4	•	•	•	
2	5	7	8	9		2	5	•	•	•	
3	4	8	9	11		3	4	•	•	•	
3	5	6	7	11		3	5	•	•	•	
4	5	6	9	10		4	5	•	•	•	

$\underset{\sim}{D}$ $\qquad\qquad$ $\underset{\sim}{D}^*$

The dots represent the points 6,7,8,9,10,11 and hence each of the remaining blocks can be completed with one of $\binom{6}{3} = 20$ possible triples. Of primary interest is the triple to complete block (i) of $\underset{\sim}{D}^*$. The elements of Aut $\underset{\sim}{D}$ which fix this skeleton and also block (i) are the identity and,

$$\alpha_1 = (1)(2)(3\ 4\ 5)(6\ 11\ 9)(7\ 8\ 10),$$

$$\alpha_2 = (1)(2)(3\ 5\ 4)(6\ 9\ 11)(7\ 10\ 8),$$

$$\alpha_3 = (1\ 2)(3)(10)(11)(4\ 5)(6\ 9)(7\ 8),$$

$$\alpha_4 = (1\ 2)(5)(6)(8)(3\ 4)(9\ 11)(7\ 10),$$

$$\alpha_5 = (1\ 2)(4)(7)(9)(3\ 5)(6\ 11)(8\ 10).$$

Under these automorphisms the 20 triples available to complete block (i) form six equivalence classes. The triples to represent each class say D1,..., D6 are,

D1 = 6 7 8, $\quad$ D2 = 6 7 9, $\quad$ D3 = 6 7 10,

D4 = 6 7 11, $\quad$ D5 = 6 9 11, $\quad$ D6 = 7 8 10.

With the top two blocks of $\underset{\sim}{D}^*$ now complete, some other elements of Aut $\underset{\sim}{D}$ appropriate to some classes are,

$$\beta_1 = (1)(2)(7)(3\ 6)(4\ 9)(5\ 11)(8\ 10),$$

$$\beta_2 = (1)(2)(8)(3\ 9)(4\ 11)(5\ 6)(7\ 10),$$

$$\beta_3 = (1)(2)(10)(3\ 11)(4\ 6)(7\ 8)(5\ 9),$$

$$\beta_4 = (1\ 2)(10\ 8\ 7)(9\ 3\ 6\ 4\ 11\ 5),$$

$$\beta_5 = (1\ 2)(8\ 10\ 7)(11\ 4\ 6\ 3\ 9\ 5),$$

$$\beta_6 = (7)(8)(10)(1\ 2)(3\ 11)(4\ 9)(5\ 6).$$

As these fix the top two blocks and not the whole skeleton, their most profitable use is in eliminating isomorphs once the designs have been created. The α_i and β_i permutations are of no use when considering classes D1 and D2 but all are relevant to D5 and D6, and α_4 and α_5 fix D3 and D4 respectively.

The problem has now been reduced to the systematic selection of the remaining eight triples to complete each of the six skeletons. This can be done manually or treated as an exercise in computer programming. The second means was used and the resulting program and its brief explanation appear at the conclusion of this section. An outline of the procedure used is as follows;

Step 0: Initialize all variables, control variables and arrays for the particular case being considered.

Step 1: Choose the next triple for the next incomplete block. If all triples have been tried or none give the correct number of pairs, go to *Step 5*.

Step 2: Update the pair incidence array for the new triple. If all blocks are complete go to *Step 3* otherwise go to *Step 1*.

Step 3: Increase design count by one, calculate and store design and its block types.

Step 4: Test newly obtained design against those previously obtained under all the applicable α_i and β_i permutations. If design is isomorphic, reduce design count by one.

Step 5: Update the pair incidence array and control variables to effectively remove the most recently acquired triple. If all possible selections of triples have not been tried, go to *Step 1*.

Step 6: Output all designs with their block types.

Step 7: STOP.

As each case was enumerated, permutation techniques were used to eliminate any isomorphs still remaining. Many such isomorphs existed, since permutations interchanging the blocks of $\underset{\sim}{D}$ with those of $\underset{\sim}{D}^*$ had not been considered. From the six classes, there are exactly 37 non-isomorphic reducible 2-(11,5,4) designs with repeated blocks. A breakdown of the results for the six classes is as follows,

CLASS	# OF DESIGNS PRODUCED	# OF NON-ISOMORPHIC DESIGNS ADDED
D1	72	25
D2	72	8
D3	40	1
D4	40	1
D5	16	1
D6	22	1
TOTAL	262	37

The designs that remain, identified by their computer assigned label and classified in accordance with the number of blocks of each type that they contain, are

# OF BLOCKS OF EACH TYPE (A B C R)	# OF DESIGNS	DESIGN COMPUTER ASSIGNED LABEL
0 0 0 22	1	D5 16
12 0 0 10	1	D3 4
0 0 12 10	1	D1 26
4 0 12 6	1	D1 2
0 0 16 6	1	D1 29
12 0 6 4	1	D1 22
4 0 14 4	2	D1 1 D1 27
0 0 18 4	4	D1 4 D1 10 D1 9
		D2 43

12 2 6 2	1	D2 38
10 0 10 2	1	D2 21
8 4 8 2	1	D2 68
8 0 12 2	1	D1 20
6 2 12 2	1	D1 23
4 8 8 2	1	D1 68
4 4 12 2	1	D1 3
4 2 14 2	1	D1 8
4 0 16 2	3	D2 19 D1 51 D1 19
2 2 16 2	1	D1 5
2 0 18 2	1	D1 7
0 4 16 2	2	D1 44 D1 16
0 8 12 2	2	D2 16 D4 28
0 20 0 2	1	D6 18
0 2 18 2	3	D2 10 D2 42 D1 11
0 0 20 2	4	D1 15 D1 43 D1 6
		D1 41

These 37 2-designs were extended to 3-designs. The elimination of isomorphs then revealed exactly 14 non-isomorphic reducible 3-(12,6,4) designs containing repeated blocks. These 14 3-designs' point orbits were subsequently determined and as expected they reproduced exactly the 37 non-isomorphic 2-designs, with no new designs being discovered. A list of these 3-designs and their associated point orbits and automorphism groups can be found in Part II, Section I.

```
      DIMENSION INC(20,3),IPAIR(11,11),IDES(22,5),I(9,2),NN(8),LOOP(9)
      COMMON/BLOCK1/ISTORE(200,23,6),IPERM(50,11)
      OPEN(12,FILE='DATA')
      OPEN(16,FILE='SIMPLE')
      DO 1 J1=1,11
      DO 2 J2=1,11
 2    IPAIR(J1,J2)=0
 1    CONTINUE
      READ(12,100)A
```

```
      DO 7 J=1,13
    7 READ(12,*)(IDES(J,J1),J1=1,5)
      READ(12,*)(I(J,1),J=1,9)
      READ(12,*)(I(J,2),J=1,9)
      DO 3 I1=1,20
    3 READ(12,*)(INC(I1,J),J=1,3)
      READ(12,*)NPERM                                          Step 0
      IF(NPERM.EQ.0)GO TO 14
      DO 8 J=1,NPERM
    8 READ(12,*)(IPERM(J,J1),J1=1,11)
C
   14 DO 4 I4=1,13
      DO 5 I5=1,4
      DO 6 I6=I5+1,5
    6 IPAIR(IDES(I4,I5),IDES(I4,I6))=IPAIR(IDES(I4,I5),IDES(I4,I6))+1
    5 CONTINUE
    4 CONTINUE
C
      ICOUNT=0
      NUM=1
      LOOP(1)=1
    9 DO 11 I1=LOOP(NUM),20
      DO 12 I2=1,3
      IF(IPAIR(I(NUM,1),INC(I1,I2)).EQ.4)GO TO 11
   12 IF(IPAIR(I(NUM,2),INC(I1,I2)).EQ.4)GO TO 11              Step 1
      IF(IPAIR(INC(I1,1),INC(I1,2)).EQ.4)GO TO 11
      IF(IPAIR(INC(I1,1),INC(I1,3)).EQ.4)GO TO 11
      IF(IPAIR(INC(I1,2),INC(I1,3)).EQ.4)GO TO 11
      DO 13 I3=1,3
      IPAIR(I(NUM,1),INC(I1,I3))=IPAIR(I(NUM,1),INC(I1,I3))+1
   13 IPAIR(I(NUM,2),INC(I1,I3))=IPAIR(I(NUM,2),INC(I1,I3))+1
      IPAIR(INC(I1,1),INC(I1,2))=IPAIR(INC(I1,1),INC(I1,2))+1
      IPAIR(INC(I1,1),INC(I1,3))=IPAIR(INC(I1,1),INC(I1,3))+1
      IPAIR(INC(I1,2),INC(I1,3))=IPAIR(INC(I1,2),INC(I1,3))+1  Step 2
      LOOP(NUM)=I1
C
      NUM=NUM+1
      LOOP(NUM)=1
      IF(NUM.NE.10)GO TO 9
      ICOUNT=ICOUNT+1
      DO 153 J3=1,13
      DO 154 J4=1,5
  154 ISTORE(ICOUNT,J3,J4)=IDES(J3,J4)
  153 CONTINUE
      DO 155 J5=14,22
      ISTORE(ICOUNT,J5,1)=I(J5-13,1)
      ISTORE(ICOUNT,J5,2)=I(J5-13,2)
      DO 156 J6=3,5
  156 ISTORE(ICOUNT,J5,J6)=INC(LOOP(J5-13),J6-2)
  155 CONTINUE
C
      DO 31 J=1,4
   31 ISTORE(ICOUNT,23,J)=0
      DO 32 KX=1,8
   32 NN(KX)=0
      DO 48 LL=1,22
      DO 41 II=1,22
      N=0
      DO 42 J=1,5
      DO 43 K=1,5
      IF(ISTORE(ICOUNT,LL,K).EQ.ISTORE(ICOUNT,II,J))N=N+1      Step 3
   43 CONTINUE
   42 CONTINUE
      DO 44 L=1,6
      JJ=L-1
```

```
      IF(N.EQ.JJ)GO TO 41
 44   CONTINUE
 41   NN(L)=NN(L)+1
      IF(NN(1).EQ.1.AND.NN(4).EQ.5)ISTORE(ICOUNT,LL,6)=1
      IF(NN(2).EQ.3.AND.NN(4).EQ.6)ISTORE(ICOUNT,LL,6)=2
      IF(NN(2).EQ.2.AND.NN(5).EQ.1)ISTORE(ICOUNT,LL,6)=3
      IF(NN(3).EQ.20.AND.NN(6).EQ.2)ISTORE(ICOUNT,LL,6)=4
      ISTORE(ICOUNT,23,ISTORE(ICOUNT,LL,6))=
     *ISTORE(ICOUNT,23,ISTORE(ICOUNT,LL,6))+1
      DO 49 MM=1,6
 49   NN(MM)=0
 48   CONTINUE
C
      IF(ICOUNT.EQ.1)GO TO 168
      IF(NPERM.EQ.0)GO TO 168
      IDUM=ICOUNT+1
      DO 158 J8=1,NPERM
      DO 159 J9=14,22
      DO 160 J0=1,5
 160  ISTORE(IDUM,J9,J0)=IPERM(J8,ISTORE(ICOUNT,J9,J0))
 159  CONTINUE
      DO 161 J1=1,ICOUNT-1
      DO 162 J2=1,4
 162  IF(ISTORE(J1,23,J2).NE.ISTORE(ICOUNT,23,J2))GO TO 161
      DO 163 J3=14,22
      DO 164 J4=14,22
      IF(ISTORE(ICOUNT,J3,6).NE.ISTORE(J1,J4,6))GO TO 164
      DO 165 J5=1,5
      DO 166 J6=1,5
 166  IF(ISTORE(IDUM,J3,J5).EQ.ISTORE(J1,J4,J6))GO TO 165
      GO TO 164
 165  CONTINUE
      GO TO 163
 164  CONTINUE
      GO TO 161
 163  CONTINUE
      ICOUNT=ICOUNT-1
      GO TO 201
 161  CONTINUE
 158  CONTINUE
 168  WRITE(1,1006)(ISTORE(ICOUNT,14,J),J=1,6)
C
 11   CONTINUE
 201  NUM=NUM-1
      IF(NUM.EQ.0)GO TO 202
      DO 18 I18=1,3
      IPAIR(I(NUM,1),INC(LOOP(NUM),I18))=
     *IPAIR(I(NUM,1),INC(LOOP(NUM),I18))-1
 18   IPAIR(I(NUM,2),INC(LOOP(NUM),I18))=
     *IPAIR(I(NUM,2),INC(LOOP(NUM),I18))-1
      IPAIR(INC(LOOP(NUM),1),INC(LOOP(NUM),2))=
     *IPAIR(INC(LOOP(NUM),1),INC(LOOP(NUM),2))-1
      IPAIR(INC(LOOP(NUM),1),INC(LOOP(NUM),3))=
     *IPAIR(INC(LOOP(NUM),1),INC(LOOP(NUM),3))-1
      IPAIR(INC(LOOP(NUM),2),INC(LOOP(NUM),3))=
     *IPAIR(INC(LOOP(NUM),2),INC(LOOP(NUM),3))-1
      LOOP(NUM)=LOOP(NUM)+1
      GO TO 9
C
 202  DO 301 J1=1,ICOUNT
      WRITE(16,1004) A,J1
      WRITE(16,1005)(ISTORE(J1,23,J2),J2=1,4)
      DO 303 J3=1,22
 303  WRITE(16,1006)(ISTORE(J1,J3,J4),J4=1,6)
 301  CONTINUE
C
```

Step 4

Step 5

Step 6

```
  100 FORMAT(A4)
 1004 FORMAT(1X,A3,I6)
 1005 FORMAT(1X,4I3)                                      Step 7
 1006 FORMAT(1X,5I3,I6)
      CLOSE(12)
      CLOSE(15)
      CLOSE(16)
      STOP
      END
```

The correlation between the program and the previously given procedure is demonstrated by the grouping of the code into the relevant sections. Input information is read from the file 'DATA' and any resulting designs output to the file 'SIMPLE'. The purpose of some of the more important variables and arrays is as follows;

INC(,)	stores the 20 possible triples.
IPAIR(a,b)	contains the number of incidences of the pair (a,b).
IDES(,)	temporary storage for a newly created design.
I(,)	contains the 9 pairs of $\underset{\sim}{D}^*$.
LOOP(a)	contains the index of the triple completing block a.
ISTORE(, ,)	stores all the required designs.
IPERM(,)	stores all the relevant α_i and β_i permutations.
NUM	is the index of the current incomplete block being considered.
ICOUNT	the present number of designs.

THE DATA

Below is a copy of the data used to enumerate the designs for equivalence class D5. The added comments should make its format reasonably self-explanatory.

```
D5                      Equivalence class
1,2,3,4,5
1,2,6,9,11
1,3,7,9,10
1,4,6,7,8
1,5,8,10,11
2,3,6,8,10                 The 13 blocks of the 2-(11,5,4) design.
2,4,7,10,11                Note that points must be given in
2,5,7,8,9                  ascending numerical value.
3,4,8,9,11
3,5,6,7,11
```

```
4,5,6,9,10
1,2,3,4,5
1,2,6,9,11
```

```
1,1,1,2,2,2,3,3,4
3,4,5,3,4,5,4,5,5
```

Pairs for blocks of $\underset{\sim}{D}^*$

```
6,7,8
6,7,9
6,7,10
6,7,11
6,8,9
6,8,10
6,8,11
6,9,10
6,9,11
6,10,11
7,8,9
7,8,10
7,8,11
7,9,10
7,9,11
7,10,11
8,9,10
8,9,11
8,10,11
9,10,11
```

The 20 possible completing triples.
(Also in ascending numerical value).

```
11
```

= NPERM = # of relevant α,β permutations

```
1,2,4,5,3,11,8,10,6,7,9
1,2,5,3,4,9,10,7,11,8,6
2,1,3,5,4,9,8,7,6,10,11
2,1,4,3,5,6,10,8,11,7,9
2,1,5,4,3,11,7,10,9,8,6
1,2,6,9,11,3,7,10,4,8,5
1,2,9,11,6,5,10,8,3,7,4
1,2,11,6,9,4,8,7,5,10,3
2,1,6,11,9,4,10,7,3,8,5
2,1,9,6,11,3,8,10,5,7,4
2,1,11,9,6,5,7,8,4,10,3
```

α and β permutations for this case. To demonstrate the format used, the first permutation represents the point mappings,

$$\begin{array}{ccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 1 & 2 & 4 & 5 & 3 & 11 & 8 & 10 & 6 & 7 & 9 \end{array}$$

CHAPTER 3

THE 3-(12,6,4) AND 2-(11,5,4) DESIGNS WITHOUT REPEATED BLOCKS

If blocks of type R(2 0 0 20 0 0 2) are disallowed then the blocks of the 3-(12,6,4) design must be of type AC (1 1 5 30 5 1 1) or type B(1 0 9 24 9 0 1). Thus this chapter is composed of two main sections. In Section I all non-isomorphic 3-(12,6,4) designs containing at least one AC type block are created. The remaining section then documents the development of the non-isomorphic 3-(12,6,4) designs not containing any AC type blocks, i.e. those consisting entirely of B type blocks. The task is completed by determining the point orbits of the resulting 3-designs. The non-isomorphic 2-(11,5,4) designs without repeated blocks, are then available by taking the appropriate point restrictions.

Chapter 3: Section I

THE DESIGNS CONTAINING AT LEAST ONE AC TYPE BLOCK

Section I·1: The Skeleton for an AC Type Block of a 3-(12,6,4) Design

As $n_0 = 1$ and $n_5 = 1$ for an AC type block, such blocks occur in easily discernable, distinct quartets. Therefore, without any loss of generality, let a 3-(12,6,4) design contain the quartet of AC type blocks

[a b c d e f] [0 1 2 3 4 5]

[a b c d e 0] [f 1 2 3 4 5] .

As these blocks effectively partition the point set, it is convenient hereafter to represent a point from {a,b,c,d,e} and {1,2,3,4,5} by an '*' and a '•' respectively. Each of these blocks must intersect the remaining blocks of the design five times at a pair of points, thirty times at a triple of points, and five times at a quadruple of points.

The skeleton is balanced for 0 and f by the addition of ten blocks containing both points; ten with 0 only; ten with f only; and a further ten blocks devoid of 0 or f. If the blocks containing one of these points are to avoid an illegal five-point intersection with an AC type block, they must be completed with a pair of *'s and a triple of •'s, or vica-versa. In either case a four-point intersection with <u>one</u> of the AC type blocks results. Thus, all four-point intersections for each of the AC type blocks occurs in these blocks of the design. This then forces the remaining blocks of the design to contain an equal number of *'s and •'s.

These requirements and the self-complementary property produce the following basic structure for a 3-(12,6,4) design with a quartet of AC type blocks.

```
[    a    b    c  |  d    e  |  f  ]     [ 0    1    2  |  3    4    5  ]
[    1    2    3  |  4    5  |  f  ]     [ 0    a    b  |  c    d    e  ]
[ |  *    *    *  |  •    •  |  f  ]     [ 0 |  *    *  |  •    •    • | ]
[ | Complement    | five     |  f  ]     [ 0 | five     | Complement   | ]
[ | of letters    | pairs    |  f  ]     [ 0 | pairs    | of numbers   | ]
[ |  *    *    *  | from     |  f  ]     [ 0 | from     |  •    •    • | ]
[ |  *    *    *  | numbers  |  f  ]     [ 0 | letters  |  •    •    • | ]
                                                               Section A

[    0 |  *    *  |  •    •  |  f  ]     [ |  *    *    *  |  •    •    • | ]
[    0 |  *    *  |  •    •  |  f  ]     [ |  *    *    *  |  •    •    • | ]
[    0 |  *    *  |  •    •  |  f  ]     [ |  *    *    *  |  •    •    • | ]
[    0 | ten      | ten      |  f  ]     [ |  *    *    *  |  •    •    • | ]
[    0 | pairs    | pairs    |  f  ]     [ | Complement    | Complement   | ]
[    0 | from     | from     |  f  ]     [ | of letters    | of numbers   | ]
[    0 | letters  | numbers  |  f  ]     [ |  *    *    *  |  •    •    • | ]
[    0 |  *    *  |  •    •  |  f  ]     [ |  *    *    *  |  •    •    • | ]
[    0 |  *    *  |  •    •  |  f  ]     [ |  *    *    *  |  •    •    • | ]
[    0 |  *    *  |  •    •  |  f  ]     [ |  *    *    *  |  •    •    • | ]

[ |  *    *  |  •    •    •  |  f  ]     [ 0 |  *    *    *  |  •    •  | ]
[ | five     | Complement    |  f  ]     [ 0 | Complement     | five     | ]
[ | pairs    | of numbers    |  f  ]     [ 0 | of letters     | pairs    | ]
[ | from     |  •    •    •  |  f  ]     [ 0 |  *    *    *  | from     | ]
[ | letters  |  •    •    •  |  f  ]     [ 0 |  *    *    *  | numbers  | ]
                                                               Section B
```

<u>Section I·2: The Completion of Section A</u>

To keep the number of partially complete designs to a minimum, all legitimate non-equivalent ways of choosing the blocks of section A are to be determined. Some insight into the configuration of the * and • pairs is gained by further examination of the AC type blocks. In particular, a restriction on * of the AC type block [a b c d e f], must produce C(0 2 15 3 1 1) type blocks in the 2-(11,5,4) design. The five * pairs are the only source of single-point intersections, n_1, for this block. Therefore, legitimate restrictions can only be obtained if each point occurs exactly twice within these pairs. The two essentially different pair structures satisfying this condition are,

<u>A1</u>	<u>A2</u>
a b	a b
b c	a b
c d	c d
d e	c e
e a,	d e.

From the symmetry of section A, the five pairs of •'s can be similarly arranged, thus reducing the task to one of juxtaposing all possible pair sets. As there are many permutations fixing the top four blocks, only a small number of structurally different combinations are to be expected. If $\otimes$ represents the operation of aligning a particular * pair arrangement with that of a • pair arrangement, then the three cases to consider are, A1 $\otimes$ A1, A1 $\otimes$ A2 = A2 $\otimes$ A1 (by symmetry), and A2 $\otimes$ A2. By examining each possibility's relative block intersections and using permutation techniques these cases were reduced to the seven non-equivalent structures hereafter referred to as Patterns, PI, PII etc. given by:

<u>A1 $\otimes$ A1</u>

PI		PII		PIII		PIV	
1 2	a b	1 2	a b	1 2	a b	1 2	a b
2 3	b c	2 3	b c	2 3	b c	3 4	b c
3 4	c d	3 4	c d	4 5	c d	2 5	c d
4 5	d e	1 5	d e	1 5	d e	1 3	d e
1 5	e a	4 5	e a	3 4	e a	4 5	e a

A2 ⊗ A1				A2 ⊗ A2	
PV		PVa		PVI	
1 2	a b	1 2	a b	1 2	a b
3 4	a b	1 3	a b	3 4	a b
1 3	c d	4 5	c d	1 2	c d
2 5	c e	2 4	c e	3 5	c e
4 5	d e	3 5	d e	4 5	d e

A closer examination of section A constructed from PVa with emphasis on the triples containing the pair (a,b), shows that while this pair has occurred seven times within these fourteen blocks, the triple (a,b,1) has yet to occur. As $\lambda_3 = 4$ and $\lambda_2 = 10$ for the 3-(12,6,4) design, PVa cannot be correctly balanced. This indicates that the four points other than 0 and f adjacent to any repeated pair in the patterns must all be distinct, a requirement that helps reduce A2 ⊗ A2 to just one legitimate case. Thus there are just six non-equivalent ways to complete section A.

Section I.3: The Selection of Points for the *'s of Section B

Despite the different characteristics of each pattern, the analysis behind the completion of section B for each remains essentially the same. For this reason, only PI will be discussed in any detail. The fourteen blocks of section A for PI are:

PI (section A)

a b c d e f	0 1 2 3 4 5
1 2 3 4 5 f	0 a b c d e
c d e 1 2 f	0 a b 3 4 5
a d e 2 3 f	0 b c 1 4 5
a b e 3 4 f	0 c d 1 2 5
a b c 4 5 f	0 d e 1 2 3
b c d 1 5 f	0 a e 2 3 4

Hindsight has shown that the permutations which fix these blocks without interchanging *'s and •'s are the most useful. For PI ten such permutations are possible;

$$\alpha = (0)(f)(a\ b\ c\ d\ e)(1\ 2\ 3\ 4\ 5),$$

$$\beta = (0)(f)(a)(1)(b\ e)(c\ d)(2\ 5)(3\ 4),$$

$$\alpha^2, \alpha^3, \alpha^4, \alpha\beta, \alpha^2\beta, \alpha^3\beta, \alpha^4\beta, \beta^2 .$$

An incidence count of (*,*,f) triples in PI gives,

<u>Triple</u>.	abf	acf	adf	aef	bcf	bdf	bef	cdf	cef	def
<u>No</u>.	3	2	2	3	3	2	2	3	2	3

As four occurrences of each triple are required, the remaining fifteen * pairs with f are,

a b	a e	b e
a c	b c	c d
a c	b d	c e
a d	b d	c e
a d	b e	d e .

Ten of these pairs are required for the [0 * * • • f] blocks while the remaining five appear in the bottom [* * • • • f] blocks of section B. As no (0,*,f) triples have been formed yet, the ten * pairs for the [0 * * • • f] blocks must contain each point exactly four times. Thus the remaining five * pairs for the [* * • • • f] blocks must contain each point exactly twice. The selection of five pairs from the above fifteen so that each point occurs exactly twice, produces the seventeen possibilities;

a b	a b	a b	a b	a b	a b	a b	a c	a c
a c	a c	a d	a d	a d	a e	a e	a c	a d
b d	b e	b c	b d	b e	b c	b d	b d	b c
c e	c d	c e	c e	c d	c d	c d	b e	b e
d e	d e	d e	c e	c e	d e	c e	d e	d e

a c	a c	a c	a c	a c	a d	a d	a d
a d	a d	a e	a e	a e	a d	a e	a e
b d	b e	b c	b d	b d	b c	b c	b c
b e	b e	b d	b d	b e	b e	b d	b e
c e	c d	d e	c e	c d	c e	c e	c d.

The ten α and β permutations put these into five equivalence classes T1,..., T5. A representative 'Type' from each class is given below. The structure of the ten pairs occurring with (0,f) clearly indicates their dissimilarities.

	T1	T2	T3	T4	T5
with (0,f)	a c	a c	a c	a c	a b
	a d	a d	a c	a c	a c
	a d	a d	a d	a d	a d
	a e	a e	a e	a d	a e
	b c	b c	b c	b d	b c
	b d	b d	b d	b d	b d
	b e	b d	b e	b e	b e
	b e	b e	b e	b e	c d
	c d	c e	c d	c e	c e
	c e	c e	d e	c e	d e
with f	a b	a b	a b	a b	a c
	a c	a c	a d	a e	a d
	b d	b e	b d	b c	b d
	c e	c d	c e	c d	b e
	d e	d e	c e	d e	c e.

Type	Repeated pairs with (0,f)	Repeated pairs with (f)	Permutations still fixing the skeleton
T1	2	0	$\alpha\beta$ = (a e)(b d)(1 5)(2 4)
T2	3	0	$\alpha^3\beta$ = (a b)(c e)(1 2)(3 5)
T3	2	1	$\alpha^3\beta$ = (a b)(c e)(1 2)(3 5)
T4	5	0	all ten
T5	0	0	all ten

Thus there are just five non-equivalent ways to complete the skeleton with a PI top, for the *'s.

Section I·4: The Selection of Points for the •'s of Section B.

The final balancing of the skeleton and hence the design, is accomplished by the selection and positioning of the points to occupy the • regions of section B. The approach taken to complete section B for the •'s is similar to that used for the *'s, and begins by noting the number of (0,•,•) triples in PI;

Triple.	012	013	014	015	023	024	025	034	035	045
No.	3	2	2	3	3	2	2	3	2	3 .

As each triple must occur four times, the remaining fifteen incomplete blocks containing 0 are to be filled by the • pairs,

1 2	1 5	2 5
1 3	2 3	3 4
1 3	2 4	3 5
1 4	2 4	3 5
1 4	2 5	4 5 .

These must be positioned so that each (0,*,•) triple occurs exactly four times throughout the design. The top fourteen blocks of PI already contain the (0,*,•) triples according to the scheme,

* • 0	1	2	3	4	5
a	0	1	2	2	1
b	1	0	1	2	2
c	2	1	0	1	2
d	2	2	1	0	1
e	1	2	2	1	0

whose entries give the number of appearances of the appropriate triple.

Two general methods were proposed for the final balancing of the design.

(a) In a similar fashion to the *'s the fifteen • pairs can be partitioned into seventeen ways of distributing them amongst the [0 * * • • f] and [0 * * * • •] blocks. This is done by ensuring each • point appears four times in the [0 * * • • f] blocks, so that the correct numbers of (0,•,f) triples result. For each combination of the • pair sets against the Types T1 to T5, every alignment of the constituent pairs must be tested for the correct occurrence of (0,*,•) triples. Self-complementarity ensures that the correct balance for these triples also results in the correct numbers of (*,•,f) triples, and hence the structure is a 3-(12,6,4) design.

(b) Alternatively, each of the Types T1,..., T5 is taken in turn, and the independent positionings of each of the • points to balance the (0,*,•) triples is noted. This process is assisted by the absence of any (0,•,f) triples. Thus, each • point must occur exactly four times within the [0 * * • • f] blocks and hence just twice within the blocks containing 0 only. Every possible combination of these positionings is then developed and those resulting in the occurrence of the fifteen • pairs are 3-(12,6,4) designs.

Note that in both these methods no attention is given to the balancing of the (*,*,•) and (*,•,•) triples and yet the structure is deemed to be a 3-(12,6,4) design. A closer inspection reveals that the methods actually produce the 22 blocks of a 2-(11,5,4) design with the point 0 added to each block. When the complements of these blocks are added the result is the extension by complementation of a 2-(11,5,4) design and this has been shown to be a 3-(12,6,4) design.

Method (b) appeared to be more efficient and easier to administer. Its manual implementation was used to successfully complete PI, but the process proved very tedious and somewhat susceptible to human error. Thus a computer program was constructed and the results obtained for PI were invaluable as testing grounds for the algorithm. An outline of the procedure adopted by this algorithm is discussed in the following sub-section.

At this stage it is convenient to remind the reader that the * and • pairs forming PII and PIII are the same as those constituting PI. As the variation is due entirely to the different alignment of these pair sets, only the incidences of (0,*,•) triples are altered. Thus, the above construction for section B is almost identical for these three cases. Further variation occurs as PII and PIII are fixed by only one non-trivial permutation. The effect of this is to increase the number of Types from five to eleven.

Section I·5: The Algorithm for the Completion of Section B

As many of the Types are fixed by one or more of the α or β permutations, a reduction in the amount of computational effort may be achieved by regarding the top [0 * * • • f] block of section B as fixed. A permutation which fixes the Type and a particular * pair can then be used to reduce the number of non-equivalent • pairs for this block. If no * pair is fixed or no such permutation(s) exist, the * pair can be arbitrarily chosen and all ten • pairs will need to be examined. The performance of the algorithm can be checked by using two equivalent • pairs and testing that the designs from one pair are isomorphic to those of the other. For any particular Type, the * pair and its associated •

pairs to complete the fixed block, are predetermined and given as input information.

While the above technique precludes the production of many isomorphic designs, it may not represent the exhaustive use of any or all the applicable permutations. Thus a secondary elimination process to fully utilise these permutations will need to be added.

The overall structure of the algorithm is represented by the following procedure:

Step 0 : Read all input information concerning this Pattern and Type. Includes the skeleton, the position of any repeated pairs, the • pairs to be considered, the (0,*,•) triple matrix X, etc.

Step 1 : Consider the next • pair for the fixed block. If all pairs have been processed go to *Step 12*.

Step 2 : Update the incidence matrix X for the (0,*,•) triples contained in the fixed block.

Step 3 : Find and store all 'Positionings' for each of the first four • points against the *'s, to give the correct number of (0,*,•) triples (the fifth • point is forced once the other four have been positioned). Including the fixed block each point occurs four times in the [0 * * • • f] blocks (these blocks have indices 1-10) and twice in the [0 * * • • •] blocks (indices 11-15).

Step 4 : Re-initialize X ready for the next • pair.

Step 5 : Systematically test all possible combinations of the Positionings for the four points until a set is found such that the resulting • pairs are correct, i.e. the (0,•,•) triples are balanced. If no/no more sets are found go to *Step 11*.

Step 6 : A design has been found so increase the design count by one. Convert, and store the Positionings as a 2×15 array where the columns from left to right contain the • pairs from top to bottom, which complete the design.

Step 7 : If the design has repeated blocks, go to *Step 10*.

Step 8 : If the presence of a repeated * pair means that this design is equivalent to one already produced, then go to *Step 10*.

Step 9 : Adjust the pointers so that the next combination of Positionings will be tested. Go to *Step 5*.

Step 10: Decrease the design count by one. Go to *Step 9*.

Step 11: Output the 2×15 arrays representing the created designs. Go to *Step 1*.

Step 12: Rewind the output file containing the 2×15 arrays. Read any relevant permutation data. If no permutations are given, STOP.

Step 13: Test all 2×15 arrays and eliminate those representing isomorphic designs under the given α_i and β_i permutation(s). Output the surviving 2×15 arrays to another file. STOP.

As the correlation between a step of the procedure and the section of code to perform it will readily assist in understanding the algorithm, it is convenient that a listing and brief discussion of the FORTRAN program appear next. The reader may wish to avoid this discussion for the present and proceed directly to the summary of the results beginning on page 45.

In understanding the following program it is important to note that during the creative stages of the algorithm, the points of any block or partially complete block invariably appear in ascending numerical value. Also, for computational purposes the *'s assume the values 1,..., 5 while the •'s are eventually assigned 6,..., 10. The point 0 is unchanged and f is represented as 11.

The program consists of three sections; **MAIN** and the two associated subroutines **IWORK( )** and **IFIT( )**. After reading the input information for the particular Pattern and Type, **MAIN** invokes the subroutine **IWORK( )** to develop all possible designs with the fixed block containing the first • pair. Any designs produced are output to the file IYES1 in the abbreviated form of a 2×15 array. Once all the • pairs have been processed, the file IYES1 containing all the designs for this case is rewound in preparation for the exhaustive elimination of isomorphs under the applicable α_i and β_i permutations. Where such permutation(s) exist, input information supplied by DATAREAD incites **MAIN** to complete this elimination and the resulting 2×15 arrays are output to 'SOLUT'. If there are no applicable permutations the program stops and the results are obtainable from IYES1.

It is important to note that while **IWORK( )** is producing and combining the • 'Positionings', these points are actually being represented by 1,..., 5, and only once a design has been found is there a conversion to 6,..., 10 via the mapping $x \to x+5$. The input data must therefore be appropriately tailored to allow for this facet of the program.

Lastly, a brief description of the overall function of the subroutine **IFIT** (IN1,ITEST,IN2,IPOINT) should help clarify the method used to execute *Step 5* of the procedure. The pair of variables (IN1,ITEST) reference the Positioning for the • point IN1, stored at the location IYES(1-6,IN1,ITEST). The (IN2,IPOINT) pair is similarly defined. The effect of calling this subroutine is to produce the first IN2 Positioning after IPOINT which, when taken in combination with that defined by (IN1,ITEST), gives the correct number of • pairs, i.e. it balances the (0,IN1,IN2) triples. When this is possible the result is passed back as a new index value for IPOINT, otherwise this variable is returned with a 0 value.

Because of the extensive branching involved in the algorithm used to complete *Step 5* of the procedure, a flow chart of this process has been included. In this diagram each diamond represents a call of the subroutine **IFIT( )** to search for a compatible Positioning for the second point against the specified Positioning of the first point. The three possible outcomes denoted by the letters S, D and N indicate the occurrence of the following events;

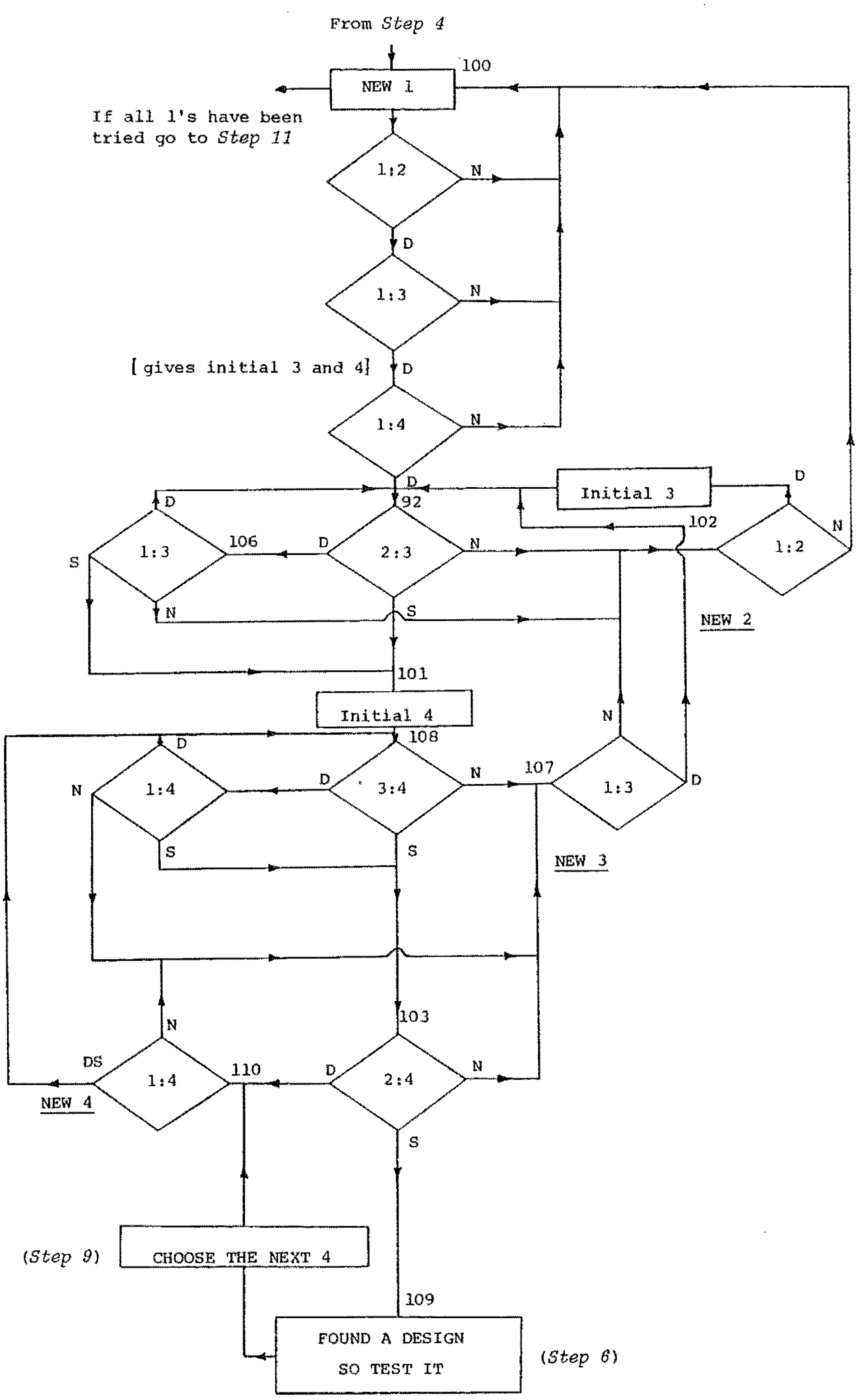
S = SAME : Represents success in that the currently held index for the variable Positioning is compatible.

D = DIFFERENT: The currently held index for the variable Positioning is incorrect and must be updated to the next possible value returned through IPOINT.

N = NONE : IPOINT = 0 so including the currently held index, none of the remaining variable Positionings are compatible.

If S fails to occur as an option, it is because the currently held index for the variable Positioning is known to be inappropriate and the next possibility is to be found. Note that this flow chart is incomplete as the steps updating control and storage variables have been ignored. This was a deliberate move so that the mechanism of solution is presented as simply and comprehensibly as possible. It is implicitly assumed that the correct updates have been made following each decision.

Flowchart for *Step 5*



Before presenting the program, the purposes of some of its more important variables and arrays are given below.

ISPEC( )	Contains the • pairs for the fixed block.
X(a,b)	Contains the number of (0,a,b) triples where a ∈ *'s and b ∈ •'s.
IPERM1(,) IPERM2(,)	Contains α_i and β_i permutation information.
IPAIRS(1-15,1-2,a)	Contains the ath 15 • pairs to complete the skeleton.
IYES(1-6,a,b)	Contains the bth Positioning for the • point a.
ICON(a,1-2)	Contains the ath * pair for the [0 * * • • f] blocks.
ICON2(a,1-3)	Contains the ath * triple for the [0 * * * • •] blocks.
IALLOW(ab)	Contains the number of (0,a,b) triples required for balance where a,b ∈ •'s.
IREP(1-5:6)	IREP(6) stores the number of repeated * pairs. As the program requires repeated * pairs to occur consecutively, IREP(1-5) gives the indices of the first * pair of each set.
ICOUNT(a)	Counts the number of Positionings for the • point a.
JTIMES	Assumes the values 1 to 4 each representing a • point (for the purpose of developing the Positionings).
IVALUE(a)	Stores the index for the Positioning of • point a currently being examined.
NUMBER	Current number of designs.

As the program requires forty data files and these contribute little to the understanding of the text they have been listed and explained in Appendix 1.

MAIN

```
INTEGER*4 ISPEC(10),X(5,5),IPERM1(15,30),IPERM2(10,30)
COMMON/BLOCK1/IPAIRS(15,2,800),IYES(6,4,200),KTEST1(5),KTEST2(5)
COMMON/BLOCK2/ICON(10,2),ICON2(5,3),IALLOW(45),ICOUNT(4),IREP(6)
COMMON/FILE/LP1,LP2,LP3,RD1,RD2,RD3
OPEN(LP1,FILE = 'IYES1')
OPEN(RD1,FILE = 'DATAREAD')
OPEN(LP2,FILE = 'SOLUT')
READ(RD1,*) X
```

Step 0

```
      READ(RD1,*) ISPEC
      READ(RD1,*) IALLOW(12),IALLOW(13),IALLOW(14),IALLOW(15),IALLOW(23)
      READ(RD1,*) IALLOW(24),IALLOW(25),IALLOW(34),IALLOW(35),IALLOW(45)
      READ(RD1,*) NOFSP
      READ(RD1,*) ICON
      READ(RD1,*) ICON2
      READ(RD1,*) IREP
      DO 100 ITOTAL=1,NOFSP                                          Step 1
         CALL IWORK(ISPEC,ITOTAL,X)
  100    CONTINUE
C
      REWIND LP1
      READ(RD1, *)NPERM                                              Step 12
      IF(NPERM.EQ.0)GO TO 600
      DO 1 I=1,NPERM
    1 READ(RD1,*)(IPERM1(J1,I),J1=1,15),(IPERM2(J2,I),J2=6,10)
      NUMB=0
    2 NUMB=NUMB+1
      READ(LP1,150,END=600)(IPAIRS(K,1,NUMB),K=1,15)
      READ(LP1,151)(IPAIRS(L,2,NUMB),L=1,15)
      IF(NUMB.EQ.1)GO TO 3
      IDUM=NUMB+1
      DO 4 N1=1,NPERM
      DO 5 N2=1,15
      IPAIRS(N2,1,IDUM)=IPERM2(IPAIRS(IPERM1(N2,N1),1,NUMB),N1)
      IPAIRS(N2,2,IDUM)=IPERM2(IPAIRS(IPERM1(N2,N1),2,NUMB),N1)
      IF(IPAIRS(N2,1,IDUM).LT.IPAIRS(N2,2,IDUM))GO TO 5
      ITEMP=IPAIRS(N2,2,IDUM)
      IPAIRS(N2,2,IDUM)=IPAIRS(N2,1,IDUM)
      IPAIRS(N2,1,IDUM)=ITEMP                                        Step 13
    5 CONTINUE
C
      DO 301 LPR=1,NUMB-1
      IF(IREP(6).EQ.0)GO TO 303
      DO 300 KPR=1,IREP(6)
      KTEST1(KPR)=IPAIRS(IREP(KPR),1,IDUM)*10+IPAIRS(IREP(KPR),2,
     *IDUM)
      KTEST2(KPR)=IPAIRS(IREP(KPR)+1,1,IDUM)*10+IPAIRS(IREP(KPR)+1,2,
     *IDUM)
  300 CONTINUE
      DO 302 MPR=1,IREP(6)
      IEQU=0
      KOLD1=IPAIRS(IREP(MPR),1,LPR)*10+IPAIRS(IREP(MPR),2,LPR)
      KOLD2=IPAIRS(IREP(MPR)+1,1,LPR)*10+IPAIRS(IREP(MPR)+1,2,LPR)
      IF(KTEST1(MPR).EQ.KOLD1.OR.KTEST1(MPR).EQ.KOLD2)IEQU=IEQU+1
      IF(KTEST2(MPR).EQ.KOLD1.OR.KTEST2(MPR).EQ.KOLD2)IEQU=IEQU+1
  302 IF(IEQU.NE.2)GO TO 301
  303 DO 304 NPR=1,15
      IF(IREP(6).EQ.0)GO TO 306
      DO 305 NPR1=1,IREP(6)
  305 IF(NPR.EQ.IREP(NPR1).OR.NPR.EQ.IREP(NPR1)+1)GO TO 304
  306 NPTEST=IPAIRS(NPR,1,IDUM)*10+IPAIRS(NPR,2,IDUM)
      NDUMT=IPAIRS(NPR,1,LPR)*10+IPAIRS(NPR,2,LPR)
      IF(NPTEST.NE.NDUMT)GO TO 301
  304 CONTINUE
      NUMB=NUMB-1
      GO TO 2
  301 CONTINUE
    4 CONTINUE
    3 WRITE(LP2,150)(IPAIRS(J1,1,NUMB),J1=1,15)
      WRITE(LP2,151)(IPAIRS(J2,2,NUMB),J2=1,15)
      GO TO 2
  150 FORMAT(1X,15(I4))
  151 FORMAT(1X,15(I4),/)
  152 FORMAT(1X,////////,I4)
C
```

```
600   CLOSE(LP1)
      CLOSE(LP2)
      CLOSE(RD1)
      STOP
      END
```

IWORK( )

```
      SUBROUTINE IWORK(ISPEC,ITOTAL,X)
      INTEGER*4 X(5,5),ISPEC(10),IVALUE(4)
      COMMON/BLOCK1/IPAIRS(15,2,800),IYES(6,4,200),KTEST1(5),KTEST2(5)
      COMMON/BLOCK2/ICON(10,2),ICON2(5,3),IALLOW(45),ICOUNT(4),IREP(6)
      COMMON/FILE/LP1,LP2,LP3,RD1,RD2,RD3
      III=ISPEC(ITOTAL)/10                                          Step 2
      JJJ=ISPEC(ITOTAL)-10*III
      X(ICON(1,1),III)=X(ICON(1,1),III)+1
      X(ICON(1,1),JJJ)=X(ICON(1,1),JJJ)+1
      X(ICON(1,2),III)=X(ICON(1,2),III)+1
      X(ICON(1,2),JJJ)=X(ICON(1,2),JJJ)+1
      DO 1000 JTIMES=1,4
      ICOUNT(JTIMES)=0
      IIIJJJ=0
      IF(III.EQ.JTIMES.OR.JJJ.EQ.JTIMES) IIIJJJ=1
      DO 1 I=2,7+IIIJJJ
        X(ICON(I,1),JTIMES)=X(ICON(I,1),JTIMES)+1
        X(ICON(I,2),JTIMES)=X(ICON(I,2),JTIMES)+1
        II=I+1
       DO 2 J=II,8+IIIJJJ
         IF(X(ICON(J,1),JTIMES).EQ.4.OR.X(ICON(J,2),JTIMES).EQ.4)
     *   GO TO 2
         X(ICON(J,1),JTIMES)=X(ICON(J,1),JTIMES)+1
         X(ICON(J,2),JTIMES)=X(ICON(J,2),JTIMES)+1
         JJ=J+1
        DO 3 K=JJ,9+IIIJJJ
          IF(X(ICON(K,1),JTIMES).EQ.4.OR.X(ICON(K,2),JTIMES).EQ.4)
     *    GO TO 3
          X(ICON(K,1),JTIMES)=X(ICON(K,1),JTIMES)+1
          X(ICON(K,2),JTIMES)=X(ICON(K,2),JTIMES)+1
          KK=K+1
          IF(IIIJJJ.EQ.1) GO TO 33
         DO 4 L=KK,10
           IF(X(ICON(L,1),JTIMES).EQ.4.OR.X(ICON(L,2),JTIMES).EQ.4)
     *     GO TO 4
           X(ICON(L,1),JTIMES)=X(ICON(L,1),JTIMES)+1
           X(ICON(L,2),JTIMES)=X(ICON(L,2),JTIMES)+1
 33       DO 5 M=1,4
            IF(X(ICON2(M,1),JTIMES).EQ.4.OR.X(ICON2(M,2),JTIMES)
     *      .EQ.4) GO TO 5
            IF(X(ICON2(M,3),JTIMES).EQ.4) GO TO 5
           DO 55 MI=1,3
 55          X(ICON2(M,MI),JTIMES)=X(ICON2(M,MI),JTIMES)+1
             MM=M+1
            DO 6 N=MM,5
              IF(X(ICON2(N,1),JTIMES).EQ.4.OR.X(ICON2(N,2),JTIMES)
     *        .EQ.4) GO TO 6
              IF(X(ICON2(N,3),JTIMES).EQ.4) GO TO 6
              IF(IIIJJJ.EQ.1) L=1                                   Step 3
              ICOUNT(JTIMES)=ICOUNT(JTIMES)+1
              IF(ICOUNT(JTIMES).GT.200)GO TO 87
             DO 7 IJK=1,15
               IF(IJK.NE.I) GO TO 8
               IYES(1,JTIMES,ICOUNT(JTIMES))=IJK
               GO TO 7
 8             IF(IJK.NE.J) GO TO 9
               IYES(2,JTIMES,ICOUNT(JTIMES))=IJK
```

```
                 GO TO 7
 9               IF(IJK.NE.K) GO TO 10
                 IYES(3,JTIMES,ICOUNT(JTIMES))=IJK
                 GO TO 7
10               IF(IJK.NE.L) GO TO 11
                 IYES(4,JTIMES,ICOUNT(JTIMES))=IJK
                 GO TO 7
11               IF(IJK-10.NE.M) GO TO 12
                 IYES(5,JTIMES,ICOUNT(JTIMES))=IJK-10
                 GO TO 7
12               IF(IJK-10.NE.N) GO TO 7
                 IYES(6,JTIMES,ICOUNT(JTIMES))=IJK-10
 7               CONTINUE
6                CONTINUE
                DO 66 NI=1,3
  66              X(ICON2(M,NI),JTIMES)=X(ICON2(M,NI),JTIMES)-1
   5              CONTINUE
                  IF(IIIJJJ.EQ.1)GO TO 15
                 X(ICON(L,1),JTIMES)=X(ICON(L,1),JTIMES)-1
                 X(ICON(L,2),JTIMES)=X(ICON(L,2),JTIMES)-1
   4             CONTINUE
  15            X(ICON(K,1),JTIMES)=X(ICON(K,1),JTIMES)-1
                X(ICON(K,2),JTIMES)=X(ICON(K,2),JTIMES)-1
   3            CONTINUE
              X(ICON(J,1),JTIMES)=X(ICON(J,1),JTIMES)-1
              X(ICON(J,2),JTIMES)=X(ICON(J,2),JTIMES)-1
   2          CONTINUE
           X(ICON(I,1),JTIMES)=X(ICON(I,1),JTIMES)-1
           X(ICON(I,2),JTIMES)=X(ICON(I,2),JTIMES)-1
   1       CONTINUE
 1000 CONTINUE
      X(ICON(1,1),III)=X(ICON(1,1),III)-1
      X(ICON(1,1),JJJ)=X(ICON(1,1),JJJ)-1                    Step 4
      X(ICON(1,2),III)=X(ICON(1,2),III)-1
      X(ICON(1,2),JJJ)=X(ICON(1,2),JJJ)-1
C
      NUMBER=0
      DO 100 I1S=1,ICOUNT(1)
      IN1 =1
      IVALUE(1)=I1S
      DO 90 INDEX=2,4
      IPOINT=0
      CALL IFIT(IN1,I1S,INDEX,IPOINT)
      IF(IPOINT.EQ.0)GO TO 100
      IVALUE(INDEX)=IPOINT
   90 CONTINUE
      INIV4=IVALUE(4)
      INIV3=IVALUE(3)
  92  IN1=2
      IN2=3
      IDUM=IVALUE(3)-1
      CALL IFIT(IN1,IVALUE(2),IN2,IDUM)
      IF(IDUM.EQ.IVALUE(3))GO TO 101
      IF(IDUM.EQ.0.)GO TO 102
      IVALUE(3)=IDUM
  106 IN1=1
      IDUM=IVALUE(3)-1
      CALL IFIT(IN1,I1S,IN2,IDUM)
      IF(IDUM.EQ.IVALUE(3))GO TO 101
      IF(IDUM.EQ.0)GO TO 102
      IVALUE(3)=IDUM
      GO TO 92
 102  IN1=1
      IN2=2
      CALL IFIT(IN1,I1S,IN2,IVALUE(2))
      IF(IVALUE(2).EQ.0)GO TO 100
      IVALUE(3)=INIV3
      GO TO 92
```

Step 5

```
 101   IVALUE(4)=INIV4
 108   IN1=3
       IN2=4
       IDUM=IVALUE(4)-1
       CALL IFIT(IN1,IVALUE(3),IN2,IDUM)
       IF(IDUM.EQ.IVALUE(4))GO TO 103
       IF(IDUM.EQ.0)GO TO 107
       IVALUE(4)=IDUM
       IN1=1
       IDUM=IVALUE(4)-1
       CALL IFIT(IN1,I1S,IN2,IDUM)
       IF(IDUM.EQ.IVALUE(4))GO TO 103
       IF(IDUM.EQ.0)GO TO 107
       IVALUE(4)=IDUM
       GO TO 108
 107   IN2=3
       IN1=1
       CALL IFIT(IN1,I1S,IN2,IVALUE(3))
       IF(IVALUE(3).EQ.0)GO TO 102
       GO TO 92
 103   IN1=2
       IN2=4
       IDUM=IVALUE(4)-1
       CALL IFIT(IN1,IVALUE(2),IN2,IDUM)
       IF(IDUM.EQ.IVALUE(4))GO TO 109
       IF(IDUM.EQ.0)GO TO 107
       IVALUE(4)=IDUM
  110  IN1=1
       IDUM=IVALUE(4)-1
       CALL IFIT(IN1,I1S,IN2,IDUM)
       IF(IDUM.EQ.0)GO TO 107
       IVALUE(4)=IDUM
       GO TO 108
C
109    NUMBER =NUMBER+1
       DO 180 JUM=1,15
       DO 181 JUN=1,2
 181   IPAIRS(JUM,JUN,NUMBER)=10
 180   CONTINUE
       IBOT5=0
       DO 200 M6=1,6
       IF(M6.GT.4)IBOT5=10
 200   IPAIRS(IBOT5+IYES(M6,1,IVALUE(1)),1,NUMBER)=6
       IBOT5=0
       DO 201 M7=1,6
       IF(M7.GT.4)IBOT5=10
       IF(IPAIRS(IBOT5+IYES(M7,2,IVALUE(2)),1,NUMBER).EQ.10)GO TO 202
       IPAIRS(IBOT5+IYES(M7,2,IVALUE(2)),2,NUMBER)=7
       GO TO 203
 202   IPAIRS(IBOT5+IYES(M7,2,IVALUE(2)),1,NUMBER)=7
 203   IBOT5=0
  201  CONTINUE
       DO 204 M8=1,6
       IF(M8.GT.4)IBOT5=10
       IF(IPAIRS(IBOT5+IYES(M8,3,IVALUE(3)),1,NUMBER).EQ.10)GO TO 205
       IF(IPAIRS(IBOT5+IYES(M8,3,IVALUE(3)),2,NUMBER).NE.10)GO TO 308
       IPAIRS(IBOT5+IYES(M8,3,IVALUE(3)),2,NUMBER)=8
       GO TO 206
 205   IPAIRS(IBOT5+IYES(M8,3,IVALUE(3)),1,NUMBER)=8
 206   IBOT5=0
 204   CONTINUE
       DO 207 M9=1,6
       IF(M9.GT.4)IBOT5=10
       IF(IPAIRS(IBOT5+IYES(M9,4,IVALUE(4)),1,NUMBER).EQ.10)GO TO 208
       IF(IPAIRS(IBOT5+IYES(M9,4,IVALUE(4)),2,NUMBER).NE.10)GO TO 308
       IPAIRS(IBOT5+IYES(M9,4,IVALUE(4)),2,NUMBER)=9
       GO TO 209
```

Step 6

```
 208   IPAIRS(IBOT5+IYES(M9,4,IVALUE(4)),1,NUMBER)=9
 209   IBOT5=0
 207   CONTINUE
C
       IF(IREP(6).EQ.0)GO TO 306
       DO 300 KPR=1,IREP(6)
       KTEST1(KPR)=IPAIRS(IREP(KPR),1,NUMBER)*10+IPAIRS(IREP(KPR),2,
      *NUMBER)
       KTEST2(KPR)=IPAIRS(IREP(KPR)+1,1,NUMBER)*10+IPAIRS(IREP(KPR)+1,2,
      *NUMBER)
  300  IF(KTEST1(KPR).EQ.KTEST2(KPR))GO TO 308
       IF(NUMBER.EQ.1)GO TO 306
       DO 301 LPR=1,NUMBER-1
       DO 302 MPR=1,IREP(6)
       IEQU=0
       KOLD1=IPAIRS(IREP(MPR),1,LPR)*10+IPAIRS(IREP(MPR),2,LPR)
       KOLD2=IPAIRS(IREP(MPR)+1,1,LPR)*10+IPAIRS(IREP(MPR)+1,2,LPR)
       IF(KTEST1(MPR).EQ.KOLD1.OR.KTEST1(MPR).EQ.KOLD2)IEQU=IEQU+1
       IF(KTEST2(MPR).EQ.KOLD1.OR.KTEST2(MPR).EQ.KOLD2)IEQU=IEQU+1
  302  IF(IEQU.NE.2)GO TO 301
       DO 304 NPR=1,15
       DO 305 NPR1=1,IREP(6)
  305  IF(NPR.EQ.IREP(NPR1).OR.NPR.EQ.IREP(NPR1)+1)GO TO 304
       NPTEST=IPAIRS(NPR,1,NUMBER)*10+IPAIRS(NPR,2,NUMBER)
       NDUMT=IPAIRS(NPR,1,LPR)*10+IPAIRS(NPR,2,LPR)
       IF(NPTEST.NE.NDUMT)GO TO 301
 304   CONTINUE
  308  NUMBER=NUMBER-1
       GO TO 306
  301  CONTINUE
C
  306  IVALUE(4)=IVALUE(4)+1
       GO TO 110
 100   CONTINUE
       DO 400 IJOUT=1,NUMBER
       WRITE(LP1,210)(IPAIRS(IJJ,1,IJOUT),IJJ=1,15)
       WRITE(LP1,210)(IPAIRS(IKK,2,IJOUT),IKK=1,15)
  400  WRITE(LP1,211)
  130  FORMAT(1X,4(I6))
 210   FORMAT(1X,15(I4))
 211   FORMAT(1X)
       RETURN
 87    WRITE(LP1,215)
 215   FORMAT(1X,'ARRAY DIMENSION LIMIT OVERRUN')
       STOP
       END
```

Step 7

Step 8

Step 10

Step 9

Step 11

IFIT( )

```
       SUBROUTINE IFIT(IN1,ITEST,IN2,IPOINT)
       COMMON/BLOCK1/IPAIRS(15,2,800),IYES(6,4,200),KTEST1(5),KTEST2(5)
       COMMON/BLOCK2/ICON(10,2),ICON2(5,3),IALLOW(45),ICOUNT(4),IREP(6)
       COMMON/FILE/LP1,LP2,LP3,RD1,RD2,RD3
       INDEX=IN1*10+IN2
       INTERS=0
       ISTART=IPOINT+1
       IF(ISTART.GT.ICOUNT(IN2))GO TO 7
       DO 1 I=ISTART,ICOUNT(IN2)
       DO 3 II=1,4
       DO 4 JJ=1,4
       IF(IYES(JJ,IN2,I).NE.IYES(II,IN1,ITEST))GO TO 4
       INTERS=INTERS+1
 4     CONTINUE
 3     CONTINUE
       DO 5 L=5,6
       IF(IYES(L,IN2,I).EQ.IYES(5,IN1,ITEST))INTERS=INTERS+1
    5  IF(IYES(L,IN2,I).EQ.IYES(6,IN1,ITEST))INTERS=INTERS+1
       IF(INTERS.NE.IALLOW(INDEX))GO TO 6
       IPOINT=I
```

```
      RETURN
6     INTERS=0
1     CONTINUE
    7 IPOINT=0
      RETURN
      END
```

Section I·6: The Results

As no permutations interchanging *'s and •'s have been used throughout the development of these designs, while those which fix these two point sets have been fully utilized, the designs are expected to have been created in isomorphic pairs. In particular, the symmetry exhibited by PI, PII, PIII, PIV, and PVI, requires that a table of the Types for the * pairs against the Types for the • pairs be commutative in each case. This provides a useful check on the program and input data, as a comparison between independent runs can be made. Some of the Types are difficult to distinguish between, so for the purpose of constructing these tables, they will be regrouped in accordance with the position of any repeated pairs. Each Type is re-classified using the ordered pair (a,b) where a is the number of repeated pairs occurring in the blocks containing (0,f) (for the •'s, columns 1 to 10 of the 2×15 output array), and b counts those in the remaining blocks of section B (for the •'s, columns 11 to 15 of the output array). From the results, the following tables verifying the correctness of the algorithm, were constructed.

PI

		•				
		(2,0)	(3,0)	(2,1)	(5,0)	(0,0)
*	(2,0) T1	6	1	1	0	1
	(3,0) T2	1	1	0	0	0
	(2,1) T3	1	0	4	0	3
	(5,0) T4	0	0	0	0	0
	(0,0) T5	1	0	3	0	24

PII

		•				
		(2,0)	(3,0)	(2,1)	(5,0)	(0,0)
*	(2,0) T1,T5,T7	46	12	16	0	21
	(3,0) T2,T3,T10	12	9	7	0	2
	(2,1) T4,T8,T11	16	7	25	0	27
	(5,0) T6	0	0	0	0	0
	(0,0) T9	21	2	27	0	151

PIII

		•				
		(2,0)	(3,0)	(2,1)	(5,0)	(0,0)
	(2,0) T1,T4,T10	79	33	40	0	40
	(3,0) T2,T6,T11	33	26	19	1	12
*	(2,1) T3,T8,T9	40	19	33	0	39
	(5,0) T5	0	1	0	0	1
	(0,0) T7	40	12	39	1	178

PIV

		•				
		(2,0)	(3,0)	(2,1)	(5,0)	(0,0)
	(2,0) T1	85	62	41	5	46
	(3,0) T2	62	42	27	4	24
*	(2,1) T3	41	27	14	3	15
	(5,0) T4	5	4	3	2	3
	(0,0) T5	46	24	15	3	46

PVI

		•	
		(2,0)	(3,1)
*	(2,0) T2,T4	76	25
	(3,1) T1,T3	25	14

The following table lists the number of designs produced by each of the forty data sets, and represents a pool of designs from which isomorphs are to be sieved.

Pattern	Subcases or Types											
	T1	T2	T3	T4	T5	T6	T7	T8	T9	T10	T11	Total
I	9	2	8	0	28	-	-	-	-	-	-	47
II	19	4	20	21	22	0	54	29	201	6	25	401
III	54	46	72	66	2	21	270	44	15	77	24	691
IV	239	159	100	17	134	-	-	-	-	-	-	649
V	25	127	60	162	-	-	-	-	-	-	-	374
VI	12	31	27	70	-	-	-	-	-	-	-	140
Total												2302

To reference any particular design, the notation P# - T# N# will be adopted where, P# refers to the Pattern, T# to the particular Type, and N# indicates the $\#^{th}$ 2×15 array produced by the program. Each of these designs was tested to ensure that its blocks complied with those of a 3-(12,6,4) design.

The possible occurrence of blocks of three different types in the 2-(11,5,4) designs, and their non-self-complementary nature, makes processing the 3-designs by a comparison of the 2-designs they contain, a useful means of sorting. As well as working with smaller designs, this approach has the advantage that the transitivity sets of the 3-designs are necessarily ascertained. To this end, a computer program was constructed to list each 3-design as it was developed; the block type being given beside each block. More importantly, it was also used to provide a similar listing of the twelve 2-(11,5,4) designs embedded as point restrictions.

This information was used to classify and eliminate isomorphs amongst the 3-designs. The criterion for discarding any particular design was the existence of a permutation relating it to a previously produced design. In this way the 2302 3-designs were reduced to 392 non-isomorphic 3-(12,6,4) designs, whose point restrictions contain 3509 non-isomorphic 2-(11,5,4) designs.

As the comparison of these designs was done manually, this method is susceptible to some degree of human error. While no designs have been lost, the often high degree of similarity between various designs and the difficulties involved in obtaining the point orbits (largest transitivity sets), prompted the introduction of a second, independently based, classification system.

Thus, let q_i be the number of quadruples appearing exactly i times throughout a 3-(12,6,4) design. Then counting occurrences of quadruples,

$$q_1 + 2q_2 + 3q_3 + 4q_4 = b \cdot \binom{6}{2} = 660 . \qquad (1)$$

Also,

$$q_0 + q_1 + q_2 + q_3 + q_4 = \binom{12}{4} = 495 . \tag{2}$$

Therefore,

$$-q_0 + q_2 + 2q_3 + 3q_4 = 165 . \tag{3}$$

As all 3-(12,6,4) designs have a corresponding solution to equation (3), the four q_i variables present another method for classifying these designs. As q_2 is probably large, only q_0,q_3 and q_4 and their corresponding constituent quadruples will be used.

Under this system the 392 non-isomorphic 3-designs were separated into 141 equivalence classes. Further, within each equivalence class, the number of times each point occurred in the sets of quadruples constituting each of q_0,q_3 and q_4, was usually enough to prove any two designs were non-isomorphic. An unforeseen advantage of this approach was the relative ease involved in developing each design's full automorphism group from its sets of quadruples. The overall effect however was to strongly confirm the results obtained earlier.

As the twelve reducible 3-(12,6,4) designs all contain at least one quartet of AC type blocks, these designs must necessarily have been produced. This was the case, and a copy of each is represented by PIV-T1 N235, PIV-T1 N12, PIV-T1 N5, PIV-T1 N111, PIV-T5 N90, PIV-T1 N13, PIV-T1 N69, PIV-T1 N21, PIV-T1 N123, PIV-T1 N4, PIV-T1 N11, PIV-T1 N20.

To maintain some degree of continuity the listing of these designs will be deferred until all non-isomorphic 3-(12,6,4) designs have been produced. If the reader wishes to peruse them now, a key providing a copy of each of the 392 representative designs can be found in Part II, Section II. Also included there are two tables showing the division of the designs under each classification system.

Chapter 3: Section II

THE DESIGNS CONSISTING ENTIRELY OF B TYPE BLOCKS

With the enumeration of the designs containing AC type blocks completed, the immediate task has been reduced to determining all the designs consisting entirely of B type blocks. Thus in creating these designs it is now possible to exclude any construction or partially complete design if it is found to contain an AC type block. This should greatly reduce the number of possibilities to be examined.

Section II·1: A Skeleton for the B Type Blocks

A restriction on any point of a B(1 0 9 24 9 0 1) type block of a 3-(12,6,4) design will yield a B(0 3 12 6 0 1) type block in the corresponding 2-(11,5,4) design. Thus for legitimate restrictions each point in a B type block of the 3-design must occur three times within the nine two-point intersections with other blocks, twelve times within the twenty-four three-point intersections, and six times within the nine four-point intersections. This and the self-complementary nature of the 3-(12,6,4) design were used to derive the following skeleton for the B type block:-

(i)

*	*	*	*	*	*
*	*	•	•	•	•
*	*	•	•	•	•
(iii)		(v)			
9 pairs		9 quadruples			
Each		Each point			
point		six times			
three		•	•	•	•
times		•	•	•	•
*	*	•	•	•	•

(ii)

•	•	•	•	•	•
*	*	*	*	•	•
*	*	*	*	•	•
(iv)				(vi)	
Complements				Complements	
w.r.t *'s.				w.r.t •'s.	
Each point				Each point	
six times				three times	
*	*	*	*	•	•
*	*	*	*	•	•

Section A

[	* * *	• • •	]	[	* * *	• • •	]
[	* * *	• • •	]	[	* * *	• • •	]
[	* * *	• • •	]	[	* * *	• • •	]
[	* * *	• • •	]	[	* * *	• • •	]
[	(vii)	(ix)	]	[	(viii)	(x)	]
[	12 triples	12 triples	]	[	Complements	Complements	]
[	Each point	Each point	]	[	w.r.t *'s.	w.r.t •'s.	]
[	six times	six times	]	[	Each point	Each point	]
[	* * *	• • •	]	[	six times	six times	]
[	* * *	• • •	]	[	* * *	• • •	]
[	* * *	• • •	]	[	* * *	• • •	]
[	* * *	• • •	]	[	* * *	• • •	]

Section B

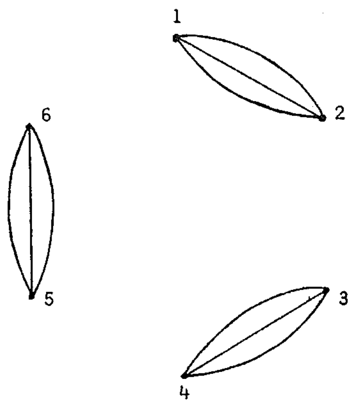
To break the problem down into manageable pieces, firstly consider the choice of points for the * regions. The top block (i) can be chosen arbitrarily and assigning it to the set {1,2,3,4,5,6} produces the complementary block [7 8 9 10 11 12] for region (ii). The nine pairs for region (iii) must now be chosen from {1,2,3,4,5,6} so that each point occurs exactly three times. The possibilities can be grouped into one of seven categories in accordance with the number of repeated pairs present. Having done this it is easy to show that only nine non-equivalent 'Patterns' are available, i.e.,

Pairs repeated thrice	Pairs repeated twice	Pairs occurring once	Number of Non-equivalent forms
3	0	0	1
1	2	2	1
1	0	6	1
0	3	3	1
0	2	5	2
0	1	7	1
0	0	9	2

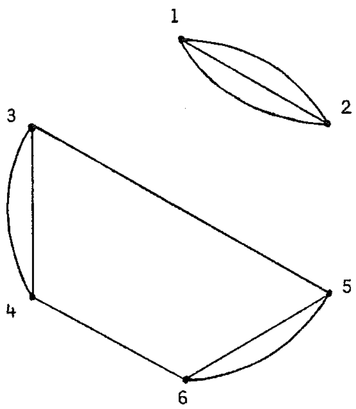
In developing these designs the nine representative patterns below will be used to complete region (iii). The graph of each pattern has also been included as these provide a useful means for comparison and identification. They are all the non-isomorphic graphs of degree 3 on 9 points.

PI	PII	PIII	PIV	PV	PVa	PVI	PVII	PVIII
1 2	1 2	1 2	1 2	1 2	1 2	1 2	1 2	1 2
1 2	1 2	1 2	1 2	1 2	1 2	1 2	1 3	1 3
1 2	1 2	1 2	3 4	3 4	3 4	1 3	1 4	1 4
3 4	3 4	3 4	3 4	3 4	3 4	2 4	2 5	2 3
3 4	3 4	3 5	5 6	1 5	1 5	3 5	2 6	2 5
3 4	5 6	3 6	5 6	2 6	2 5	3 6	3 5	3 6
5 6	5 6	4 5	1 3	3 5	3 6	4 5	3 6	4 5
5 6	3 5	4 6	2 5	4 6	4 6	4 6	4 5	4 6
5 6	4 6	5 6	4 6	5 6	5 6	5 6	4 6	5 6

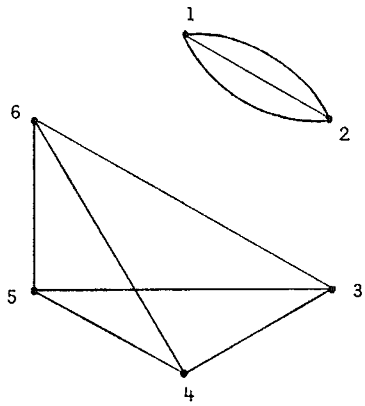
PI



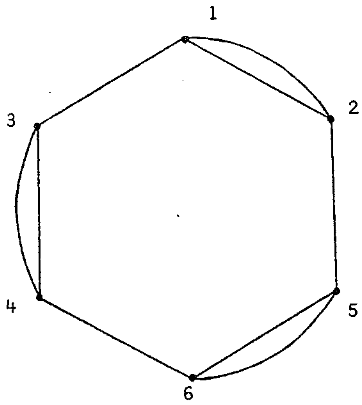
PII



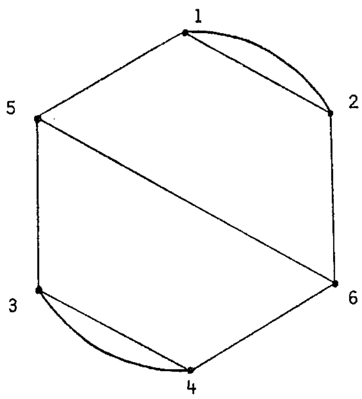
PIII



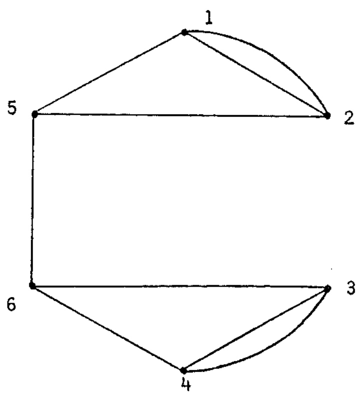
PIV



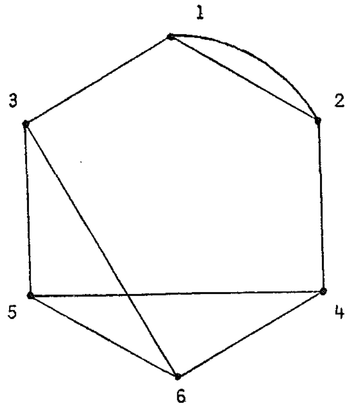
PV



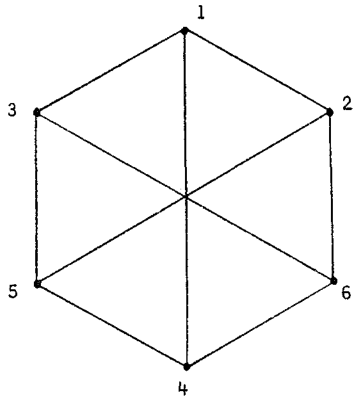
PVa



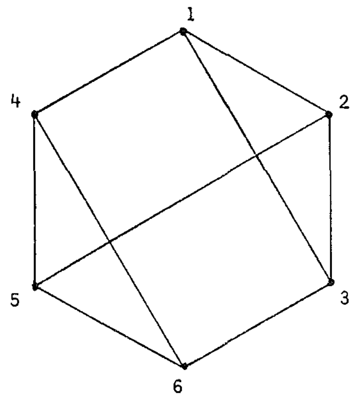
PVI



PVII



PVII



A closer inspection of PVa and in particular the blocks,

[1 2 3 4 5 6]	[• • • • • •]
[1 2 • • • •]	[3 4 5 6 • •]
[1 2 • • • •]	[3 4 5 6 • •]
[1 5 • • • •]	[2 3 4 6 • •]
[2 5 • • • •]	[1 3 4 6 • •]

shows that this pattern is impossible to balance as the triple (3,4,6) has already occurred <u>five</u> times.

Once one of the remaining eight non-equivalent sets of nine pairs has been assigned to region (iii), the corresponding quadruples for region (iv) are readily obtained by taking the complements of these with respect to the point set {1,2,3,4,5,6}. As all triples from this set must each occur four times throughout the design, the triples occupying regions (vii) and (viii) of section B are also defined. Thus requiring the block [1 2 3 4 5 6] to be B type, results in just eight non-equivalent ways to complete the skeleton for these points.

<u>Section II·2: The Completion of Section A</u>

The eight half-skeletons are to be taken in turn and completed with {7,8,9,10,11,12} subject to the occurrence of B type blocks only. If considered independently, the remaining regions of the skeleton can also be completed in eight non-equivalent ways. This property will not be used in the balancing process for these skeletons but will provide a useful check on any results obtained. Because of the size and repetitive nature of this problem it was considered necessary to employ a computer program to solve it.

The nine pairs for region (vi), and hence by complementation the quadruples for region (v), are to be determined first. Here all legitimate non-equivalent placements of the •'s against the known *'s of regions (iii) and (iv) are required. Each point from {7,8,9,10,11,12} must occur exactly three times in region (vi), therefore relative to the * quadruples it must assume one of $\binom{9}{3}$ = 84 different 'positionings.' If a * quadruple is repeated twice or thrice in region (iv), then all positionings including more than one of such quadruples must be automatically rejected in order to prevent the occurrence of a five-point intersection and hence an AC type block. The reduction in the number of positionings is performed by subroutine **TESTI( )** and depending on the Pattern being examined is quite considerable. The number eliminated by this algorithm was checked and found to be entirely consistent with the theoretically calculated value, i.e. for Patterns PI,PII,..., PVIII the 84 positionings were reduced to 27,51,65,63,70,77,84 and 84 respectively. Because of the simplistic nature of **TESTI( )** little explanation of it is required. It suffices to say that this subroutine scans the three index values of each positioning, and instructs **MAIN** to reject it if any two of these values coincide with any of the pairs of illegal index values which were supplied as input data. The remaining positionings are now to be taken six at a time, and every possible combination is to be tested to determine those that define a set of nine pairs in region (vi), and hence a completed form for section A. This task is performed by the program **MAIN**, which uses an incidence array IPST( ) whose elements store the number of • points occurring with each * quadruple under a particular set of positionings. All possible sets of positionings are systematically tested and those resulting in the correct incidence count of two for each of IPST( )s' nine applicable elements, are referred to subroutine

PAIRS( ) for further scrutiny. **MAIN** is best described in terms of the following procedure.

<u>Procedure for **MAIN**</u>

Step 0: Read input information. Most of the arrays are stored in COMMON to permit direct access by the two subroutines.

Step 1: Calculate the next positioning and store in INDEX(,).

Step 2: Call **TESTI( )** to test this positioning. If it is disallowed then arrange for its overwriting by the next one. If all positionings have been created then go to *Step 3* otherwise go to *Step 1*.

Step 3: Write the total number of legitimate positionings to the terminal.

Step 4: Choose the next 1st positioning; adjust IPTS( ) to include it, and write its number to the terminal. If all have been tried go to *Step 11*.

Step 5: Choose the next 2nd positioning and adjust IPTS( ) to include it. If all have been tried then remove the current 1st positioning and go to *Step 4*.

Step 6: Choose the next 3rd positioning and adjust IPTS( ) to include it. If none are legitimate or all have been tried then remove the current 2nd positioning and go to *Step 5*.

Step 7: Choose the next 4th positioning and adjust IPTS( ) to include it. If none are legitimate or all have been tried then remove the current 3rd positioning and go to *Step 6*.

Step 8: Choose the next 5th positioning and adjust IPTS( ) to include it. If none are legitimate or all have been tried then remove the current 4th positioning and go to *Step 7*.

Step 9: Choose the next 6th positioning. If none are legitimate or all have been tried then remove the current 5th positioning and go to *Step 8*.

Step 10: A set of nine pairs has been found so increase the count by one and call subroutine **PAIRS( )** to process this 'pairing'. When control is returned from **PAIRS( )** go to *Step 9*.

Step 11: Close all files and STOP.

The code used to implement this procedure can be found at the end of this section.

The task of completing section A is now reduced to finding all the legitimate non-equivalent forms amongst the sets of nine pairs, (hereafter referred to as pairings). Subroutine **PAIRS( )** does this by converting the sets of positionings into pairings, and then subjecting these to a series of tests.

The first and most efficient eliminating factor is due to the availability of permutations which fix the * quadruples and the point set {7,8,9,10,11,12}. Some manual calculations using these produce only a small number of non-equivalent ways to select the first three or four pairs of any pairing. Here, when choosing a representative from an equivalence class it is crucial to retain the pair set which is produced first by the construction algorithm **MAIN.** These representative pair sets are input to the array IOUT(, ,) and produce the greatest reduction in the number of possibilities.

Further possibilities are removed by the rejection of those pairings whose corresponding blocks are found to contain AC type blocks. Their blocks also undergo a triple incidence count to ensure that they are consistent with those of a 3-(12,6,4) design.

A pairing that survives these tests is then classified according to which of the eight non-equivalent forms it exhibits. This will greatly assist in the next, more complicated, and final elimination process. More specifically, to provide a simpler and possibly more economical implementation, only those permutations which fix blocks (i) and (ii) without interchanging them will be utilised. Under these conditions the above classification process means that no two pairings from different classes could possibly be equal.

The method for determining two equivalent pairings within a class is the focal point of the algorithm. Intuitively there are 6! = 720 possible permutations on {7,8,9,10,11,12}, but applying these directly would be grossly inefficient. Therefore the more attractive approach of comparing the points' positionings will be adopted. This method is demonstrated in the following example where two test pairings are to be compared with a previously obtained pairing. It is important to note that this example is constructed to illustrate the innermost comparison mechanism for the pairings, and in practice the creation process negates the existence of Test 2 as it is already present as the fixed pairing.

The Comparison of Pairings

Fixed * quadruples	Incidence index	Fixed pairing	Test 1 pairing	Test 2 pairing
3 4 5 6	2	7 8	u v	u v
2 4 5 6	3	8 10	v w	v w
2 3 5 6	4	9 11	x y	x y
1 3 4 6	5	7 12	u z	u z
1 3 4 5	6	10 12	w x	w z
1 2 4 6	7	7 9	v y	u x
1 2 4 5	8	8 11	u x	v y
1 2 3 6	9	9 10	w z	w x
1 2 3 5	10	11 12	y z	y z

Vector representation of positionings for:

Fixed pairing / Test 1 pairing / Test 2 pairing

$$7 = \begin{pmatrix}2\\5\\7\end{pmatrix},\ 8 = \begin{pmatrix}2\\3\\8\end{pmatrix}, \qquad u = \begin{pmatrix}2\\5\\8\end{pmatrix},\ v = \begin{pmatrix}2\\3\\7\end{pmatrix}, \qquad u = \begin{pmatrix}2\\5\\7\end{pmatrix},\ v = \begin{pmatrix}2\\3\\8\end{pmatrix},$$

$$9 = \begin{pmatrix}4\\7\\9\end{pmatrix},\ 10 = \begin{pmatrix}3\\6\\9\end{pmatrix}, \qquad w = \begin{pmatrix}3\\6\\9\end{pmatrix},\ x = \begin{pmatrix}4\\6\\8\end{pmatrix}, \qquad w = \begin{pmatrix}3\\6\\9\end{pmatrix},\ x = \begin{pmatrix}4\\7\\9\end{pmatrix},$$

$$11 = \begin{pmatrix}4\\8\\10\end{pmatrix},\ 12 = \begin{pmatrix}5\\6\\10\end{pmatrix}, \qquad y = \begin{pmatrix}4\\7\\10\end{pmatrix},\ z = \begin{pmatrix}5\\9\\10\end{pmatrix}, \qquad y = \begin{pmatrix}4\\8\\10\end{pmatrix},\ z = \begin{pmatrix}5\\6\\10\end{pmatrix},$$

where $\{u,v,w,x,y,z\} \in \{7,8,9,10,11,12\}$.

With everything fixed, it is straightforward to compare the positionings for each regardless of what values u,v,w,x,y, and z actually assume. For instance, in the fixed pairing the point 7 has a positioning vector $(2\ 5\ 7)^T$ but for Test 1 no such vector exists. Therefore as 7 has no corresponding point in Test 1, their two pairings are non-equivalent (under the previously stated conditions). An examination of the positioning vectors for Test 2 however will quickly show the equivalence of this to the fixed pairing with u $\Leftrightarrow$ 7, v $\Leftrightarrow$ 8, w $\Leftrightarrow$ 10, x $\Leftrightarrow$ 9, y $\Leftrightarrow$ 11, and z $\Leftrightarrow$ 12.

As mentioned earlier this process forms the basis for comparison between pairings. The duplication it is required to eliminate is caused by permutations fixing the * quadruples and/or repeated * quadruples. In either case the effect of these can be represented by a corresponding adjustment in the blocks' index values. For instance, the * quadruples of the previous example are fixed by the point permutation (1)(5)(6)(2 3 4), and this has the effect of permuting the index values according to (2 3 4)(5 7 9)(6 8 10). Thus to test pairings under this permutation their positionings are adjusted using the index permutation and then the test is applied in its standard form. Repeated * quadruples are dealt with in a similar manner. If a particular * quadruple appears at indices r and s then the permutation used fixes all indices except r and s which it interchanges. The information pertaining to the * quadruples is predetermined and given as input data.

This concludes the discussion of the final elimination process used in PAIRS( ). If a pairing is found to be equivalent to a previously developed one then it is discarded by reducing NUM by one and control is returned to MAIN. Should the pairing survive then all permutations fixing section A which do not interchange {1,2,3,4,5,6} with {7,8,9,10,11,12} are calculated using the final elimination stage of PAIRS( ) on a copy of the pairing. This information along with the top twenty blocks is then output to the file SOLUT ready for processing by the program which completes the design. Control is then returned to MAIN.

The following procedure gives a summarized form of the algorithm used in subroutine PAIRS( ). The three program segments are then listed with each piece of code being sectioned and labelled with the step of the procedure that it executes. A complete listing of each of the eight data files used by **MAIN** is available from Appendix 2.

Procedure for Subroutine **PAIRS( )**

Step 0: Receive control from **MAIN**. Most information is stored in COMMON but the number of the pairing and its' positionings are given as arguments.

Step 1: Store the positionings in a single array LS(,) and also as a 9×2 entry in ISTORE(, ,).

Step 2: If the first few pairs do not correspond to one of the non-equivalent sets in IOUT(, ,) go to *Step 16*.

Step 3: If the top twenty blocks contain any AC type blocks then go to *Step 16*.

Step 4: Obtain and store the twenty blocks of section A in II(,) and if any triple occurs more than four times go to *Step 16*.

Step 5: Put the pairing into one of eight classes in accordance with the structural pattern that it exhibits.

Step 6: Initialize control variables for elimination phase.

Step 7: Perform the next adjustment of the positionings indices to allow for permutations which fix the * quadruples. If all have been tried, go to *Step 14*.

Step 8: Perform the next adjustment of the positionings indices to allow for the occurrence of repeated * quadruples. If all have been tried then reset to the first case and go to *Step 7*.

Step 9: Consider the next previously obtained pairing. If all have been tested then reset to the first and go to *Step 8*.

Step 10: If this pairing is not in the same class as the new pairing go to *Step 9*.

Step 11: If this pairing is not equivalent to the new pairing go to *Step 9*.

Step 12: The two pairings are equivalent. If in the elimination phase, then reject the new pairing by reducing NUM by one, and go to *Step 16*.

Step 13: An automorphism for section A has been identified so calculate and output it to SOLUT. Go to *Step 9*.

Step 14: The pairing is different from all those previously retained. If its automorphisms have already been found then go to *Step 16*.

Step 15: Write section A to SOLUT and initialize the control variables to those for the automorphism finding phase. Go to *Step 7*.

Step 16: Reduce NUM by one and return control to **MAIN**.

MAIN

```
      COMMON/GR0/IOUT(100,5,2),LINT(2),INS(12,12,12),II(7,44)
      COMMON/GR1/ISTORE(10000,10,2),IDIS(2,10),INDEX(85,3)
      COMMON/GR2/NREP,IREP(300,10),IDIS2(3,7)
      COMMON/GR3/IPERM2(100,10),NPERM2,ICON2(100),ISTCON(100,10,6)
      DIMENSION IPTS(10)
      OPEN(12,FILE='DATA')
      OPEN(15,FILE='SOLUT')
      DATA IPTS/0,0,0,0,0,0,0,0,0,0/
      READ(12,*)(II(M1,1),M1=1,6)
      READ(12,*)(II(M2,11),M2=1,6)
      DO 1 M3=1,2
    1 READ(12,*)(II(M3,M4),M4=2,10)
      DO 2 M5=1,4
    2 READ(12,*)(II(M5,M6),M6=12,20)
      DO 5 M11=1,10
    5 READ(12,*)IDIS(1,M11),IDIS(2,M11)
      READ(12,*)IDIS2(1,7)
      IF(IDIS2(1,7).EQ.0)GO TO 22
      DO 21 M21=1,IDIS2(1,7)
   21 READ(12,*)(IDIS2(M22,M21),M22=1,3)                  Step 0
   22 READ(12,*)NREP
      DO 13 M13=1,NREP
   13 READ(12,*)(IREP(M13,M14),M14=2,10)
      READ(12,*)NPERM2
      DO 6 M15=1,NPERM2
    6 READ(12,*)(IPERM2(M15,M16),M16=2,10)
      READ(12,*)(ICON2(M16),M16=1,NPERM2)
      DO 7 M17=1,NPERM2
      DO 8 M18=1,ICON2(M17)
    8 READ(12,*)(ISTCON(M17,M18,M19),M19=1,6)
    7 CONTINUE
      READ(12,*)LINT(1),LINT(2)
      DO 23 M21=1,LINT(1)
      DO 9 M20=1,LINT(2)
    9 READ(12,*)IOUT(M21,M20,1),IOUT(M21,M20,2)
   23 CONTINUE
C
      CLOSE(12)
C
      ICOUNT=0
      IFLAG=1
      DO 10 I1=2,8                                        Step 1
      DO 11 I2=I1+1,9
      DO 12 I3=I2+1,10
      ICOUNT=ICOUNT+1
      INDEX(ICOUNT,1)=I1
      INDEX(ICOUNT,2)=I2
      INDEX(ICOUNT,3)=I3
      CALL TEST1(ICOUNT,IFLAG)
      IF(IFLAG.EQ.0)ICOUNT=ICOUNT-1
   12 CONTINUE                                            Step 2
   11 CONTINUE
   10 CONTINUE
C
      NUM=0                                               Step 3
      WRITE(1,*)ICOUNT
      DO 30 L1=1,ICOUNT-5
      WRITE(1,*)L1,NUM                                    Step 4
      IPTS(INDEX(L1,1))=IPTS(INDEX(L1,1))+1
      IPTS(INDEX(L1,2))=IPTS(INDEX(L1,2))+1
      IPTS(INDEX(L1,3))=IPTS(INDEX(L1,3))+1
      DO 40 L2=L1,ICOUNT-4
        IPTS(INDEX(L2,1))=IPTS(INDEX(L2,1))+1             Step 5
        IPTS(INDEX(L2,2))=IPTS(INDEX(L2,2))+1
        IPTS(INDEX(L2,3))=IPTS(INDEX(L2,3))+1
```

```
      DO 50 L3=L2,ICOUNT-3
        GO TO (51,51,50),IPTS(INDEX(L3,1))+1
 51     GO TO (52,52,50),IPTS(INDEX(L3,2))+1
 52     GO TO (53,53,50),IPTS(INDEX(L3,3))+1
 53     IPTS(INDEX(L3,1))=IPTS(INDEX(L3,1))+1
        IPTS(INDEX(L3,2))=IPTS(INDEX(L3,2))+1
        IPTS(INDEX(L3,3))=IPTS(INDEX(L3,3))+1
      DO 60 L4=L3,ICOUNT-2
          GO TO (61,61,60),IPTS(INDEX(L4,1))+1
 61       GO TO (62,62,60),IPTS(INDEX(L4,2))+1
 62       GO TO (63,63,60),IPTS(INDEX(L4,3))+1
 63       IPTS(INDEX(L4,1))=IPTS(INDEX(L4,1))+1
          IPTS(INDEX(L4,2))=IPTS(INDEX(L4,2))+1
          IPTS(INDEX(L4,3))=IPTS(INDEX(L4,3))+1
      DO 70 L5=L4,ICOUNT-1
          GO TO (71,71,70),IPTS(INDEX(L5,1))+1
 71       GO TO (72,72,70),IPTS(INDEX(L5,2))+1
 72       GO TO (73,73,70),IPTS(INDEX(L5,3))+1
 73       IPTS(INDEX(L5,1))=IPTS(INDEX(L5,1))+1
          IPTS(INDEX(L5,2))=IPTS(INDEX(L5,2))+1
          IPTS(INDEX(L5,3))=IPTS(INDEX(L5,3))+1
      DO 80 L6=L5,ICOUNT
            GO TO (81,81,80),IPTS(INDEX(L6,1))+1
 81         GO TO (82,82,80),IPTS(INDEX(L6,2))+1
 82         GO TO (83,83,80),IPTS(INDEX(L6,3))+1
C
 83           NUM=NUM+1
              CALL PAIRS(L1,L2,L3,L4,L5,L6,NUM)
C
  80        CONTINUE
            IPTS(INDEX(L5,1))=IPTS(INDEX(L5,1))-1
            IPTS(INDEX(L5,2))=IPTS(INDEX(L5,2))-1
            IPTS(INDEX(L5,3))=IPTS(INDEX(L5,3))-1
 70       CONTINUE
          IPTS(INDEX(L4,1))=IPTS(INDEX(L4,1))-1
          IPTS(INDEX(L4,2))=IPTS(INDEX(L4,2))-1
          IPTS(INDEX(L4,3))=IPTS(INDEX(L4,3))-1
 60     CONTINUE
        IPTS(INDEX(L3,1))=IPTS(INDEX(L3,1))-1
        IPTS(INDEX(L3,2))=IPTS(INDEX(L3,2))-1
        IPTS(INDEX(L3,3))=IPTS(INDEX(L3,3))-1
 50    CONTINUE
       IPTS(INDEX(L2,1))=IPTS(INDEX(L2,1))-1
       IPTS(INDEX(L2,2))=IPTS(INDEX(L2,2))-1
       IPTS(INDEX(L2,3))=IPTS(INDEX(L2,3))-1
 40   CONTINUE
      IPTS(INDEX(L1,1))=IPTS(INDEX(L1,1))-1
      IPTS(INDEX(L1,2))=IPTS(INDEX(L1,2))-1
      IPTS(INDEX(L1,3))=IPTS(INDEX(L1,3))-1
 30   CONTINUE
C
      CLOSE(12)
      CLOSE(15)
      STOP
      END
```

Step 6

Step 7

Step 8

Step 9

Step 10

Part of *Steps* *4,5,6,7,8,9*

Step 11

PAIRS( )

```
      SUBROUTINE PAIRS(L1,L2,L3,L4,L5,L6,NUM)
      COMMON/GR0/IOUT(100,5,2),LINT(2),INS(12,12,12),II(7,44)
      COMMON/GR1/ISTORE(10000,10,2),IDIS(2,10),INDEX(85,3)
      COMMON/GR2/NREP,IREP(300,10),IDIS2(3,7)
      COMMON/GR3/IPERM2(100,10),NPERM2,ICON2(100),ISTCON(100,10,6)
      DIMENSION LS(6,3),ITYPE(6),LS1(6,3),IPERM1(6,2),LS2(6,3)
C ALL TEST ARE ON THE TOP 20 BLOCKS OR NINE PAIRS FORMING THEM.
```

Step 0

```
C FORM THE NINE PAIRS.
      DO 7 IO=1,3
      LS(1,IO)=INDEX(L1,IO)
      LS(2,IO)=INDEX(L2,IO)
      LS(3,IO)=INDEX(L3,IO)
      LS(4,IO)=INDEX(L4,IO)
      LS(5,IO)=INDEX(L5,IO)
      LS(6,IO)=INDEX(L6,IO)
    7 CONTINUE                                                  Step 1
      DO 1 I1=1,10
      DO 2 I2=1,2
    2 ISTORE(NUM,I1,I2)=0
    1 CONTINUE
      DO 3 I3=1,3
    3 ISTORE(NUM,LS(1,I3),1)=7
      DO 4 I4=2,6
      DO 5 I5=1,3
      IF(ISTORE(NUM,LS(I4,I5),1).EQ.0)GO TO 6
      ISTORE(NUM,LS(I4,I5),2)=I4+6
      GO TO 5
    6 ISTORE(NUM,LS(I4,I5),1)=I4+6
    5 CONTINUE
    4 CONTINUE
C
C
C  TOP PAIRS MUST BE ONE OF NONISOMORHIC SETS IN IOUT().
      DO 9 M2=1,LINT(1)
      DO 10 M1=1,LINT(2)                                        Step 2
      IF(ISTORE(NUM,M1+1,1).NE.IOUT(M2,M1,1).OR.
     * ISTORE(NUM,M1+1,2).NE.IOUT(M2,M1,2))GO TO 9
   10 CONTINUE
      GO TO 12
    9 CONTINUE
      GO TO 14
C
C
C TEST FOR A/C-TYPE BLOCK OCCURRENCES.                          Step 3
   12 IF(IDIS2(1,7).EQ.0)GO TO 42
      DO 21 M1=1,IDIS2(1,7)
      IF(ISTORE(NUM,IDIS2(1,M1),1).EQ.ISTORE(NUM,IDIS2(2,M1),1).AND.
     * ISTORE(NUM,IDIS2(1,M1),2).EQ.ISTORE(NUM,IDIS2(2,M1),2))GO TO 14
      IF(ISTORE(NUM,IDIS2(1,M1),1).EQ.ISTORE(NUM,IDIS2(3,M1),1).AND.
     * ISTORE(NUM,IDIS2(1,M1),2).EQ.ISTORE(NUM,IDIS2(3,M1),2))GO TO 14
      IF(ISTORE(NUM,IDIS2(2,M1),1).EQ.ISTORE(NUM,IDIS2(3,M1),1).AND.
     * ISTORE(NUM,IDIS2(2,M1),2).EQ.ISTORE(NUM,IDIS2(3,M1),2))GO TO 14
   21 CONTINUE
C
C  TEST THAT NO TRIPLE OCCURS MORE THAN FOUR TIMES.
   42 DO 51 K1=1,10
      DO 52 K2=K1+1,11
      DO 53 K3=K2+1.12
   53 INS(K1,K2,K3)=0
   52 CONTINUE                                                  Step 4
   51 CONTINUE
C
      DO 54 K4=2,10
      NSOL=2
      DO 55 K5=7,12
      IF(ISTORE(NUM,K4,1).NE.K5)GO TO 60
      II(5,K4+10)=K5
      GO TO 55
   60 IF(ISTORE(NUM,K4,2).NE.K5)GO TO 61
      II(6,K4+10)=K5
      GO TO 55
   61 NSOL=NSOL+1
      II(NSOL,K4)=K5
   55 CONTINUE
   54 CONTINUE
```

```
      DO 56 K6=1,20
      DO 57 K7=1,4
      DO 58 K8=K7+1,5
      DO 59 K9=K8+1,6
      IF(INS(II(K7,K6),II(K8,K6),II(K9,K6)).EQ.4)GO TO 14
      INS(II(K7,K6),II(K8,K6),II(K9,K6))=INS(II(K7,K6),II(K8,K6),
     * II(K9,K6))+1
   59 CONTINUE
   58 CONTINUE
   57 CONTINUE
   56 CONTINUE
C
C  CATAGORIES AS ONE OF 8 NONISOMORPHIC TYPES.
      DO 20 J0=1,6
   20 ITYPE(J0)=0
      DO 41 J1=2,10
      IDUM=0
      DO 22 J2=2,10
      IF(ISTORE(NUM,J1,1).EQ.ISTORE(NUM,J2,1).AND.
     * ISTORE(NUM,J1,2).EQ.ISTORE(NUM,J2,2))IDUM=IDUM+1
   22 CONTINUE
      ITYPE(IDUM)=ITYPE(IDUM)+1
   41 CONTINUE
      GO TO(23,24,25,26,24,27,28,29,24,30),ITYPE(1)+1
   23 ISTORE(NUM,1,1)=8
      GO TO 15
   25 ISTORE(NUM,1,1)=7
      GO TO 15                                           Step 5
   26 ISTORE(NUM,1,1)=6
      GO TO 15
   28 ISTORE(NUM,1,1)=4
      GO TO 15
   29 ISTORE(NUM,1,1)=3
      GO TO 15
   27 ISTORE(NUM,1,1)=5
      GO TO 15
   30 IDUM=0
      DO 33 J3=2,10
      IF(ISTORE(NUM,J3,1).NE.7)GO TO 33
      IDUM=IDUM+1
      ITYPE(IDUM)=ISTORE(NUM,J3,2)
   33 CONTINUE
      DO 34 J4=2,10
      DO 35 J5=1,3
   35 IF(ISTORE(NUM,J4,1).EQ.ITYPE(J5))GO TO 36
      GO TO 34
   36 DO 37 J7=1,3
      IF(J7.EQ.J5)GO TO 37
      IF(ISTORE(NUM,J4,2).EQ.ITYPE(J7))GO TO 38
   37 CONTINUE
   34 CONTINUE
      ISTORE(NUM,1,1)=2
      GO TO 15
   38 ISTORE(NUM,1,1)=1
C
C  ELIMINATE IF ISOMORPHIC TO A PREVIOUS CASE.
   15 IF(NUM.EQ.1)GO TO 157
      IGAT=1                                             Step 6
      ISLT=2
      INTIL=1
   16 NUM1=NUM
      NUM=NUM+1
      ISTORE(NUM,1,1)=ISTORE(NUM1,1,1)
```

```
      DO 111 K11=1,NPERM2
      DO 113 K13=1,6
      DO 114 K14=1,3
 114  LS1(K13,K14)=IPERM2(K11,LS(K13,K14))
 113  CONTINUE
      DO 115 K15=1,NREP
      DO 116 K16=1,6
      DO 117 K17=1,3
 117  LS2(K16,K17)=IREP(K15,LS1(K16,K17))
 116  CONTINUE
      DO 100 MAST=INTIL,NUM-ISLT
      IF(ISTORE(NUM,1,1).NE.ISTORE(MAST,1,1))GO TO 100
      DO 101 K1=1,6
      IPERM1(K1,1)=ISTORE(MAST,LS2(K1,1),1)
      IPERM1(K1,2)=ISTORE(MAST,LS2(K1,1),2)
      DO 102 K2=2,3
      DO 103 K3=1,2
 103  IF(IPERM1(K1,K3).NE.ISTORE(MAST,LS2(K1,K2),1).AND.
     * IPERM1(K1,K3).NE.ISTORE(MAST,LS2(K1,K2),2))IPERM1(K1,K3)=0
      IF(IPERM1(K1,1).EQ.0.AND.IPERM1(K1,2).EQ.0)GO TO 100
 102  CONTINUE
 101  CONTINUE
C
C  A PERMUTATION HAS BEEN FOUND.
      IF(IGAT.NE.1)GO TO 104
      NUM=NUM-1
      GO TO 14
C
C  CODE USED WHEN FINDING AUTOMORPHISMS FOR TOP 20 BLOCKS.
 104  WRITE(15,1002)ICON2(K11)
      DO 105 N5=1,ICON2(K11)
 105  WRITE(15,1003)(ISTCON(K11,N5,N6),N6=1,6)
C
      DO 121 N21=1,2
      IF(IPERM1(1,N21).EQ.0)GO TO 121
      DO 122 N22=1,2
      IF(IPERM1(2,N22).EQ.0)GO TO 122
      DO 123 N23=1,2
      IF(IPERM1(3,N23).EQ.0)GO TO 123
      DO 124 N24=1,2
      IF(IPERM1(4,N24).EQ.0)GO TO 124
      DO 125 N25=1,2
      IF(IPERM1(5,N25).EQ.0)GO TO 125
      DO 126 N26=1,2
      IF(IPERM1(6,N26).EQ.0)GO TO 126
      JDUM=IPERM1(1,N21)*IPERM1(2,N22)*IPERM1(3,N23)*IPERM1(4,N24)*
     * IPERM1(5,N25)*IPERM1(6,N26)
      IF(JDUM.NE.665280)GO TO 126
      WRITE(15,1003)IPERM1(1,N21),IPERM1(2,N22),IPERM1(3,N23)
     * ,IPERM1(4,N24),IPERM1(5,N25),IPERM1(6,N26)
 126  CONTINUE
 125  CONTINUE
 124  CONTINUE
 123  CONTINUE
 122  CONTINUE
 121  CONTINUE
C
 100  CONTINUE
 115  CONTINUE
 111  CONTINUE
C
C  NONISOMORPHIC. INITIALIZE TO FIND AUTOMORPHISMS.
      IF(IGAT.EQ.2)WRITE(15,1004)
      IF(IGAT.EQ.2)GO TO 14
      NUM=NUM-1
```

Step 7

Step 8

Step 9

Step 10

Step 11

Step 12

Step 13

Part of *Steps 7,8,9*

Step 14

```
157   DO 158 N4=1,10
158   WRITE(15,1001)(II(N5,N4),N5=1,6),(II(N6,N4+10),N6=1,6)
      INTIL=NUM
      IGAT=2
      ISLT=1
      WRITE(1,*)NUM
      GO TO 16
C
 14   NUM=NUM-1
      RETURN
1001  FORMAT(1X,6(I3),5X,6(I3))
1002  FORMAT(1X,I3,'   0')
1003  FORMAT(1X,6(I3))
1004  FORMAT(1X,'   0')
 24   WRITE(15,1005)
1005  FORMAT(1X,'SOMETHING IS INCORRECT')
      END
```

Step 15

Step 16

TESTI( )

```
      SUBROUTINE TEST1(J,IFLAG)
      COMMON/GR0/IOUT(100,5,2),LINT(2),INS(12,12,12),II(7,44)
      COMMON/GR1/ISTORE(10000,10,2),IDIS(2,10),INDEX(85,3)
      COMMON/GR2/NREP,IREP(300,10),IDIS2(3,7)
      COMMON/GR3/IPERM2(100,10),NPERM2,ICON2(100),ISTCON(100,10,6)
      IFLAG=1
      IDUM=0
      IF(IDIS(1,10).EQ.0)GO TO 6
      DO 1 J1=1,IDIS(1,10)
      DO 2 J2=1,2
      DO 3 J3=1,3
    3 IF(INDEX(J,J3).EQ.IDIS(J2,J1))GO TO 4
      GO TO 2
    4 IDUM=IDUM+1
    2 CONTINUE
      IF(IDUM.EQ.2)GO TO 5
      IDUM=0
    1 CONTINUE
      RETURN
    5 IFLAG=0
    6 RETURN
      END
```

Section II·3: Completion of the Design

It is important to note that a solution for section A does not guarantee the existence of blocks to complete section B. It does however provide the two sets of twelve complementary pairs of triples for the remaining regions of section B. Thus the final balancing of the design becomes a matter of the relative alignment of the triples from these two sets.

For simplicity the eight non-equivalent configurations PI to PVIII are again dealt with individually. As well as splitting the overall workload some advantage may be gained from the resulting invariance (for each run) of regions (vii) and (viii). One possibility for utilizing this property is to take each solution of section A in turn and generate the corresponding triples for regions (ix) and (x). Every possible alignment of these triples against the fixed triples is then tested for balance. The direct implementation of this approach would require a maximum of $\binom{24}{12} \approx 3$ million combinations to be tested for each solution of section A. This indicates a need for caution, but it is pertinent to add that the estimate is unrealistically large as it has been calculated by wrongly assuming that the placing of each triple is an independent event. This approach was not used in the hope that a conceptionally simpler, though possibly less efficient program would result from the adoption of the following method.

This method views the problem in terms of the selection of twelve pairs of complementary blocks from the universal set of $\binom{12}{6} = 924$ blocks. One significant advantage of this method is that any blocks with illegal intersections with the blocks of section A, need only be detected and discarded once at the start of the balancing process. Of primary interest are the twelve blocks containing the point 1 to be chosen from the $\binom{11}{5} = 462$ available sextuples. The program **FINISH** generates each of these and refers it to the subroutine **TSTP21( )** where a comparison with the blocks of section A is made.

TSTP21( ) performs two tests on each sextuple. The first ensures that no sextuple is accepted if it contains a triple which has already occurred four times in section A. As an incidence count of the triples in the blocks of section A has been stored in the array INS(12,12,12), this is simply a matter of examining the $\binom{6}{3} = 20$ array elements relevant to the given sextuple. Secondly, if any five- or six-point intersection with a block of section A is detected then the sextuple is rejected in order to prevent the occurrence of an AC or R type block.

The resulting legitimate sextuples are stored in ISTOR(1,*,1-6) while their complements are positioned at ISTOR(2,*,1-6). The method now requires this array to be systematically scanned by twelve pointers to choose the combinations of sextuples which avoid five-point intersections, and which give a correct triple incidence count for INS(, ,). Throughout this selection process the pointer values were always adjusted to begin at the first sextuple containing the next triple required for balance. This ensures that only sextuples with a correct set of triples for regions (vii) and (viii) are tested.

The balancing program is composed of the four interactive sections; **FINISH**, **TSTP22( )**, **PUTIN( )** and **PUTOUT( )**. **FINISH** is responsible for overall control, and manipulates the pointer values and its subroutines **PUTIN( )** and **PUTOUT( )** in accordance with information supplied by its testing subroutine **TSTP22( )**. As their names imply, **PUTIN( )** and **PUTOUT( )** update the triple incidence array INS(, ,) to respectively accept a new and reject a previously obtained sextuple. These two subroutines also recalculate the 'lowest' triple required for balance.

The focal point of the balancing operation is the three tests performed on a possible sextuple by the subroutine **TSTP22( )**. The first of these ensures that the lowest triple required for balance is contained by the sextuple. The second ensures that no triple will occur more than four times, while the last detects any five-point intersections with the previously accepted sextuples (or their complements). In all events the result is passed back to **FINISH** via the argument IFLAG.

The calculation of a set of twelve pointers and hence a design causes all permutations fixing the twenty blocks of the current section A to be read from the input file SOLUT. After manipulating these into a more convenient form the subroutine **TAUT( )** is used to check their validity. The failure of any permutation to fix the blocks of section A will halt the program and an error message is printed in the output file COMPLT. The most common cause of failure is through incorrect permutation data being supplied to **MAIN**. This checking process is only performed when a section A is found to produce designs. The first design found activates it, and any subsequent designs with this top have their bottom 24 blocks tested against the previously retained designs under the given permutations. Any isomorphs are discarded by adjusting a control variable NDES to NDES-1.

If a design is found to be non-isomorphic to the previously obtained designs the subroutine **BLOCK( )** is invoked to provide a copy of it, listing the blocks and their corresponding type. This subroutine is an adaption of that used to process the designs with AC type blocks. It provides a useful check on the program by illustrating that only B type blocks are present.

As all blocks are of the same type, there is some difficulty involved in removing further isomorphs from the pool of designs created. Thus the classification system used for the designs with AC type blocks which is based on the quadruple incidences throughout the design, will be the most useful here. As before only the q_0,q_3 and q_4 quadruples are required, so the information concerning these was calculated and output by **BLOCK( )**. For the present case the following lemma shows the exact relationship between these three variables.

<u>Lemma 1</u>: If q_i represents the number of quadruples occurring exactly i times throughout a 3-(12,6,4) design consisting entirely of B type blocks then,

$$q_0 + q_3 + 3q_4 = 33 .$$

<u>Proof</u>: Equation (3) page 48 holds for any 3-(12,6,4) design.

$$q_2 + 2q_3 + 3q_4 - q_0 = 165 . \qquad (3)$$

Now count the total number of four-point intersections between the blocks of a 3-(12,6,4) design consisting entirely of B type blocks, using two different methods. Firstly, $n_4 = 9$ and $n_5 = 0$ for a B type block, and there are 44 such blocks in all. Thus there are $(44 \times 9)/2$ four-point intersections between blocks (the division by 2 balances the effect of having counted each intersection twice). Equating this to the corresponding expression in terms of the q_i variables gives,

$$\binom{2}{2}q_2 + \binom{3}{2}q_3 + \binom{4}{2}q_4 = (44 \times 9)/2$$

$$q_2 + 3q_3 + 6q_4 = 198 . \qquad (4)$$

Eliminating q_2 by subtracting (3) from (4) produces the required result. □

When control is returned from **BLOCK( )** the code used to test whether the design was isomorphic to an earlier one, can now be used to determine if any of the permutations fixing section A are actually automorphisms of the new design. This is done by adjusting some control variables and any automorphisms are output to COMPLT.

When all possible pointer positions have been tested the program reads the next solution for section A from SOLUT and the process begins again. Because the program does not compare designs as a whole or those resulting from different solutions of section A, and as the eight cases are treated independently, many isomorphs are expected. The amount of effort involved in their subsequent manual elimination using information supplied by their q_i variables is not expected to present a problem.

As usual a simplified procedure for **FINISH** will now be presented followed by the code used in the program. The six accompanying subroutines have also been listed. All of the programs data is obtained directly from **MAINS**'s output file SOLUT.

<u>Procedure for **FINISH**</u>

Step 0: Declare all arrays, and open files.

Step 1: Initialize the triple incidence array INS(, ,) to zero.

Step 2: Read the twenty blocks for the next section A and update INS(, ,) for their triples. If no more data STOP.

Step 3: Develop and test using subroutine **TSTP21( )** the 462 sextuples containing the point 1.

Step 4: Initialize control variables.

Step 5: Systematically examine all possible combinations of the twelve pointers for those resulting in a design. This is done using twelve nested Do Loops each representing the choice of a sextuple. The loops are practically identical with each operating in the following way: the sextuple is referred to **TSTP22( )** for testing and one of the following three events will result depending on the IFLAG value.

a) IFLAG = 1. The sextuple is acceptable so update the incidence array using **PUTIN( )**. Store the pointer value and activate the next loop.

b) IFLAG = 2. The sextuple is not acceptable so reactivate this loop using the next pointer value.

c) IFLAG = 3. The sextuple is not acceptable as the lowest triple of the sextuple is higher than the lowest triple required for balance. So reactivate the previous loop by removing its currently accepted sextuple using **PUTOUT( )**.

When a design is found go to *Step 6*.
When all possibilities have been tested go to *Step 12*.

Step 6: Increase the design count NDES by one and store the pointer values. If this is not the first design for the present section A go to *Step 9*.

Step 7: Read and manipulate any permutation information concerning this section A.

Step 8: Test the validity of this permutation information and if it is incorrect print an error message and STOP.

Step 9: Test this new design against all the previously retained designs under all the given permutations. If this is an isomorph, decrease NDES by one and go to the innermost loop of *Step 5*.

Step 10: Call BLOCK( ) to analyse and output the new design.

Step 11: Initialize the control variables so that the code of *Step 9* can be used to determine if any of the permutations are in fact automorphisms of the new design. Rerun this code outputting any automorphisms. When finished go to the innermost loop of *Step 5*.

Step 12: Ensure that the next data to be read from SOLUT is the top twenty blocks of the next section A. Go to *Step 2*.

FINISH

```
      COMMON/BLOCK1/NQUAD(12,12,12,12)
      COMMON/BLOCK2/INS(12,12,12),ISTOR(2,500,6)
      COMMON/BLOCK3/IAUT(1000,12),II(7,44),LOOP(12),LINT(2)        Step 0
      DIMENSION IDES(500,12),ITV(6),IPOS(6),IDUMP(6)
      OPEN(12,FILE='SOLUT')
      OPEN(14,FILE='COMPLT')
  901 DO 20 M0=1,10
      DO 21 M1=M0+1,11
      DO 22 M2=M1+1,12                                              Step 1
   22 INS(M0,M1,M2)=0
   21 CONTINUE
   20 CONTINUE
C
      DO 1 I1=1,10
      I5=I1+10
      READ(12,1000,END=900)(II(I2,I1),I2=1,6),(II(I3,I5),I3=1,6)
    5 DO 2 I2=1,4
      DO 3 I3=I2+1,5
      DO 4 I4=I3+1,6                                                Step 2
      INS(II(I2,I1),II(I3,I1),II(I4,I1))=
     * INS(II(I2,I1),II(I3,I1),II(I4,I1))+1
    4 INS(II(I2,I5),II(I3,I5),II(I4,I5))=
     * INS(II(I2,I5),II(I3,I5),II(I4,I5))+1
    3 CONTINUE
    2 CONTINUE
    1 CONTINUE
C
      NUM=0
      NUM1=0
      DO 11 L1=1,7
      DO 12 L2=L1+1,8
      DO 13 L3=L2+1,9
      DO 14 L4=L3+1,10
      DO 15 L5=L4+1,11
      DO 16 L6=L5+1,12
      NUM1=NUM1+1
      IF(NUM1.EQ.463)GO TO 17
      NUM=NUM+1                                                     Step 3
      ISTOR(1,NUM,1)=L1
      ISTOR(1,NUM,2)=L2
      ISTOR(1,NUM,3)=L3
      ISTOR(1,NUM,4)=L4
      ISTOR(1,NUM,5)=L5
      ISTOR(1,NUM,6)=L6
      CALL TSTP21(NUM)
   16 CONTINUE
   15 CONTINUE
   14 CONTINUE
   13 CONTINUE
   12 CONTINUE
   11 CONTINUE
C
```

```
17    LINT(1)=ISTOR(1,1,2)
      LINT(2)=ISTOR(1,1,3)
      IFLAG=1
      NDES=0
      NMB=6
      NMIND=44
      DO 51 N1=1,NUM-11
         CALL PUTIN(N1)
         LOOP(1)=N1
      DO 52 N2=N1+1,NUM-10
         CALL TSTP22(N2,1,IFLAG)
         GO TO (91,52,81),IFLAG
 91      CALL PUTIN(N2)
         LOOP(2)=N2
      DO 53 N3=N2+1,NUM-9
         CALL TSTP22(N3,2,IFLAG)
         GO TO (92,53,82),IFLAG
 92      CALL PUTIN(N3)
         LOOP(3)=N3
      DO 54 N4=N3+1,NUM-8
         CALL TSTP22(N4,3,IFLAG)
         GO TO (93,54,83),IFLAG
 93      CALL PUTIN(N4)
         LOOP(4)=N4
      DO 55 N5=N4+1,NUM-7
         CALL TSTP22(N5,4,IFLAG)
         GO TO (94,55,84),IFLAG
 94      CALL PUTIN(N5)
         LOOP(5)=N5
      DO 56 N6=N5+1,NUM-6
         CALL TSTP22(N6,5,IFLAG)
         GO TO (95,56,85),IFLAG
 95      CALL PUTIN(N6)
         LOOP(6)=N6
      DO 57 N7=N6+1,NUM-5
         CALL TSTP22(N7,6,IFLAG)
         GO TO (96,57,86),IFLAG
 96      CALL PUTIN(N7)
         LOOP(7)=N7
      DO 58 N8=N7+1,NUM-4
         CALL TSTP22(N8,7,IFLAG)
         GO TO (97,58,87),IFLAG
 97      CALL PUTIN(N8)
         LOOP(8)=N8
      DO 59 N9=N8+1,NUM-3
         CALL TSTP22(N9,8,IFLAG)
         GO TO (98,59,88),IFLAG
 98      CALL PUTIN(N9)
         LOOP(9)=N9
      DO 60 N10=N9+1,NUM-2
         CALL TSTP22(N10,9,IFLAG)
         GO TO (99,60,89),IFLAG
 99      CALL PUTIN(N10)
         LOOP(10)=N10
      DO 61 N11=N10+1,NUM-1
         CALL TSTP22(N11,10,IFLAG)
         GO TO (100,61,90),IFLAG
100      CALL PUTIN(N11)
         LOOP(11)=N11
      DO 62 N12=N11+1,NUM
         CALL TSTP22(N12,11,IFLAG)
         GO TO (101,62,910),IFLAG
101      LOOP(12)=N12
C
```

Step 4

Step 5

```
      NDES=NDES+1
      DO 120 M1=1,12
  120 IDES(NDES,M1)=LOOP(M1)
      IF(NDES.NE.1)GO TO 300
      NAUT=0
      READ(12,102)IPOS(1)
  203 IV1=IPOS(1)
      DO 202 IV2=1,IV1
  202 READ(12,102)(IAUT(NAUT+IV2,IV3),IV3=1,6)
      READ(12,102)(IPOS(IV4),IV4=1,6)
      IF(IPOS(1).EQ.0)GO TO 201
      IF(IPOS(2).EQ.0)GO TO 203
  210 DO 205 IV5=1,IV1
      NAUT=NAUT+1
      IF(NAUT.GT.1000)GO TO 851
      DO 206 IV6=1,6
  206 IAUT(NAUT,IV6+6)=IPOS(IV6)
  205 CONTINUE
      READ(12,102)(IPOS(IV7),IV7=1,6)
      IF(IPOS(1).EQ.0)GO TO 201
      IF(IPOS(2).EQ.0)GO TO 203
      DO 208 IV8=1,IV1
      DO 209 IV9=1,6
  209 IAUT(NAUT+IV8,IV9)=IAUT(NAUT+1-IV8,IV9)
  208 CONTINUE
      GO TO 210
C
  201 IFLAG=0
      DO 211 IV11=1,NAUT
      DO 230 IV30=1,12
  230 IF(IAUT(IV11,IV30).NE.IV30)GO TO 231
      IDENTY=IV11
      GO TO 211
 231  DO 213 IV13=1,10
      DO 214 IV14=1,6
  214 IDUMP(IV14)=IAUT(IV11,II(IV14,IV13))
      CALL TAUT(IFLAG,IDUMP)
      IF(IFLAG.EQ.0)GO TO 850
  213 CONTINUE
  211 CONTINUE
      GO TO 400
C
  300 IFD1=1
      IFD2=NDES-1
      IFD3=0
  310 DO 302 M2=IFD1,IFD2
      DO 303 M3=1,NAUT
      IF(M3.EQ.IDENTY)GO TO 303
      DO 304 M4=1,12
      DO 305 M5=1,6
  305 ITV(M5)=IAUT(M3,ISTOR(1,IDES(NDES,M4),M5))
      DO 306 M6=1,12
      INDE=1
      DO 307 M7=1,6
      DO 308 M8=1,6
  308 IF(ITV(M7).EQ.ISTOR(INDE,IDES(M2,M6),M8))GO TO 307
      IF(INDE.EQ.2)GO TO 306
      IF(M7.GT.1)GO TO 306
      INDE=2
  307 CONTINUE
      GO TO 304
  306 CONTINUE
      GO TO 303
  304 CONTINUE
      IF(IFD3.EQ.0)GO TO 309
      WRITE(14,103)(IAUT(M3,M9),M9=1,12)
  303 CONTINUE
  302 CONTINUE
```

Step 6

Step 7

Step 8

Step 9

```
      IF(IFD3.EQ.1)GO TO 62                                Step 10
  400 CALL BLOCK(NMB,NMIND)
      IFD1=NDES
      IFD2=NDES
      IFD3=1                                                Step 11
      GO TO 310
  309 NDES=NDES-1
C
C
 62      CONTINUE
 910  CALL PUTOUT(N11)
 61      CONTINUE
  90  CALL PUTOUT(N10)
 60      CONTINUE
  89  CALL PUTOUT(N9)
 59      CONTINUE
  88  CALL PUTOUT(N8)
 58      CONTINUE
  87  CALL PUTOUT(N7)                               Part of Step 5
 57      CONTINUE
  86  CALL PUTOUT(N6)
 56      CONTINUE
  85  CALL PUTOUT(N5)
 55      CONTINUE
  84  CALL PUTOUT(N4)
 54      CONTINUE
  83  CALL PUTOUT(N3)
 53      CONTINUE
  82  CALL PUTOUT(N2)
 52      CONTINUE
  81  CALL PUTOUT(N1)
 51      CONTINUE
C
      IF(NDES.NE.0)GO TO 901
  893 READ(12,102) IPOS(1)                                 Step 12
      IF(IPOS(1).EQ.0)GO TO 901
      GO TO 893
C
  851 WRITE(14,105)
  850 WRITE(14,104)(IAUT(IV11,MMM),MMM=1,12)
 1000 FORMAT(1X,6(I3),5X,6(I3))
  102 FORMAT(1X,6(I3))                                     Stop
  103 FORMAT(1X,12(I3))
  104 FORMAT(1X,'THIS DOES NOT HOLD',/1X,12(I3))
  105 FORMAT(1X,'TO MANY AUTOMORPHISM')
  900 CLOSE(12)
      CLOSE(14)
      STOP
      END
```

TSTP21( )

```
      SUBROUTINE TSTP21(NUM)
      COMMON/BLOCK1/NQUAD(12,12,12,12)
      COMMON/BLOCK2/INS(12,12,12),ISTOR(2,500,6)
      COMMON/BLOCK3/IAUT(1000,12),II(7,44),LOOP(12),LINT(2)
      I=1
   22 DO 1 I1=1,4
      DO 2 I2=I1+1,5
      DO 3 I3=I2+1,6
      GO TO (3,3,3,4),INS(ISTOR(I,NUM,I1),ISTOR(I,NUM,I2),
     * ISTOR(I,NUM,I3))
    3 CONTINUE
  2   CONTINUE
  1   CONTINUE
```

```
      GO TO 5
    4 NUM=NUM-1
      RETURN
C
    5 DO 11 J1=1,10
      IDUM=0
      DO 12 J2=1,6
      DO 13 J3=1,6
      IF(ISTOR(I,NUM,J2).LT.II(J3,J1))GO TO 12
      IF(ISTOR(I,NUM,J2).GT.II(J3,J1))GO TO 13
      IDUM=IDUM+1
      GO TO 12
   13 CONTINUE
   12 CONTINUE
      IF(IDUM.GE.5.OR.IDUM.LE.1)GO TO 4
   11 CONTINUE
      IF(I.EQ.2)GO TO 23
C
      I=2
      IND1=0
      IND2=0
      IND3=0
   20 IND1=IND1+1
   21 IND2=IND2+1
      IF(IND2.EQ.13)GO TO 22
      IF(IND1.EQ.7)GO TO 24
      IF(ISTOR(1,NUM,IND1).EQ.IND2)GO TO 20
   24 IND3=IND3+1
      ISTOR(I,NUM,IND3)=IND2
      GO TO 21
   23 RETURN
      END
```

TSTP22( )

```
      SUBROUTINE TSTP22(IND,NLP,IFLAG)
      COMMON/BLOCK1/NQUAD(12,12,12,12)
      COMMON/BLOCK2/INS(12,12,12),ISTOR(2,500,6)
      COMMON/BLOCK3/IAUT(1000,12),II(7,44),LOOP(12),LINT(2)
      IFLAG=1
      ITRY1=10*LINT(1)+LINT(2)
      ITRY2=10*ISTOR(1,IND,2)+ISTOR(1,IND,3)
      IF(ITRY1.LT.ITRY2)GO TO 14
      IF(ITRY1.GT.ITRY2)GO TO 4
      DO 1 J1=1,4
      DO 2 J2=J1+1,5
      DO 3 J3=J2+1,6
      IF(INS(ISTOR(1,IND,J1),ISTOR(1,IND,J2),ISTOR(1,IND,J3)).EQ.4)
     * GO TO 4
    3 IF(INS(ISTOR(2,IND,J1),ISTOR(2,IND,J2),ISTOR(2,IND,J3)).EQ.4)
     * GO TO 4
    2 CONTINUE
    1 CONTINUE
      GO TO 5
    4 IFLAG=2
      RETURN
C
    5 DO 11 I1=1,NLP
      IDUM=0
      DO 12 I2=1,6
      DO 13 I3=1,6
      IF(ISTOR(1,IND,I2).LT.ISTOR(1,LOOP(I1),I3))GO TO 12
      IF(ISTOR(1,IND,I2).GT.ISTOR(1,LOOP(I1),I3))GO TO 13
      IDUM=IDUM+1
      GO TO 12
```

```
13   CONTINUE
12   CONTINUE
     IF(IDUM.EQ.5.OR.IDUM.EQ.1)GO TO 4
11   CONTINUE
     RETURN
14   IFLAG=3
     RETURN
     END
```

PUTIN( )

```
      SUBROUTINE PUTIN(L)
      COMMON/BLOCK1/NQUAD(12,12,12,12)
      COMMON/BLOCK2/INS(12,12,12),ISTOR(2,500,6)
      COMMON/BLOCK3/IAUT(1000,12),II(7,44),LOOP(12),LINT(2)
      DO 1 J1=1,4
      DO 2 J2=J1+1,5
      DO 3 J3=J2+1,6
      INS(ISTOR(1,L,J1),ISTOR(1,L,J2),ISTOR(1,L,J3))=
     * INS(ISTOR(1,L,J1),ISTOR(1,L,J2),ISTOR(1,L,J3))+1
 3    INS(ISTOR(2,L,J1),ISTOR(2,L,J2),ISTOR(2,L,J3))=
     * INS(ISTOR(2,L,J1),ISTOR(2,L,J2),ISTOR(2,L,J3))+1
 2    CONTINUE
 1    CONTINUE
      DO 4 J4=LINT(1),6
      DO 5 J5=J4+1,6
 5    IF(INS(1,J4,J5).NE.4)GO TO 6
 4    CONTINUE
      RETURN
 6    LINT(1)=J4
      LINT(2)=J5
      RETURN
      END
```

PUTOUT( )

```
      SUBROUTINE PUTOUT(L)
      COMMON/BLOCK1/NQUAD(12,12,12,12)
      COMMON/BLOCK2/INS(12,12,12),ISTOR(2,500,6)
      COMMON/BLOCK3/IAUT(1000,12),II(7,44),LOOP(12),LINT(2)
      DO 1 J1=1,4
      DO 2 J2=J1+1,5
      DO 3 J3=J2+1,6
      INS(ISTOR(1,L,J1),ISTOR(1,L,J2),ISTOR(1,L,J3))=
     * INS(ISTOR(1,L,J1),ISTOR(1,L,J2),ISTOR(1,L,J3))-1
 3    INS(ISTOR(2,L,J1),ISTOR(2,L,J2),ISTOR(2,L,J3))=
     * INS(ISTOR(2,L,J1),ISTOR(2,L,J2),ISTOR(2,L,J3))-1
 2    CONTINUE
 1    CONTINUE
      DO 4 J4=2,LINT(1)
      DO 5 J5=J4+1,6
 5    IF(INS(1,J4,J5).NE.4)GO TO 6
 4    CONTINUE
      RETURN
 6    LINT(1)=J4
      LINT(2)=J5
      RETURN
      END
```

BLOCK( )

```
      SUBROUTINE BLOCK(NMB,NMIND)
      COMMON/BLOCK1/NQUAD(12,12,12,12)
      COMMON/BLOCK2/INS(12,12,12),ISTOR(2,500,6)
      COMMON/BLOCK3/IAUT(1000,12),II(7,44),LOOP(12),LINT(2)
      DIMENSION NN(8)
      DO 33 IN3=1,9
      DO 34 IN4=IN3+1,10
      DO 35 IN5=IN4+1,11
      DO 36 IN6=IN5+1,12
   36 NQUAD(IN3,IN4,IN5,IN6)=0
   35 CONTINUE
   34 CONTINUE
   33 CONTINUE
      DO 31 IN1=1,12
      DO 32 IN2=1,6
      II(IN2,IN1+20)=ISTOR(1,LOOP(IN1),IN2)
   32 II(IN2,IN1+32)=ISTOR(2,LOOP(IN1),IN2)
   31 CONTINUE
C
      DO 22 KX=1,8
   22 NN(KX)=0
      DO 8 LL=1,NMIND
      DO 1 I=1,NMIND
      N=0
      DO 2 J=1,NMB
      DO 3 K=1,NMB
      IF(II(K,LL).EQ.II(J,I))N=N+1
    3 CONTINUE
    2 CONTINUE
      MNINB=NMB+1
      DO 4 L=1,MNINB
      JJ=L-1
      IF(N.EQ.JJ)GO TO 1
    4 CONTINUE
    1 NN(L)=NN(L)+1
      IF(MNINB.EQ.7)GO TO 25
      IF(NN(1).EQ.1.AND.NN(4).EQ.5)II(6,LL)=1
      IF(NN(2).EQ.3.AND.NN(4).EQ.6)II(6,LL)=2
      IF(NN(2).EQ.2.AND.NN(5).EQ.1)II(6,LL)=3
      IF(NN(3).NE.15.AND.NN(3).NE.12)II(6,LL)=4
      GO TO 30
   25 IF(NN(3).EQ.9.AND.NN(4).EQ.24)II(7,LL)=1
      IF(NN(3).EQ.5.AND.NN(4).EQ.30)II(7,LL)=2
      IF(NN(3).NE.9.AND.NN(3).NE.5)II(7,LL)=3
   30 DO 9 MM=1,MNINB
    9 NN(MM)=0
    8 CONTINUE
      IF(MNINB.EQ.7)GO TO 40
      RETURN
   40 WRITE(14,50)
      DO 12 MMM=1,10
      MMN=MMM+10
   12 WRITE(14,400)(II(J,MMM),J=1,7),(II(I,MMN),I=1,7)
      DO 13 MM2=21,32
      MM3=MM2+12
   13 WRITE(14,400)(II(J,MM2),J=1,7),(II(I,MM3),I=1,7)
C
      WRITE(14,500)
      DO 61 L1=1,44
      DO 62 L2=1,3
      DO 63 L3=L2+1,4
      DO 64 L4=L3+1,5
      DO 65 L5=L4+1,6
```

```
 65   NQUAD(II(L2,L1),II(L3,L1),II(L4,L1),II(L5,L1))
     * =NQUAD(II(L2,L1),II(L3,L1),II(L4,L1),II(L5,L1))+1
 64   CONTINUE
 63   CONTINUE
 62   CONTINUE
 61   CONTINUE
C
      DO 66 L6=1,9
      DO 67 L7=L6+1,10
      DO 68 L8=L7+1,11
      DO 69 L9=L8+1,12
      GO TO (70,69,69,71,72),NQUAD(L6,L7,L8,L9)+1
 70   WRITE(14,501)L6,L7,L8,L9,NQUAD(L6,L7,L8,L9)
      GO TO 69
 71   WRITE(14,502)L6,L7,L8,L9,NQUAD(L6,L7,L8,L9)
      GO TO 69
 72   WRITE(14,503)L6,L7,L8,L9,NQUAD(L6,L7,L8,L9)
 69   CONTINUE
 68   CONTINUE
 67   CONTINUE
 66   CONTINUE
      RETURN
500   FORMAT('1','NUM OF QUADS IS',/)
501   FORMAT(1X,4(I3),3X,I3)
502   FORMAT(1X,22X,4(I3),3X,I3)
503   FORMAT(1X,44X,4(I3),3X,I3)
 50   FORMAT('1','THE 3-(12,6,4) DESIGN IS'/1X,26('-'))
 400  FORMAT('0',6I3,2X,I2,10X,6I3,2X,I2)
      END
```

TAUT( )

```
      SUBROUTINE TAUT(IFLAG,IDUM)
      COMMON/BLOCK1/NQUAD(12,12,12,12)
      COMMON/BLOCK2/INS(12,12,12),ISTOR(2,500,6)
      COMMON/BLOCK3/IAUT(1000,12),II(7,44),LOOP(12),LINT(2)
      DIMENSION IDUM(6)
      IFLAG=1
      DO 1 L1=1,20
      DO 2 L2=1,6
      DO 3 L3=1,6
 3    IF(IDUM(L2).EQ.II(L3,L1))GO TO 2
      GO TO 1
 2    CONTINUE
      RETURN
 1    CONTINUE
      IFLAG=0
      RETURN
      END
```

Section II·4: The Resulting Designs

The two programs MAIN and FINISH were run consecutively so that the often lengthy intermediate data file SOLUT could be destroyed as soon as possible. A policy of eliminating isomorphs immediately after a case is completed was also adopted to lessen storage requirements. It was also found that the manually determined non-equivalent sets of pairs for cases PV and PVI would remain non-equivalent (under the conditions stated) even once the entire pairing was known. Thus to avoid many unnecessary comparisons, each non-equivalent pair set for these two cases was run separately. As no such decomposition exists for PVII and PVIII, much longer run times were recorded for these cases.

The designs were labelled with the case name followed by a number corresponding to their relative order of discovery by the program. As each case from PI through to PVIII was completed, any isomorphs were discovered and eliminated using information provided by their constituent quadruples. In particular there were sixteen solutions to the equation of Lemma 1, so the designs were automatically partitioned into sixteen equivalence classes. Then by considering the points composing the q_0, q_3 and q_4 quadruples of two designs from the same class, it was a relatively simple procedure to find a permutation relating them or to prove that such a permutation did not exist. In this way the 869 designs created were reduced to just 34 non-isomorphic 3-(12,6,4) designs.

The following table summarises the results from the various runs which were made using a Prime 750 computer.

Case	# of Designs Created	# of Non-isomorphic Designs Left
PI	4	4
PII	18	11
PIII	45	7
PIV	4	2
PV·1	0	0
PV·2	5	3
PV·3	0	0
PV·4	4	0
PV·5	22	4
PV·6	11	0
PV·7	17	2
PVI·1	1	0
PVI·2	0	0
PVI·3	58	0
PVI·4	70	0
PVI·5	145	0
PVI·6	20	0
PVII	38	1
PVIII	407	0
TOTAL	869	34

At this point it is useful to recall that no permutations swapping the fixed pairings with the variable pairings of section A were utilised. This lack of interchange between block (i) and block (ii) coupled with the otherwise exhaustive use of permutations which fix these two blocks, indicates that most designs should have been produced in isomorphic pairs. More specifically a table mapping the structure of the fixed against the variable pairing should be commutative. The following table constructed from the resulting designs is consistent with this requirement and thus provides some evidence that the program and its data are correct.

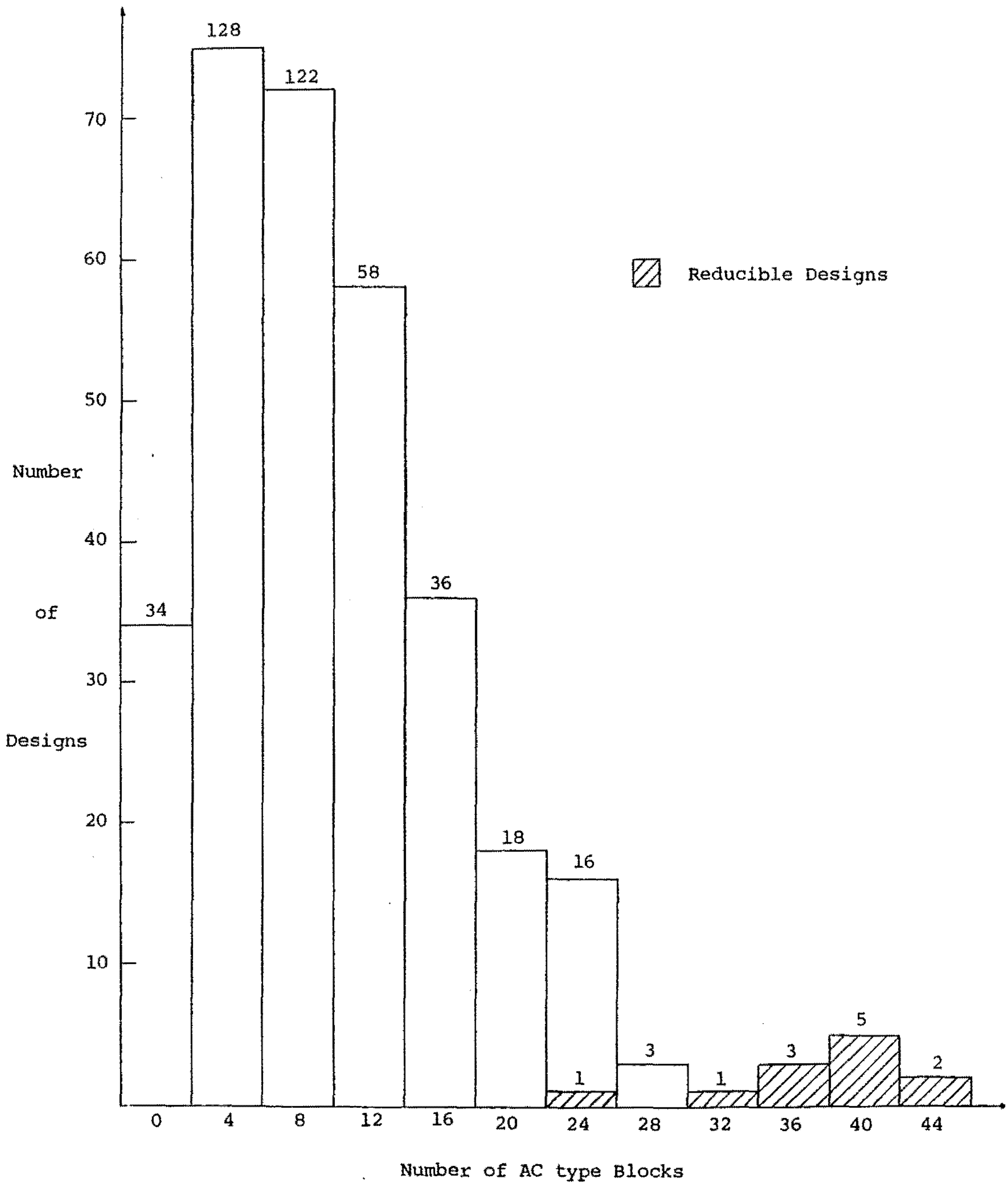
Case i.e. Fixed pairing	Variable Pairing Structure is equivalent to; PI	PII	PIII	PIV	PV	PVI	PVII	PVIII
PI	0	0	0	0	0	0	0	4
PII	0	0	1	0	2	3	0	12
PIII	0	1	10	0	5	20	2	7
PIV	0	0	0	0	0	1	0	3
PV	0	2	5	0	2	23	0	27
PVI	0	3	20	1	23	131	6	110
PVII	0	0	2	0	0	6	2	28
PVIII	4	12	7	3	27	110	28	216

From this table it is evident that the 254 designs forming the lower triangle can be automatically discarded as each of these is an isomorphic copy of a design in the upper triangle. Indeed considerable computational effort would be saved by eliminating variable pairings of section A which were known to have been examined in previous runs. This objective can be simply achieved by adjusting the code in **PAIRS( )** which performs *Step 5* of the procedure, so that any pairing with an equivalent structure to an earlier run case is rejected. This was not done as the author wished to maintain the independence between runs so that the correct functioning of the program would be demonstrated.

To conclude this chapter the non-isomorphic 2-(11,5,4) designs embedded in each of the 34 non-isomorphic 3-(12,6,4) designs were determined. Each 3-design's point orbits were found by producing all of its automorphisms. This was not overly difficult as a great deal of information was provided by the designs constituent quadruples. The result was a further 228 2-designs to be added to the 3509 designs already determined in Section I. Thus it is the author's assertion that in the absence of repeated blocks there are exactly 426 non-isomorphic 3-(12,6,4) designs whose point restrictions contain exactly 3737 non-isomorphic 2-(11,5,4) designs.

The following bar graph gives a breakdown of the 426 non-isomorphic 3-(12,6,4) designs by categorizing them according to the number of the AC and B type blocks that they contain. Where a category contains reducible designs the number of these has also been given. The relatively low proportion of reducible to non-reducible designs is clearly evident and shows the very restrictive nature of this property.

Graph of 3-(12,6,4) Designs without Repeated Blocks



It is again convenient to delay the presentation of the designs determined in this section. A Catalogue and representative copy of each design is available in Part II, Section III. The three transitive designs PII-N6, PIII-N30, and a restriction on the point 5 of PVII-N18 are discussed in Chapter 5.

CHAPTER 4

THE 3-(12,6,4) AND 2-(11,5,4) DESIGNS WITH REPEATED BLOCKS

In the preceding chapter the designs were created subject to the non-occurrence of repeated blocks. Thus a knowledge of the designs developed there is of little use in determining or restricting the number of designs in this last category. However, some insight into a possible solution may be gained from the method used in Chapter two, where the reducible designs with repeated blocks were enumerated. This view is reinforced by the fact that there is no analytical advantage in considering the R(2 0 0 40 0 0 2) type block of the 3-(12,6,4) design, over the corresponding R(0 0 20 0 0 2) type block of the 2-(11,5,4) design.

Two advantages in dealing with the smaller 2-designs are that relatively less storage space and computational effort will be required, and that the previously developed program may possibly be adapted with only minor structural changes. Upon determining all the non-isomorphic 2-(11,5,4) designs the extension and restriction processes can be used to generate all the non-isomorphic 3-(12,6,4) designs. The additional effort required to do this phase is comparatively small and does not affect the viability of the overall approach. As the 2-designs must pack exhaustively into 3-designs with no new 2-designs being discovered, this secondary exercise also provides a useful check on the construction process.

Section I: A Skeleton for the Designs with Repeated Blocks

Without loss of generality let [1 2 3 4 5] be an R(0 0 20 0 0 2) type block of a 2-(11,5,4) design. This block must therefore occur twice and intersect each of the remaining twenty blocks in exactly two points. To achieve pairwise balance each of the $\binom{5}{2} = 10$ pairs from the repeated blocks must occur twice as a two-point intersection. If • represents a point to be chosen from {6,7,8,9,10,11} then the skeleton must be of the form

1 2 3 4 5		2 3 • • •
1 2 3 4 5		2 4 • • •
1 2 • • •	(i)	2 4 • • •
1 2 • • •	(ii)	2 5 • • •
1 3 • • •	(iii)	2 5 • • •
1 3 • • •	(iv)	3 4 • • •
1 4 • • •	(v)	3 4 • • •
1 4 • • •		3 5 • • •
1 5 • • •		3 5 • • •
1 5 • • •		4 5 • • •
2 3 • • •		4 5 • • • .

It is important to note here the skeleton's highly regular form. A consequence of this is that a large number of isomorphic designs will be created if care is not taken to eliminate equivalent partially complete designs as soon as they arise. Thus the early elimination of equivalent configurations will be foremost in reducing the amount of computational effort required for the final balancing stage of the construction process. Therefore it is proposed that the blocks (i) to (iv) be systematically completed to produce a minimal set of all possible non-equivalent configurations for the top six blocks of the 2-(11,5,4) design.

Firstly, the six • points available to complete block (i) are equivalent and so the triple (6,7,8) will be arbitrarily assigned to this block. This choice now forces the • points into the two equivalence classes, {6,7,8} and {9,10,11}. So by choosing zero, one, two, or three points from one class and the rest from the other, block (ii) can be completed in four non-equivalent ways. The representatives for these four cases are,

I	II	III	IV
1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5
1 2 3 4 5	1 2 3 4 5	1 2 3 4 5	1 2 3 4 5
1 2 6 7 8	1 2 6 7 8	1 2 6 7 8	1 2 6 7 8
1 2 6 7 8	1 2 6 7 9	1 2 6 9 10	1 2 9 10 11 .

Before moving to the selection of points for the next two incomplete blocks, it is convenient to show that two configurations of points cannot occur within the blocks of any 2-(11,5,4) design.

<u>LEMMA 1</u>: A 2-(11,5,4) design cannot contain any set of three blocks exhibiting an equivalent configuration to

$$[1\ 2\ 6\ 7\ *],\ [1\ 2\ 6\ 7\ *],\ [1\ 6\ 7\ *\ *]\ ,$$

where the *'s represent any other points.

<u>Proof</u>: Suppose a 2-(11,5,4) design contains the given set of blocks, and let a_i be the number of remaining blocks containing exactly i of the points $\{1,2,6,7\}$. The block intersection numbers for a C type block require that $a_4 = 0$. Then counting blocks we have

$$22 - 3 = 19 = a_0 + a_1 + a_2 + a_3 \,. \tag{1}$$

Counting single-point incidences for these four points

$$4r - 11 = 29 = a_1 + 2a_2 + 3a_3 \,. \tag{2}$$

Counting the pair incidences for these four points

$$\binom{4}{2}\lambda - \left[2\binom{4}{2} + \binom{3}{2}\right] = 9 = a_2 + 3a_3 \,. \tag{3}$$

Eliminating a_1 and a_2 from these equations produces

$$a_0 + a_3 = -1 \,.$$

But $a_0, a_3 \geqslant 0$; therefore this configuration of points cannot occur. □

<u>LEMMA 2</u>: A 2-(11,5,4) design cannot contain any set of three blocks exhibiting an equivalent configuration to

$$[1\ 2\ 6\ 7\ 8],\ [1\ 2\ 6\ 7\ 9],\ [1\ 6\ 8\ 9\ *]\ ,$$

where * represents any other point.

Proof: Suppose a 2-(11,5,4) design contains the given set of blocks. From Lemma 1 no three of the points {1,2,6,7} can occur together in any of the remaining blocks of the design. For pairwise balance for these four points

$\binom{4}{2}\lambda - \left[2\binom{4}{2} + 1\right] = 24 - 13 = 11$ of the remaining blocks must contain exactly two of them, while $4r - (2.11 + 10) = 40 - 32 = 8$ i.e. all the rest, must contain a singleton from this set.

If a_1 and a_2 are the number of times the point 8 must occur with a pair and singleton from {1,2,6,7} respectively, then pairwise balance for 8 is obtained if

$$4.4 - 6 = 10 = a_1 + 2a_2 .$$

Also

$$r - 2 = 8 = a_1 + a_2 .$$

Thus $a_1 = 6$ and $a_2 = 2$. But by symmetry the point 9 must also occur six times with a singleton from this set. As there are just eight singletons available throughout the remaining blocks of the design, the pair (8,9) must occur at least four times. Its presence already in the three given blocks thus violates the $\lambda = 4$ condition and hence this configuration cannot occur. □

Section II: Completing Blocks (iii) and (iv)

Firstly, consider the choice of points for blocks (iii) and (iv) of Case I. As blocks (i) and (ii) are identical in Case I, all other blocks of the design must intersect them in exactly two points. Thus blocks (iii) and (iv) must contain one point from {6,7,8} and two from {9,10,11}. As the points within each set are equivalent at this stage, only one non-equivalent triple is available to complete block (iii). The choice of (6,9,10) as this triple, now puts the points into the equivalence classes, {6}, {7,8}, {9,10}, and {11}. These classes and the required occurrences of one point from {6,7,8} and two from {9,10,11}, can be used to restrict block (iv) to four non-equivalent triples, i.e., Case I

Triple for block (iii)	Triples for block (iv)
6 9 10	6 9 10 6 9 11 7 9 10 7 9 11 .

Notice that in choosing between two equivalent triples a policy of retaining the one with the smallest first point, or if these are equal, the smallest second point, etc., will be adopted. Although not necessary, this practice eliminates any randomness in the selection process, so the resulting configurations will form a standard reference set which can be readily checked or recalculated if required. The algorithm for the final balancing of the skeleton will also produce results consistent with this approach.

An incidence count of the points forming these four configurations will quickly show that they are different. This is the exception rather than the rule as permutations interchanging block (iii) with block (iv), or blocks (i) and (ii) with blocks (iii) and (iv) have not been considered. It is also important to note that with the completion of Case I, any occurrence of a repeated triple in blocks (iii) and (iv) of the subsequent cases, may be automatically discarded as having already been obtained.

The block by block approach used to complete blocks (iii) and (iv) of Case I will now be applied to Case II. Of the $\binom{6}{3} = 20$ triples available to these blocks, 6 7 8, 6 7 9, 6 7 10, and 6 7 11 can be eliminated using Lemma 1, while 6 8 9 and 7 8 9 are removed by Lemma 2. This leaves the fourteen legitimate triples,

Case II

6 8 10	6 8 11	6 9 10	6 9 11	6 10 11	7 8 10	7 8 11
7 9 10	7 9 11	7 10 11	8 9 10	8 9 11	8 10 11	9 10 11 .

The top four blocks of Case II put the six points into the equivalence classes {6,7}, {8,9}, and {10,11}. Using these and restricting our attention to the triples for block (iii), the fourteen possibilities produce the following four equivalence classes,

{6 8 10, 6 8 11, 6 9 10, 6 9 11, 7 8 10, 7 8 11, 7 9 10, 7 9 11}

{6 10 11, 7 10 11}

{8 9 10, 8 9 11}

{8 10 11, 9 10 11} .

In accordance with the previously stated rule, the first triple listed will represent the whole class.

The fourteen triples available to each subcase for the completion of block (iv) provide 56 configurations for scrutiny. These are to be examined under the six eliminating processes explained briefly below. Each process has been assigned a code to be used as a reference suffix.

A - Automatically discarded through equivalence with Case I.

I - Discarded as blocks are identical to a previous configuration.

L1 - Discarded using Lemma 1.

L2 - Discarded using Lemma 2.

P1 - Discarded as this configuration is equivalent to a previous one under a permutation which fixes its top four blocks.

P2 - Discarded as this configuration is equivalent to a previous one under a permutation which interchanges blocks (i) and (ii) with blocks (iii) and (iv).

Working systematically through the subcases resulted in a further fifteen non-equivalent configurations to be added to the four already obtained from Case I. A full list of the triples to complete blocks (iii) and (iv) of Case II appears below. The process used to eliminate a particular configuration is indicated by the suffix appended to the triple for block (iv).

Case II

Triple for block (iii)	Triples for block (iv)				
6 8 10	6 8 10-A	6 8 11-L1	6 9 10	6 9 11	6 10 11
	7 8 10-L2	7 8 11	7 9 10	7 9 11	7 10 11
	8 9 10	8 9 11	8 10 11	9 10 11	

6 10 11	6 8 10-I	6 8 11-P1	6 9 10-P1	6 9 11-P1	6 10 11-A
	7 8 10-P2	7 8 11-P1	7 9 10-P1	7 9 11-P1	7 10 11
	8 9 10	8 9 11-P1	8 10 11	9 10 11-P1 .	
8 9 10	6 8 10-I	6 8 11-P2	6 9 10-P1	6 9 11-P1	6 10 11-I
	7 8 10-P1	7 8 11-P1	7 9 10-P1	7 9 11-P1	7 10 11-P1
	8 9 10-A	8 9 11-P2	8 10 11-P2	9 10 11-P1 .	
8 10 11	6 8 10-I	6 8 11-P1	6 9 10-P2	6 9 11-P1	6 10 11-I
	7 8 10-P1	7 8 11-P1	7 9 10-P1	7 9 11-P1	7 10 11-P1
	8 9 10-I	8 9 11-P1	8 10 11-A	9 10 11 .	

The completion of these two cases is now used to restrict the possibilities available in Case III. Namely, repetition of these earlier cases is avoided if the triples for blocks (iii) and (iv) are required to intersect in fewer than two points. This and the previously described methods of elimination, were used to reduce Case III and subsequently Case IV to a set of new non-equivalent configurations to be added to those already obtained. The additional 19 configurations are composed of the following triples,

Case III

Triple for block (iii)	Triples for block (iv)
6 7 8	6 9 10 6 9 11 7 9 10 7 9 11 9 10 11 .
6 7 9	6 8 10 6 8 11 7 8 10 7 8 11 7 10 11 8 10 11 .
6 7 11	7 9 10 8 9 10 8 9 11 9 10 11 .
6 9 11	none remaining.
7 8 9	none remaining.
7 8 11	9 10 11 .
7 9 11	8 10 11 .

Case IV

Triple for block (iii)	Triples for block (iv)
6 7 8	9 10 11 .
6 7 9	8 10 11 .

Thus, there are just 38 non-equivalent ways to complete these four blocks of the given skeleton. It is now convenient to list the quartets of blocks forming these configurations and assign to each a reference label:-

D1	D2	D3	D4	D5
1 2 6 7 8	1 2 6 7 8	1 2 6 7 8	1 2 6 7 8	1 2 6 7 8
1 2 6 7 8	1 2 6 7 8	1 2 6 7 8	1 2 6 7 8	1 2 6 7 9
1 3 6 9 10	1 3 6 9 10	1 3 6 9 10	1 3 6 9 10	1 3 6 8 10
1 3 6 9 10	1 3 6 9 11	1 3 7 9 10	1 3 7 9 11	1 3 6 9 10

D6	D7	D8	D9	D10
1 2 6 7 8	1 2 6 7 8	1 2 6 7 8	1 2 6 7 8	1 2 6 7 8
1 2 6 7 9	1 2 6 7 9	1 2 6 7 9	1 2 6 7 9	1 2 6 7 9
1 3 6 8 10	1 3 6 8 10	1 3 6 8 10	1 3 6 8 10	1 3 6 8 10
1 3 6 9 11	1 3 6 10 11	1 3 7 8 11	1 3 7 9 10	1 3 7 9 11

D11	D12	D13	D14	D15
1 2 6 7 8	1 2 6 7 8	1 2 6 7 8	1 2 6 7 8	1 2 6 7 8
1 2 6 7 9	1 2 6 7 9	1 2 6 7 9	1 2 6 7 9	1 2 6 7 9
1 3 6 8 10	1 3 6 8 10	1 3 6 8 10	1 3 6 8 10	1 3 6 8 10
1 3 7 10 11	1 3 8 9 10	1 3 8 9 11	1 3 8 10 11	1 3 9 10 11

D16	D17	D18	D19	D20
1 2 6 7 8	1 2 6 7 8	1 2 6 7 8	1 2 6 7 8	1 2 6 7 8
1 2 6 7 9	1 2 6 7 9	1 2 6 7 9	1 2 6 7 9	1 2 6 9 10
1 3 6 10 11	1 3 6 10 11	1 3 6 10 11	1 3 8 10 11	1 3 6 7 8
1 3 7 10 11	1 3 8 9 10	1 3 8 10 11	1 3 9 10 11	1 3 6 9 10

D21	D22	D23	D24	D25
1 2 6 7 8	1 2 6 7 8	1 2 6 7 8	1 2 6 7 8	1 2 6 7 8
1 2 6 9 10	1 2 6 9 10	1 2 6 9 10	1 2 6 9 10	1 2 6 9 10
1 3 6 7 8	1 3 6 7 8	1 3 6 7 8	1 3 6 7 8	1 3 6 7 9
1 3 6 9 11	1 3 7 9 10	1 3 7 9 11	1 3 9 10 11	1 3 6 8 10

D26	D27	D28	29	30
1 2 6 7 8	1 2 6 7 8	1 2 6 7 8	1 2 6 7 8	1 2 6 7 8
1 2 6 9 10	1 2 6 9 10	1 2 6 9 10	1 2 6 9 10	1 2 6 9 10
1 3 6 7 9	1 3 6 7 9	1 3 6 7 9	1 3 6 7 9	1 3 6 7 9
1 3 6 8 11	1 3 7 8 10	1 3 7 8 11	1 3 7 10 11	1 3 8 10 11

31	32	33	34	35
1 2 6 7 8	1 2 6 7 8	1 2 6 7 8	1 2 6 7 8	1 2 6 7 8
1 2 6 9 10	1 2 6 9 10	1 2 6 9 10	1 2 6 9 10	1 2 6 9 10
1 3 6 7 11	1 3 6 7 11	1 3 6 7 11	1 3 6 7 11	1 3 7 8 11
1 3 7 9 10	1 3 8 9 10	1 3 8 9 11	1 3 9 10 11	1 3 9 10 11

36	37	38
1 2 6 7 8	1 2 6 7 8	1 2 6 7 8
1 2 6 9 10	1 2 9 10 11	1 2 9 10 11
1 3 7 9 11	1 3 6 7 8	1 3 6 7 9
1 3 8 10 11	1 3 9 10 11	1 3 8 10 11 .

Upon closer inspection of D24 with respect to the C type blocks [1 2 6 7 8] and [1 3 6 7 8], it can be shown that the skeleton with this configuration is impossible to balance. More specifically, the application of Lemma 1 to this pair of blocks requires that each of the four remaining incomplete blocks with 1, contain less than two points from $\{6,7,8\}$. A contradiction is provided by the fact that collectively these points must occur five more times within these four blocks to give pairwise balance. A similar approach can be used to discard configuration 37. Here, the application of Lemma 1 to the two pairs of C type blocks, eliminates all possible triples available to the incomplete blocks containing 1. This leaves 36 configurations for further study.

Section III: Completing the Remaining Blocks

It is perhaps tempting to continue to use the techniques of Section II to produce further minimal sets of non-equivalent configurations with two additional blocks determined at each stage. However, the existence of twenty possible triples for each incomplete block will inevitably produce a rapid build-up of partially complete designs. Whether attempted manually or by computer, the accompanying storage and processing problems make this approach unattractive.

As an alternative it is proposed that the 36 configurations be completed one at a time. A disadvantage of this method is that many isomorphs will be produced. Some are due to the reasonably regular pattern of the remaining 16 incomplete blocks, but the majority can be directly attributed to the symmetry exhibited by the skeleton. While the given method provides plenty of scope for a reduction in the amount of duplication through the first of these sources, it is ineffectual in dealing with that caused by the second. The resulting increase in the number of designs to be considered should not pose any problem, as the experience gained from Chapter 3 means that a large-scale elimination of isomorphs can now be achieved reasonably efficiently.

An advantage in considering each configuration individually is that the overall algorithm for its final balancing is very similar to that used to successfully complete the reducible designs with repeated blocks. Indeed the only major differences are an increase in the number of incomplete blocks and a change in the pattern of the pairs occupying them. While the first difference is easily accommodated by increasing the number of basic loops in the algorithm, the second has more serious consequences. Namely, in the reducible case the pairs occupying the incomplete blocks were all different, so for computational purposes the blocks and hence the loops are relatively independent. But for the present case the incomplete blocks occur in identical pairs. As the choice of triple for the second block of a pair can usually be restricted by information gained in filling the first, unnecessary repetition will be avoided if the loop structure is adjusted to exploit this. Therefore the algorithm will be restructured to contain eight pairs of loops running in tandem.

The underlying process of the selection of triples for the incomplete blocks remains unchanged. Unfortunately it must be used to determine an additional seven blocks, and the corresponding increase in workload for the program gives good cause for concern. This affect has been lessened by the inclusion of extra code to facilitate in the early detection and elimination of partially complete designs which cannot be balanced.

The primary source of reduction in workload is by removing 'illegal' triples from the selection process. The triples to be removed are predetermined and given as input data. They are calculated by applying Lemmas 1 and 2 to blocks (i) to (iv) of the configurations, and any occurrence of repeated blocks here is also capitalised on. The fact that these four blocks all intersect each of the next six incomplete blocks in exactly one point is also an advantage. Namely in restricting our attention to these six blocks, simple and efficient code can be introduced. The calculation of 'illegal' triples is demonstrated in the following analysis of D1 and D2.

D1	D2
1 2 6 7 8	1 2 6 7 8
1 2 6 7 8	1 2 6 7 8
1 3 6 9 10	1 3 6 9 10
1 3 6 9 10	1 3 6 9 11

In D1 all blocks are repeated and as such must intersect the other blocks of the design in exactly two points. The next six blocks have the forms [1 4 · · ·], [1 5 · · ·] and [2 3 · · ·], i.e., the previously stated single-point intersection with each of the blocks of D1. To avoid any three-point intersections with the repeated blocks, the triples available to these blocks must not contain any two points from {6,7,8} or {6,9,10}. Thus the triples 6 7 8, 6 7 9, 6 7 10, 6 7 11, 6 8 9, 6 8 10, 6 8 11, 6 9 10, 6 9 11, 6 10 11, 7 8 9, 7 8 10, 7 8 11, 7 9 10, 8 9 10 and 9 10 11 are termed illegal and disallowed by the program by rejecting their corresponding pointer values 1,2,3,4,5,6,7,8,9,10,11,12,13,14,17, and 20.

In a similar fashion to D1, the repeated blocks (i) and (ii) of D2 exclude the triples containing 6 7, 6 8, and 7 8. As blocks (iii) and (iv) are of type C, Lemma 1 removes all triples containing 6 9, while Lemma 2 disallows 6 10 11 and 9 10 11. Thus the twenty initial triples available to the next six blocks are reduced to the six possibilities, 7 9 10, 7 9 11, 7 10 11, 8 9 10, 8 9 11, 8 10 11. As before the program achieves this reduction by rejecting the illegal triples' pointer values, which have been supplied as input data.

A secondary source of elimination is provided by applying this process in reverse. Once the program has completed the second block of an initially identical pair, they are tested for a four- or five-point intersection. A positive result promotes the application of a computationally equivalent version of Lemmas 1 and 2 for blocks found to be C type, and the two-point intersection test if they are R type. Here again, only interactions between the first six blocks to be completed, and blocks (i) to (iv) of the configuration are considered. Any extension of this idea to other blocks of the design was thought to be unduly complicated, and the additional testing required at each stage could even be detrimental to the algorithm's overall efficiency.

As many of the configurations have permutations fixing them, a further saving in computational effort was obtained by using these to restrict the choice of triple available to block (v). The triples for this block are partitioned into equivalence classes, and the previously defined rule is used to select the representative triples. Here, strict adherence to this rule is essential to the correct performance of the development process. The actual elimination is achieved by the program's rejection of 'illegal' pointer values, which are predetermined and supplied as input data.

In their order of appearance, these three sources of elimination form *Steps 3* and *7*, *Step 8*, and *Step 2* respectively in the following procedure, from which the program used to balance the remainder of the design was derived.

<u>Balancing Procedure</u>

Step 0: Read all input data and initialize arrays and variables.

Step 1: Find the next triple for the next incomplete block which gives a legitimate pair count. If no triple is acceptable, go to *Step 14*.

Step 2: If this is the first incomplete block and the calculated triple is not one of the required non-equivalent choices, go to *Step 1*.

Step 3: If this is one of the first six incomplete blocks and the calculated triple is 'illegal', go to *Step 1*.

Step 4: The triple is acceptable so update the pair incidence array to include it. As this is the first block of an identical pair, set the starting value for the next block to this pointer value.

Step 5: Find the next triple for the next incomplete block which gives a legitimate pair count. If no triple is acceptable, go to *Step 15*.

Step 6: If this is not one of the first six incomplete blocks, go to *Step 9*.

Step 7: If the calculated triple is 'illegal', go to *Step 1*.

Step 8: If the recently calculated pair of blocks intersect in four or five points, then apply the Lemma 1 and 2 or R type tests respectively. If they fail, i.e. the pair is inconsistent with blocks (i) to (iv) of the configuration, then go to *Step 5*.

Step 9: The triple for the second block of the identical pair is acceptable so update the pair incidence array.

Step 10: Update the control variables to the start values for the first block of the next identical pair of incomplete blocks. If not all blocks have been balanced, go to *Step 1*.

Step 11: Increase the design count by one and store the result as a 22×5 array.

Step 12: Calculate and store the block types for each block of this design.

Step 13: Test this new design against all the previously retained designs under the permutations fixing the top six blocks of the configuration. If it is isomorphic to any of these, decrease the design count by one.

Step 14: If all possibilities have been tried, go to *Step 16*. Otherwise remove the triple from the latest completed block (the second block of an identical pair) by adjusting the pair incidence matrix, and alter the control variables so that the next triple for this block will be tested. Go to *Step 5*.

Step 15: Remove the triple from the latest completed block (the first block of an identical pair) by adjusting the pair incidence matrix. Alter the control variables so that the next triple for this block will be tested. Go to *Step 1*.

Step 16: For printing purposes write the resulting designs to the file 'DESIGN'. For processing, output a copy to the file 'SIMPLE'.

Step 17: Close all files. STOP.

It is convenient now to list a copy of the program with the code sectioned to correspond to the steps of the procedure that it executes. So that some idea of the program's progress is available, the seventh block of any newly determined design to be retained is output to the terminal. For identification purposes each retained design is assigned a reference number. The author has endeavoured to keep the code simple and acknowledges that the program is effectual rather than efficient.

```
      DIMENSION INC(20,3),IPAIR(11,11),IDES(22,5),I(9,2,2)
      DIMENSION NN(8),IOUT1(21),IOUT2(21)
      COMMON/BLOCK1/ISTORE(2000,23,6),IPERM(50,11)
      OPEN(12,FILE='DATA')
      OPEN(15,FILE='DESIGN')
      OPEN(16,FILE='SIMPLE')
      DO 1 J1=1,11
      DO 2 J2=1,11
    2 IPAIR(J1,J2)=0
    1 CONTINUE
      READ(12,100)A
      READ(12,*)(IDES(J,1),J=1,22)
      READ(12,*)(IDES(J,2),J=1,22)
      READ(12,*)(IDES(J,3),J=1,4)
      READ(12,*)(IDES(J,4),J=1,4)
      READ(12,*)(IDES(J,5),J=1,4)
      READ(12,*)(I(J,1,1),J=1,8)
      READ(12,*)(I(J,2,1),J=1,8)
      DO 3 I1=1,20
    3 READ(12,*)(INC(I1,J),J=1,3)
      READ(12,*)(IDES(5,J),J=3,5)                          Step 0
      READ(12,*)(IDES(6,J),J=3,5)
      READ(12,*)IOUT1(21)
      READ(12,*)(IOUT1(J),J=1,IOUT1(21))
      READ(12,*)IOUT2(21)
      READ(12,*)(IOUT2(J1),J1=1,IOUT2(21))
      READ(12,*)NPERM
      DO 8 J=1,NPERM
    8 READ(12,*)(IPERM(J,J1),J1=1,11)
C
      DO 4 I4=1,6
      DO 5 I5=1,4
      DO 6 I6=I5+1,5
    6 IPAIR(IDES(I4,I5),IDES(I4,I6))=IPAIR(IDES(I4,I5),IDES(I4,I6))+1
    5 CONTINUE
    4 CONTINUE
C
      I(1,1,2)=1
      ICOUNT=0
      NUM=1
    9 DO 11 I1=I(NUM,1,2),20
      DO 12 I2=1,3
      IF(IPAIR(I(NUM,1,1),INC(I1,I2)).EQ.4)GO TO 11        Step 1
   12 IF(IPAIR(I(NUM,2,1),INC(I1,I2)).EQ.4)GO TO 11
      IF(IPAIR(INC(I1,1),INC(I1,2)).EQ.4)GO TO 11
      IF(IPAIR(INC(I1,1),INC(I1,3)).EQ.4)GO TO 11
      IF(IPAIR(INC(I1,2),INC(I1,3)).EQ.4)GO TO 11
      IF(NUM.GT.3)GO TO 20
      IF(NUM.NE.1)GO TO 50                                 Step 2
      DO 51 J1=1,IOUT1(21)
   51 IF(I1.EQ.IOUT1(J1))GO TO 11
   50 DO 19 J9=1,IOUT2(21)                                 Step 3
   19 IF(I1.EQ.IOUT2(J9))GO TO 11
   20 DO 13 I3=1,3
      IPAIR(I(NUM,1,1),INC(I1,I3))=IPAIR(I(NUM,1,1),INC(I1,I3))+1
   13 IPAIR(I(NUM,2,1),INC(I1,I3))=IPAIR(I(NUM,2,1),INC(I1,I3))+1
      IPAIR(INC(I1,1),INC(I1,2))=IPAIR(INC(I1,1),INC(I1,2))+1
      IPAIR(INC(I1,1),INC(I1,3))=IPAIR(INC(I1,1),INC(I1,3))+1    Step 4
      IPAIR(INC(I1,2),INC(I1,3))=IPAIR(INC(I1,2),INC(I1,3))+1
      I(NUM,1,2)=I1
      I(NUM,2,2)=I1
C
```

```
   10 DO 14 I4=I(NUM,2,2),20
      DO 15 I5=1,3
      IF(IPAIR(I(NUM,1,1),INC(I4,I5)).EQ.4)GO TO 14            Step 5
   15 IF(IPAIR(I(NUM,2,1),INC(I4,I5)).EQ.4)GO TO 14
      IF(IPAIR(INC(I4,1),INC(I4,2)).EQ.4)GO TO 14
      IF(IPAIR(INC(I4,1),INC(I4,3)).EQ.4)GO TO 14
      IF(IPAIR(INC(I4,2),INC(I4,3)).EQ.4)GO TO 14
      IF(NUM.GT.3)GO TO 25                                     Step 6
      DO 33 J3=1,IOUT2(21)
   33 IF(I4.EQ.IOUT2(J3))GO TO 14                              Step 7
C
      IDUM=0
      DO 24 J4=1,3
      DO 26 J6=1,3
   26 IF(INC(I(NUM,1,2),J4).EQ.INC(I4,J6))IDUM=IDUM+1
   24 CONTINUE
      IF(IDUM.LT.2)GO TO 25
      IF(IDUM.EQ.3)GO TO 27
      DO 21 J1=3,6
      IDUM=0
      DO 22 J2=3,5
      DO 23 J3=1,3
      IF(IDES(J1,J2).EQ.INC(I(NUM,1,2),J3))IDUM=IDUM+1
   23 IF(IDES(J1,J2).EQ.INC(I4,J3))IDUM=IDUM+1                 Step 8
   22 CONTINUE
      IF(IDUM.GT.3)GO TO 14
   21 CONTINUE
      GO TO 25
   27 DO 28 J8=3,6
      IDUM=0
      DO 29 J9=3,5
      DO 30 J10=1,3
   30 IF(IDES(J8,J9).EQ.INC(I4,J10))IDUM=IDUM+1
   29 CONTINUE
      IF(IDUM.NE.1) GO TO 14
   28 CONTINUE
C
   25 DO 16 I6=1,3                                             Step 9
      IPAIR(I(NUM,1,1),INC(I4,I6))=IPAIR(I(NUM,1,1),INC(I4,I6))+1
   16 IPAIR(I(NUM,2,1),INC(I4,I6))=IPAIR(I(NUM,2,1),INC(I4,I6))+1
      IPAIR(INC(I4,1),INC(I4,2))=IPAIR(INC(I4,1),INC(I4,2))+1
      IPAIR(INC(I4,1),INC(I4,3))=IPAIR(INC(I4,1),INC(I4,3))+1
      IPAIR(INC(I4,2),INC(I4,3))=IPAIR(INC(I4,2),INC(I4,3))+1
      I(NUM,2,2)=I4
C
C                                                              Step 10
      NUM=NUM+1
      I(NUM,1,2)=1
      IF(NUM.NE.9)GO TO 9
      ICOUNT=ICOUNT+1
      DO 153 J3=1,6
      DO 154 J4=1,5
  154 ISTORE(ICOUNT,J3,J4)=IDES(J3,J4)
  153 CONTINUE                                                 Step 11
      DO 155 J5=1,22
      ISTORE(ICOUNT,J5,1)=IDES(J5,1)
  155 ISTORE(ICOUNT,J5,2)=IDES(J5,2)
      IDUM=6
      DO 156 J6=1,8
      IDUM=IDUM+2
      DO 157 J7=3,5
      ISTORE(ICOUNT,IDUM-1,J7)=INC(I(J6,1,2),J7-2)
  157 ISTORE(ICOUNT,IDUM,J7)=INC(I(J6,2,2),J7-2)
  156 CONTINUE
C
```

```
      DO 31 J=1,4
 31   ISTORE(ICOUNT,23,J)=0
      DO 32 KX=1,8
 32   NN(KX)=0
      DO 48 LL=1,22
      DO 41 II=1,22
      N=0
      DO 42 J=1,5
      DO 43 K=1,5
      IF(ISTORE(ICOUNT,LL,K).EQ.ISTORE(ICOUNT,II,J))N=N+1
 43   CONTINUE
 42   CONTINUE                                              Step 12
      DO 44 L=1,6
      JJ=L-1
      IF(N.EQ.JJ)GO TO 41
 44   CONTINUE
 41   NN(L)=NN(L)+1
      IF(NN(1).EQ.1.AND.NN(4).EQ.5)ISTORE(ICOUNT,LL,6)=1
      IF(NN(2).EQ.3.AND.NN(4).EQ.6)ISTORE(ICOUNT,LL,6)=2
      IF(NN(2).EQ.2.AND.NN(5).EQ.1)ISTORE(ICOUNT,LL,6)=3
      IF(NN(3).EQ.20.AND.NN(6).EQ.2)ISTORE(ICOUNT,LL,6)=4
      ISTORE(ICOUNT,23,ISTORE(ICOUNT,LL,6))=
     *ISTORE(ICOUNT,23,ISTORE(ICOUNT,LL,6))+1
      DO 49 MM=1,6
 49   NN(MM)=0
 48   CONTINUE
C
      IF(ICOUNT.EQ.1)GO TO 168
      IDUM=ICOUNT+1
      DO 158 J8=1,NPERM
      DO 159 J9=7,22
      DO 160 J0=1,5
 160  ISTORE(IDUM,J9,J0)=IPERM(J8,ISTORE(ICOUNT,J9,J0))
 159  CONTINUE
      DO 161 J1=1,ICOUNT-1
      DO 162 J2=1,4
 162  IF(ISTORE(J1,23,J2).NE.ISTORE(ICOUNT,23,J2))GO TO 161
      DO 163 J3=7,22
      DO 164 J4=7,22
      IF(ISTORE(ICOUNT,J3,6).NE.ISTORE(J1,J4,6))GO TO 164
      DO 165 J5=1,5                                         Step 13
      DO 166 J6=1,5
 166  IF(ISTORE(IDUM,J3,J5).EQ.ISTORE(J1,J4,J6))GO TO 165
      GO TO 164
 165  CONTINUE
      GO TO 163
 164  CONTINUE
      GO TO 161
 163  CONTINUE
      ICOUNT=ICOUNT-1
      GO TO 201
 161  CONTINUE
 158  CONTINUE
 168  WRITE(1,1000)(ISTORE(ICOUNT,11,J),J=1,6)
      GO TO 201
 14   CONTINUE
C
      DO 17 I17=1,3
      IPAIR(I(NUM,1,1),INC(I(NUM,1,2),I17))=
     *IPAIR(I(NUM,1,1),INC(I(NUM,1,2),I17))-1
 17   IPAIR(I(NUM,2,1),INC(I(NUM,1,2),I17))=
     *IPAIR(I(NUM,2,1),INC(I(NUM,1,2),I17))-1               Step 15
      IPAIR(INC(I(NUM,1,2),1),INC(I(NUM,1,2),2))=
     *IPAIR(INC(I(NUM,1,2),1),INC(I(NUM,1,2),2))-1
      IPAIR(INC(I(NUM,1,2),1),INC(I(NUM,1,2),3))=
     *IPAIR(INC(I(NUM,1,2),1),INC(I(NUM,1,2),3))-1
```

```
      IPAIR(INC(I(NUM,1,2),2),INC(I(NUM,1,2),3))=
     *IPAIR(INC(I(NUM,1,2),2),INC(I(NUM,1,2),3))-1
      I(NUM,1,2)=I(NUM,1,2)+1
      GO TO 9
C
   11 CONTINUE
  201 NUM=NUM-1
      IF(NUM.EQ.0)GO TO 202
      DO 18 I18=1,3
      IPAIR(I(NUM,1,1),INC(I(NUM,2,2),I18))=
     *IPAIR(I(NUM,1,1),INC(I(NUM,2,2),I18))-1                    Step 14
   18 IPAIR(I(NUM,2,1),INC(I(NUM,2,2),I18))=
     *IPAIR(I(NUM,2,1),INC(I(NUM,2,2),I18))-1
      IPAIR(INC(I(NUM,2,2),1),INC(I(NUM,2,2),2))=
     *IPAIR(INC(I(NUM,2,2),1),INC(I(NUM,2,2),2))-1
      IPAIR(INC(I(NUM,2,2),1),INC(I(NUM,2,2),3))=
     *IPAIR(INC(I(NUM,2,2),1),INC(I(NUM,2,2),3))-1
      IPAIR(INC(I(NUM,2,2),2),INC(I(NUM,2,2),3))=
     *IPAIR(INC(I(NUM,2,2),2),INC(I(NUM,2,2),3))-1
      I(NUM,2,2)=I(NUM,2,2)+1
      GO TO 10
C
  202 ILEFT=ICOUNT-(ICOUNT/4)*4
      DO 167 J7=1,ICOUNT-ILEFT,4
      J8=J7+1
      J9=J8+1
      J10=J9+1
  203 WRITE(15,1001)(ISTORE(J7,23,J1),J1=1,4), A ,J7
     * ,(ISTORE(J8,23,J2),J2=1,4), A ,J8
      DO 170 J0=1,22
  170 WRITE(15,1000)(ISTORE(J7,J0,J1),J1=1,6),(ISTORE(J8,J0,J2),J2=1,6)
      WRITE(15,1002)(ISTORE(J9,23,J1),J1=1,4), A ,J9,
     * (ISTORE(J10,23,J2),J2=1,4), A ,J10
      DO 171 J0=1,22
  171 WRITE(15,1000)(ISTORE(J9,J0,J1),J1=1,6),(ISTORE(J10,J0,J2),J2=1,6)
  167 CONTINUE
      IF(ILEFT.EQ.0)GO TO 200
      DO 178 J8=J7,ICOUNT
      WRITE(15,1001)(ISTORE(J8,23,J1),J1=1,4), A ,J8
      DO 173 J3=1,22
  173 WRITE(15,1000)(ISTORE(J8,J3,J1),J1=1,6)
  178 CONTINUE
  200 DO 301 J1=1,ICOUNT
      WRITE(16,1004) A,J1
      WRITE(16,1005)(ISTORE(J1,23,J2),J2=1,4)
      DO 303 J3=1,22                                              Step 16
  303 WRITE(16,1006)(ISTORE(J1,J3,J4),J4=1,6)
  301 CONTINUE
      GO TO 210
C
  100 FORMAT(A4)
 1000 FORMAT(1X,5I4,I6,46X,5I4,I6)
 1001 FORMAT('1',1X,'#(A B C R)=(',4I3,' )',3X,A6,' DES.NO= ',I4,
     * 24X,'#(A B C R)=(',4I3,' )',3X,A6,' DES.NO= ',I4,/2X,26('-'),
     * 10X,7('-'),29X,26('-'),10X,7('-'))
 1002 FORMAT(1X,////////,2X,'#(A B C R)=(',4I3,' )',3X,A6,' DES.NO= ',
     * I4,24X,'#(A B C R)=(',4I3,' )',3X,A6,' DES.NO= ',I4,/2X,26('-')
     * ,10X,7('-'),29X,26('-'),10X,7('-'))
 1003 FORMAT(1X,//)
 1004 FORMAT(1X,A3,I6)
 1005 FORMAT(1X,4I3)
 1006 FORMAT(1X,5I3,I6)
  210 CLOSE(12)
      CLOSE(15)
      CLOSE(16)                                                   Step 17
      STOP
      END
```

To assist in understanding this program, the purpose of each of the more important variables and arrays is as follows:

Variable	Purpose
INC(20,3)	Contains the twenty possible triples.
IPAIR(a,b)	Contains the number of (a,b) pairs.
IDES(22,5)	Temporary storage for designs.
IOUT1(21)	Contains the pointer values of the disallowed triples for the first incomplete block.
IOUT2(21)	Contains the pointer values of the disallowed triples for the first six incomplete blocks.
ISTORE(a,23,6)	Stores the a^{th} design. The additional row and column contain information on the design's block types.
NUM	A variable to control which identical pair of blocks is being completed.
I(NUM,a,b)	If b = 1 and a = 1 or 2, this stores the pairs occupying the incomplete blocks of the skeleton. If b = 2, this stores the pointer values of the triples occupying the $(2 \times (NUM - 1) + a)^{th}$ incomplete block.

All data for the program is given on the file 'DATA', and a complete list of the data used appears in Appendix 3. While compiling the data files, it was observed that the six blocks composing configurations 29 to 38 did little to constrain the choice of triples for the remaining incomplete blocks of the design. If left unchecked the resulting increase in storage requirements and individual run times would probably cause serious problems. Thus, for these cases each of the non-equivalent triples for block (v) was run separately, as the disadvantage of having more isomorphs to eliminate was considered the lesser evil. The subcases created were distinguished by appending an alphabetic character to the configuration reference label.

In the most severe case, that of configuration 38, the above 'splitting' technique was inadequate in controlling the amount of computation involved. Fortunately, the symmetry of the skeleton and the fact that this configuration is to be completed last, provide a solution to this problem. From the symmetry aspect, once the next four blocks of the skeleton have been completed they can be collectively swapped with blocks (i) to (iv) of the configurations by permutations fixing 1 and interchanging {2,3} with {4,5}. But any legitimate pattern for these four blocks must be equivalent to that exhibited by blocks (i) to (iv) of one of the 36 configurations. As configuration 38 is the last to be completed, duplication of an earlier case will only be avoided if these next four blocks have a 38 type structure. A manual implementation of this constraint quickly showed that only 3 non-equivalent possibilities exist for these next four blocks. These will be represented by the following three patterns.

38A	38B	38C
1 4 6 7 10	1 4 6 7 10	1 4 6 8 10
1 4 8 9 11	1 4 8 9 11	1 4 7 9 11
1 5 6 7 11	1 5 6 8 9	1 5 6 9 10
1 5 8 9 10	1 5 7 10 11	1 5 7 8 11.

The modifications to the program to accommodate this additional information are simple. The three lines 49,50,51, i.e.,

```
      IF(NUM.NE.1)GO TO 50
      DO 51 J1=1,IOUT1(21)
   51 IF(I1.EQ.IOUT1(J1))GO TO 11
```

are replaced by,

```
IF(NUM.EQ.1.AND.I1.NE.IOUT1(1))GO TO 11
IF(NUM.EQ.2.AND.I1.NE.IOUT1(3))GO TO 11
```

while the code,

```
IF(NUM.EQ.1.AND.I4.NE.IOUT1(2))GO TO 14
IF(NUM.EQ.2.AND.I4.NE.IOUT1(4))GO TO 14
```

is inserted after line 70 i.e. (IF(NUM.GT.3)GO TO 25). To accompany these changes, the first four elements of IOUT1( ) are assigned data values consistent with retaining the correct triples in the next four incomplete blocks. The new information also makes lines 74 to 98 superfluous and so these are to be removed. Finally, for each of the three subcases the permutation data is constructed using all ten of their blocks.

Section IV: The Resulting 2-(11,5,4) Designs with Repeated Blocks

A Prime 750 computer and the given program and data were used to develop the 2-(11,5,4) designs with repeated blocks. After completing the first few configurations it became apparent that the number of designs created would be higher than originally predicted. Thus to minimize storage requirements a period of balancing was followed immediately by the elimination of any isomorphs created. A simple and effective program was used to eliminate most of the isomorphs, but the more difficult cases had to be examined by hand. Generally the run times for individual cases were reasonably small. However the occasional large case and the need to balance 108 cases in all, required a moderate expenditure of computer time. The following table summarizes the number of designs created and eliminated at each stage.

DATA	# Designs produced	# New Non-isomorphic Designs	DATA	# Designs produced	# New Non-isomorphic Designs
D1	9	9	D23	230	28
D2	32	23	D24	0	0
D3	57	20	D25	53	10
D4	354	14	D26	160	45
D5	42	25	D27	160	0
D6	90	42	D28	187	36
D7	92	18	29A	0	0
D8	0	0	29B	0	0
D9	90	17	29C	6	1
D10	361	124	29D	124	2
D11	452	24	29E	396	4
D12	54	0	29F	110	0
D13	452	0	29G	208	0
D14	50	0	29H	76	0
D15	404	0	29I	88	0
D16	63	0	29J	172	0
D17	544	0	29K	0	0
D18	66	0	29L	94	0
D19	20	0	29M	94	0
D20	26	10	29N	100	0
D21	69	15	29O	220	0
D22	69	8	29P	0	0

29Q	0	0
30A	16	4
30B	564	111
30C	160	2
30D	220	0
30E	510	0
30F	292	4
30G	816	0
31A	0	0
31B	217	0
31C	241	0
31D	300	0
31E	50	0
31F	268	0
32A	394	0
32B	394	0
32C	688	0
32D	1296	0
32E	510	0
32F	650	0
32G	142	0
32H	1470	0
32I	0	0
32J	0	0
33A	152	0
33B	114	0
33C	184	0
33D	478	0
33E	152	0
33F	218	0
33G	184	0
33H	534	0
33I	262	0
33J	514	0
33K	114	0
33L	570	0
33M	158	0
33N	398	0
33O	0	0
33P	0	0
33Q	0	0
33R	0	0
34A	136	0
34B	326	0
34C	260	0
34D	136	0
34E	492	0
34F	242	0
34G	42	0
34H	82	0
34I	0	0
34J	0	0
35A	399	0
35B	535	0
35C	50	0
35D	72	0
36A	191	0
36B	191	0
36C	607	0
36D	816	0
36E	208	0
38A	232	19
38B	252	40
38C	126	1
	Total # of Designs	Total # of non-isomorphic Designs
Total	24499	656

In view of these results the practicality of the introduction of split computer runs is now questionable. This technique accomplishes the intended reduction in array sizes and individual run times, but at the expense of greatly increasing the number of isomorphs to be removed by the elimination algorithm. The corresponding increase in computer time consumed by this algorithm is undesirably large. More specifically at the completion of D28, 4816 designs had been produced and reduced to 468 non-isomorphic designs. But for the next eight configurations where split computer runs had been introduced, 19,073 designs were created with only 128 being added as new non-isomorphic designs.

A better solution to the inherent problems of these configurations is provided by the handling of configuration 38. Here a knowledge of previously completed configurations and the symmetry of the skeleton were used to collectively determine a further four blocks. In applying this approach to configurations 29 to 36 it is known that configurations D1 to D28 have been completed, and that permutations interchanging {2,3} with {4,5} can be used to exclude any pattern equivalent to these when forming the next four blocks. Also for pairwise balance with 1, the number of incidences of {6,7,8,9,10,11} in the configuration directly determines their corresponding incidences in the next four blocks. This relationship is demonstrated in the following table.

Configuration	Number of {6,7,8,9,10,11} occurring i times in the configuration					The corresponding incidence pattern for the next four blocks with 1				
	i = 0	1	2	3	4	0	1	2	3	4
29	-	2	2	2	-	-	2	2	2	-
30	-	1	4	1	-	-	1	4	1	-
31	-	2	2	2	-	-	2	2	2	-
32	-	1	4	1	-	-	1	4	1	-
33	-	1	4	1	-	-	1	4	1	-
34	-	1	4	1	-	-	1	4	1	-
35	-	-	6	-	-	-	-	6	-	-
36	-	-	6	-	-	-	-	6	-	-
38	-	-	6	-	-	-	-	6	-	-

It is immediately apparent that many combinations are impossible. Let $\otimes$ represent the process of determining all non-equivalent structures resulting from the combining of two configurations. This involves fixing the first configuration and allowing the second to take all of its legitimate equivalent possibilities. These are initially limited by a correct incidence count for each of {6,7,8,9,10,11} and also by using permutations fixing the blocks of the fixed configuration. The subsequent elimination of any remaining equivalent structures is then accomplished using permutation techniques.

From the table, the task is thus reduced to determining,

$29 \otimes 29$, $29 \otimes 31$, $30 \otimes 30$, $30 \otimes 32$, $30 \otimes 33$, $30 \otimes 34$,

$31 \otimes 31$, $32 \otimes 32$, $32 \otimes 33$, $32 \otimes 34$, $33 \otimes 33$, $33 \otimes 34$,

$34 \otimes 34$, $35 \otimes 35$, $35 \otimes 36$, $35 \otimes 38$, $36 \otimes 36$, $36 \otimes 38$

($38 \otimes 38$ has already been done).

Having achieved this, further eliminations between the ten block structures may still be possible. Once these manual calculations have been done, the modified program used to balance $38 \otimes 38$ can be used to complete the resulting structures. Finally, it is important to note that the implementations of this more manual method from the beginning would greatly reduce the amount of computer usage.

A table of the 656 non-isomorphic 2-(11,5,4) designs with repeated blocks is presented in Section IV of Part II. Here the designs have been classified according to the number of blocks of each block type that they contain. The predicted occurrence of a copy of the 37 reducible 2-(11,5,4) designs with repeated blocks within this set of designs, lends support to the claim that the construction technique is exhaustive. This concludes the census of 2-(11,5,4) designs, and it is claimed that there exist exactly 3509 + 228 + 656 = 4393 such non-isomorphic designs in all.

Section V: The 3-(12,6,4) Designs with Repeated Blocks

To complete the census of 3-(12,6,4) designs, the 656 non-isomorphic 2-(11,5,4) designs with repeated blocks were packed into 3-designs. This was done by extending the 2-designs by complementation and then systematically sieving isomorphs from the pool of 3-designs created. This process produced 119 non-isomorphic 3-(12,6,4) designs which contain at least the 656 non-isomorphic 2-(11,5,4) designs as point restrictions. The subsequent determination of the automorphism groups of these designs and hence their point orbits (largest transitivity sets), showed that they contained the 656 2-designs exactly. This also provides evidence that the enumeration process is exhaustive.

It is convenient to defer the listing of these designs for the present but a full catalogue appears in Section IV of Part II. A discussion of the 2-transitive 2-(11,5,4) design D1 1, its corresponding 3-transitive extension E1 and the 1-transitive 3-(12,6,4) designs E15, E101, E105, and E119 can be found in the following chapter.

The census of 3-designs is now complete and it is claimed that there are exactly 392 + 34 + 119 = 545 non-isomorphic 3-(12,6,4) designs in all.

CHAPTER 5

THE TRANSITIVE 3-(12,6,4) AND 2-(11,5,4) DESIGNS

In the enumeration of the designs in the preceding chapters eight transitive 3-(12,6,4) designs and three transitive 2-(11,5,4) designs were discovered. A part manual, part computational approach was used to develop all the 3-designs' automorphisms so the groups of the transitive 3-designs are known in the sense that all their elements are known as point permutations. Nevertheless there is some interest in showing that these permutation groups (or their larger subgroups) are isomorphic to known groups. To this end the tables given in Coxeter and Moser [3] (hereafter contracted to C & M) are invaluable.

Note that for any 3-(12,6,4) design D a restriction on the point x gives a 2-(11,5,4) design whose automorphism group is the stabilizer of x, Aut $D_{(x)}$. This restriction process and its inverse extension, need not preserve transitivity. There are transitive 2-(11,5,4) designs which do not extend to transitive 3-(12,6,4) designs, and transitive 3-(12,6,4) designs with restrictions which are not transitive 2-(11,5,4) designs.

In the ensuing discussion S_n, A_n, D_n and C_n are respectively the symmetric, alternating, dihedral and cyclic groups of degree n.

Section I: The Transitive 3-(12,6,4) Designs

The following table lists the reference label assigned to each of the eight transitive 3-(12,6,4) designs and the area of the text where it was produced. The designs have been given in the order of their discovery. This order will be used in the subsequent listing and discussion of these designs.

Case	Reference Label	Discovered in:
I	PI-T5 N1	Chapter 3 : Section I
II	PII-N5	Chapter 3 : Section II
III	PIII-N30	Chapter 3 : Section II
IV	E1	Chapter 4
V	E15	Chapter 4
VI	E101	Chapter 4
VII	E105	Chapter 4
VIII	E119	Chapter 4

Case I: PI-T5 N1 No repeated blocks; non-decomposable.

(#AC #B #R) = (24 20 0), $(q_0 \;\; q_3 \;\; q_4) = (0 \;\; 0 \;\; 15)$.

1 2 3 4 5 11	0 6 7 8 9 10	0 1 2 6 7 11	3 4 5 8 9 10
6 7 8 9 10 11	0 1 2 3 4 5	0 1 3 6 8 11	2 4 5 7 9 10
3 4 5 6 7 11	0 1 2 8 9 10	0 1 4 6 9 11	2 3 5 7 8 10
1 4 5 7 8 11	0 2 3 6 9 10	0 1 5 6 10 11	2 3 4 7 8 9
1 2 5 8 9 11	0 3 4 6 7 10	0 2 3 7 8 11	1 4 5 6 9 10
1 2 3 9 10 11	0 4 5 6 7 8	0 2 4 7 9 11	1 3 5 6 8 10
2 3 4 6 10 11	0 1 5 7 8 9	0 2 5 7 10 11	1 3 4 6 8 9
1 3 7 9 10 11	0 2 4 5 6 8	0 3 4 8 9 11	1 2 5 6 7 10
1 4 7 8 10 11	0 2 3 5 6 9	0 3 5 8 10 11	1 2 4 6 7 9
2 4 6 8 10 11	0 1 3 5 7 9	0 4 5 9 10 11	1 2 3 6 7 8
2 5 6 8 9 11	0 1 3 4 7 10		
3 5 6 7 9 11	0 1 2 4 8 10		

Aut D, which has order 240, contains the elements

$$\alpha = (1\ 2\ 3\ 4\ 5)(6\ 7\ 8\ 9\ 10),$$

$$\beta = (0\ 7\ 4\ 3\ 10)(2\ 9\ 8\ 5\ 11),$$

and $$\gamma = (0\ 11)(1\ 6)(2\ 7)(3\ 8)(4\ 9)(5\ 10).$$

If $R = \beta^3\alpha$ and $S = \beta^2\alpha$ then

$$R^2 = S^3 = (RS)^5 = e$$

so $<\alpha,\beta> \cong A_5$ (see C & M, p67), the icosahedral group. The element γ commutes with both α and β so $<\alpha,\beta,\gamma> \cong A_5 \times C_2$. The element γ also commutes with the automorphism $\delta = (0\ 10\ 4\ 6\ 8\ 7\ 11\ 5\ 9\ 1\ 3\ 2)$. In fact Aut D $\cong S_5 \times C_2$. Aut D acts imprimitively on the pairs (0,11),(1,6),(2,7),(3,8),(4,9),(5,10).

In [2] this design is associated with a regular dodecahedron and is generalized to give a family of 3-designs.

<u>Case II: PII-N5</u> No repeated blocks; non-decomposable.

(#AC #B #R) - (0 44 0), $(q_0\ \ q_3\ \ q_4) = (12\ \ 12\ \ 3)$.

1 2 3 4 5 6	7 8 9 10 11 12	1 3 5 7 9 11	2 4 6 8 10 12
1 2 9 10 11 12	3 4 5 6 7 8	1 3 5 8 10 12	2 4 6 7 9 11
1 2 7 8 11 12	3 4 5 6 9 10	1 3 6 7 10 12	2 4 5 8 9 11
1 2 7 8 9 10	3 4 5 6 11 12	1 3 6 8 9 12	2 4 5 7 10 11
3 4 8 10 11 12	1 2 5 6 7 9	1 3 6 9 10 11	2 4 5 7 8 12
3 4 7 9 11 12	1 2 5 6 8 10	1 4 5 7 10 12	2 3 6 8 9 11
5 6 8 9 11 12	1 2 3 4 7 10	1 4 5 8 9 12	2 3 6 7 10 11
5 6 7 10 11 12	1 2 3 4 8 9	1 4 5 9 10 11	2 3 6 7 8 12
3 5 7 8 9 10	1 2 4 6 11 12	1 4 6 7 9 12	2 3 5 8 10 11
4 6 7 8 9 10	1 2 3 5 11 12	1 4 6 8 10 11	2 3 5 7 9 12
1 3 4 7 8 11	2 5 6 9 10 12	1 5 6 7 8 11	2 3 4 9 10 12

Aut D which has order 24, contains the elements

$$\alpha = (3\ 4)(5\ 6)(7\ 8)(9\ 10),$$

and

$$\beta = (1\ 3\ 10)(2\ 5\ 9)(4\ 8\ 11)(6\ 7\ 12).$$

For these $\alpha^2 = \beta^3 = (\beta^{-1}\alpha\beta\alpha)^2 = e$. Therefore Aut D $\cong A_4 \times C_2$ (see C & M, p134) where A_4 is the alternating group on four elements, the tetrahedral group. Also Aut $D_{(1)} = \langle \alpha \rangle$. Aut D acts imprimitively on the quadruples (1,2,11,12),(3,4,5,6),(7,8,9,10).

<u>Case III: PIII-N30</u> No repeated blocks; non-decomposable.

(#AC #B #R) = (0 44 0), $(q_0\ \ q_3\ \ q_4) = (12\ \ 12\ \ 3)$.

1 2 3 4 5 6	7 8 9 10 11 12	1 3 4 7 10 12	2 5 6 8 9 11
1 2 9 10 11 12	3 4 5 6 7 8	1 3 5 7 10 11	2 4 6 8 9 12
1 2 7 8 11 12	3 4 5 6 9 10	1 3 5 9 10 12	2 4 6 7 8 11
1 2 7 8 9 10	3 4 5 6 11 12	1 3 6 7 9 11	2 4 5 8 10 12
3 4 8 10 11 12	1 2 5 6 7 9	1 3 6 8 9 12	2 4 5 7 10 11
3 5 8 9 11 12	1 2 4 6 7 10	1 4 5 7 9 12	2 3 6 8 10 11
3 6 7 8 9 10	1 2 4 5 11 12	1 4 5 8 9 11	2 3 6 7 10 12
4 5 7 8 9 10	1 2 3 6 11 12	1 4 6 8 10 12	2 3 5 7 9 11
4 6 7 9 11 12	1 2 3 5 8 10	1 4 6 9 10 11	2 3 5 7 8 12
5 6 7 10 11 12	1 2 3 4 8 9	1 5 6 7 8 12	2 3 4 9 10 11
1 3 4 7 8 11	2 5 6 9 10 12	1 5 6 8 10 11	2 3 4 7 9 12

Aut D which has order 24, contains the elements

$$\alpha = (1\ 3)(2\ 6)(4\ 12)(5\ 11)(7\ 10)(8\ 9),$$

and

$$\beta = (1\ 11\ 2\ 12)(3\ 8)(4\ 10\ 5\ 9)(6\ 7).$$

These satisfy $\alpha^2 = \beta^4 = (\alpha\beta)^3 = e$. Therefore Aut D $\cong S_4$ (see C & M, p134).

Aut D acts imprimitively on the sextuples (3,6,9,10,11,12) and (1,2,4,5,7,8).

<u>Case IV: E1</u> Repeated blocks; decomposable.

(#AC #B #R) = (0 0 44), $(q_0 \; q_3 \; q_4)$ = (165 0 0).

1 2 3 4 5 12	6 7 8 9 10 11	2 3 7 10 11 12	1 4 5 6 8 9
1 2 3 4 5 12	6 7 8 9 10 11	2 4 8 9 10 12	1 3 5 6 7 11
1 2 6 7 8 12	3 4 5 9 10 11	2 4 8 9 10 12	1 3 5 6 7 11
1 2 6 7 8 12	3 4 5 9 10 11	2 5 6 9 11 12	1 3 4 7 8 10
1 3 6 9 10 12	2 4 5 7 8 11	2 5 6 9 11 12	1 3 4 7 8 10
1 3 6 9 10 12	2 4 5 7 8 11	3 4 6 8 11 12	1 2 5 7 9 10
1 4 7 9 11 12	2 3 5 6 8 10	3 4 6 8 11 12	1 2 5 7 9 10
1 4 7 9 11 12	2 3 5 6 8 10	3 5 7 8 9 12	1 2 4 6 10 11
1 5 8 10 11 12	2 3 4 6 7 9	3 5 7 8 9 12	1 2 4 6 10 11
1 5 8 10 11 12	2 3 4 6 7 9	4 5 6 7 10 12	1 2 3 8 9 11
2 3 7 10 11 12	1 4 5 6 8 9	4 5 6 7 10 12	1 2 3 8 9 11

This 3-(12,6,4) design is trivial in that it is the double of the well-known 3-(12,6,2) design obtained as the extension of the Hadamard 2-(11,5,2) design. The 2-design is 2-transitive; the 3-design is 3-transitive with a group of order 12.11.10.6 = 7920. = 11.10.9.8

<u>Case V: E15</u> Repeated blocks; non-decomposable.

(#AC #B #R) = (0 36 8), $(q_0 \; q_3 \; q_4)$ = (48 0 3).

1 2 3 4 5 12	6 7 8 9 10 11	2 3 8 9 11 12	1 4 5 6 7 10
1 2 3 4 5 12	6 7 8 9 10 11	2 4 7 10 11 12	1 3 5 6 8 9
1 2 6 7 8 12	3 4 5 9 10 11	2 4 8 9 10 12	1 3 5 6 7 11
1 2 6 7 8 12	3 4 5 9 10 11	2 5 6 9 11 12	1 3 4 7 8 10
		2 5 7 9 10 12	1 3 4 6 8 11
1 3 6 9 10 12	2 4 5 7 8 11	3 4 6 7 10 12	1 2 5 8 9 11
1 3 7 9 11 12	2 4 5 6 8 10	3 4 7 8 9 12	1 2 5 6 10 11
1 4 6 9 11 12	2 3 5 7 8 10	3 5 6 8 10 12	1 2 4 7 9 11
1 4 8 10 11 12	2 3 5 6 7 9	3 5 7 8 11 12	1 2 4 6 9 10
1 5 7 10 11 12	2 3 4 6 8 9	4 5 6 7 9 12	1 2 3 8 10 11
1 5 8 9 10 12	2 3 4 6 7 11	4 5 6 8 11 12	1 2 3 7 9 10
2 3 6 10 11 12	1 4 5 7 8 9		

Aut D which has order 48 is generated by

$$\alpha = (1\ 7\ 10\ 5\ 2\ 6\ 9\ 4\ 12\ 8\ 11\ 3),$$

$$\beta = (1\ 7\ 12\ 8\ 2\ 6)(3\ 9\ 4\ 10\ 5\ 11),$$

and

$$\gamma = (1\ 3\ 9\ 6)(2\ 4\ 11\ 7)(5\ 10\ 8\ 12).$$

These satisfy $\alpha^{12} = \beta^6 = \gamma^4 = e$, $\beta^2 = \alpha^8$ and $\gamma^2 = \alpha^6$. Aut D acts imprimitively on the triples (1,12,2),(5,3,4),(9,10,11),(8,6,7) and also on the quadruples (1,5,9,8),(12,3,10,6),(2,4,11,7). Therefore Aut D simultaneously permutes the rows and columns of the array

	$\mathfrak{R}$	$\mathfrak{S}$	$\mathfrak{T}$
a:	1	12	2
b:	5	3	4
c:	9	10	11
d:	8	6	7 .

The columns are permuted under the action of the symmetric group S_3 on R,S,T. The rows move under the action of the group generated by (a b c d) and (a b)(c d) which is the dihedral group D_4. Each of the 48 elements of Aut D provides a unique permutation of the array and Aut D $\cong S_3 \times D_4$.

<u>Case VI: E101</u> Repeated blocks; non-decomposable.

(#AC #B #R) = (0 40 4), $(q_0\ \ q_3\ \ q_4) = (0\ \ 0\ \ 15)$.

1 2 3 4 5 12	6 7 8 9 10 11	2 3 7 10 11 12	1 4 5 6 8 9
1 2 3 4 5 12	6 7 8 9 10 11	2 4 6 8 10 12	1 3 5 7 9 11
		2 4 7 9 11 12	1 3 5 6 8 10
1 2 6 7 8 12	3 4 5 9 10 11	2 5 6 8 11 12	1 3 4 7 9 10
1 2 9 10 11 12	3 4 5 6 7 8	2 5 7 9 10 12	1 3 4 6 8 11
1 3 6 7 9 12	2 4 5 8 10 11	3 4 6 9 10 12	1 2 5 7 8 11
1 3 8 10 11 12	2 4 5 6 7 9	3 4 7 8 11 12	1 2 5 6 9 10
1 4 6 7 10 12	2 3 5 8 9 11	3 5 6 9 11 12	1 2 4 7 8 10
1 4 8 9 11 12	2 3 5 6 7 10	3 5 7 8 10 12	1 2 4 6 9 11
1 5 6 7 11 12	2 3 4 8 9 10	4 5 6 10 11 12	1 2 3 7 8 9
1 5 8 9 10 12	2 3 4 6 7 11	4 5 7 8 9 12	1 2 3 6 10 11
2 3 6 8 9 12	1 4 5 7 10 11		

Aut D which has order 1440, contains the elements

$$\alpha = (1\ 2)(7\ 8),$$

$$\beta = (1\ 2\ 3\ 4\ 5\ 12)(6\ 7\ 8\ 9\ 10\ 11),$$

and $$\gamma = (1\ 7)(2\ 8)(3\ 9)(4\ 10)(5\ 11)(6\ 12).$$

These satisfy $\beta^6 = (\alpha\beta)^5 = e$ and therefore generate a subgroup of Aut D isomorphic to S_6 (see C & M, p137). The element γ commutes with both α and β, so Aut D $\cong S_6 \times C_2$. Also Aut D permutes the rows and columns of the array

1	2	3	4	5	12
7	8	9	10	11	6,

in all possible ways.

Case VII: E105 Repeated blocks; non-decomposable.

(#AC #B #R) = (0 40 4), $(q_0 \quad q_3 \quad q_4) = (12 \quad 24 \quad 3)$.

1 2 3 4 5 12	6 7 8 9 10 11	2 3 7 10 11 12	1 4 5 6 8 9
1 2 3 4 5 12	6 7 8 9 10 11	2 4 6 8 10 12	1 3 5 7 9 11
		2 4 7 9 11 12	1 3 5 6 8 10
1 2 6 7 8 12	3 4 5 9 10 11	2 5 6 9 10 12	1 3 4 7 8 11
1 2 9 10 11 12	3 4 5 6 7 8	2 5 7 8 11 12	1 3 4 6 9 10
1 3 6 7 9 12	2 4 5 8 10 11	3 4 6 9 11 12	1 2 5 7 8 10
1 3 8 10 11 12	2 4 5 6 7 9	3 4 7 8 10 12	1 2 5 6 9 11
1 4 6 7 10 12	2 3 5 8 9 11	3 5 6 8 11 12	1 2 4 7 9 10
1 4 8 9 11 12	2 3 5 6 7 10	3 5 7 9 10 12	1 2 4 6 8 11
1 5 6 7 11 12	2 3 4 8 9 10	4 5 6 10 11 12	1 2 3 7 8 9
1 5 8 9 10 12	2 3 4 6 7 11	4 5 7 8 9 12	1 2 3 6 10 11
2 3 6 8 9 12	1 4 5 7 10 11		

Aut D which has order 12, contains the elements

$$\alpha = (1\ 2\ 5\ 12\ 3\ 4)(6\ 9\ 10\ 7\ 8\ 11),$$

and

$$\beta = (1\ 6)(2\ 11)(3\ 10)(4\ 9)(5\ 8)(7\ 12).$$

For these $\alpha^6 = \beta^2 = (\alpha\beta)^2 = e$. Thus Aut D $\cong D_6 \cong S_2 \times D_3$ (see C & M, p134). Aut D acts imprimitively on the sextuples (1,2,3,4,5,12) and (6,7,8,9,10,11).

Case VIII: E119 Repeated blocks; non-decomposable.

(#AC #B #R) = (0 40 4), $(q_0 \quad q_3 \quad q_4) = (45 \quad 0 \quad 0)$.

1 2 3 4 5 12	6 7 8 9 10 11	2 3 7 9 10 12	1 4 5 6 8 11
1 2 3 4 5 12	6 7 8 9 10 11	2 4 6 10 11 12	1 3 5 7 8 9
		2 4 7 8 9 12	1 3 5 6 10 11
1 2 6 7 8 12	3 4 5 9 10 11	2 5 6 7 10 12	1 3 4 8 9 11
1 2 9 10 11 12	3 4 5 6 7 8	2 5 8 9 11 12	1 3 4 6 7 10
1 3 6 7 9 12	2 4 5 8 10 11	3 4 6 8 9 12	1 2 5 7 10 11
1 3 8 10 11 12	2 4 5 6 7 9	3 4 7 10 11 12	1 2 5 6 8 9
1 4 6 8 10 12	2 3 5 7 9 11	3 5 6 7 11 12	1 2 4 8 9 10
1 4 7 9 11 12	2 3 5 6 8 10	3 5 8 9 10 12	1 2 4 6 7 11
1 5 6 9 10 12	2 3 4 7 8 11	4 5 6 9 11 12	1 2 3 7 8 10
1 5 7 8 11 12	2 3 4 6 9 10	4 5 7 8 10 12	1 2 3 6 9 11
2 3 6 8 11 12	1 4 5 7 9 10		

Aut D which has order 1440, contains the elements

(1 9 12 7 3 10 4 6 2 11)(5 8) and (5 12)(7 9)(10 11)(6 8) and so is transitive. The stabilizer Aut $D_{(12)}$ has two point orbits, (1,2,3,4,5) and (6,7,8,9,10,11). Aut $D_{(12)}$ contains

24 elements like (1 2 3 4 5)(11)(7 9 8 10 6),

30 elements like (1)(2 3 4 5)(7)(10)(11 8 6 9),

10 elements like (1)(2)(3)(4 5)(6 10)(7 11)(8 9),

20 elements like (1)(2)(3 4 5)(7 6 8)(11 9 10),

20 elements like (1 2)(3 4 5)(7 10 8 9 6 11),

15 elements like (1)(2 3)(4 5)(8)(9)(6 11)(7 10),

and the identity. Thus $|\text{Aut } D_{(12)}| = 120$. Furthermore each permutation of 1,2,3,4,5 occurs just once. Therefore $\text{Aut } D_{(12)} \cong S_5$. The permutations on 6,7,8,9,10,11 provide another representation of S_5 as a permutation group acting on six elements.

Section II: The Transitive 2-(11,5,4) Designs

The following table gives the origin of each of the three transitive 2-(11,5,4) designs.

Case	3-Designs Reference Label	Point Restriction	Discovered in:
I	PIV-T5 N90	0	Chapter 3: Section I
II	PVII-N18	5	Chapter 3: Section II
III	E1	any	Chapter 4

The blocks of each design will be listed and its automorphism group discussed. Also, if a 2-(11,5,4) design has a transitive automorphism group then its order is a multiple of 11^{α} where $\alpha \geqslant 1$. The Sylow theorems then assure us that a subgroup of order 11 exists. This means that any transitive 2-(11,5,4) design can be generated from a pair of supplementary difference sets developed cyclically modulo 11. As verification of this property a pair of supplementary difference sets has been produced for each design.

Case I No repeated blocks; decomposable.

(#A #B #C #R) = (22 0 0 0).

1 2 6 8 11	2 4 7 8 11	6 7 8 9 10	3 4 6 8 9	2 3 5 7 8
1 3 7 9 11	2 5 6 9 11	1 2 3 4 5	4 5 7 9 10	1 3 5 6 9
1 4 6 10 11	3 4 6 7 11	1 2 8 9 10	1 5 6 7 8	1 3 4 8 10

1 5 7 10 11	3 5 8 10 11	2 3 6 7 10	2 4 5 6 10	1 2 4 7 9
2 3 9 10 11	4 5 8 9 11			

Aut D has order 110 and contains the elements $x \to ax + b \pmod{11}$, $a \neq 0$ and so is 2-transitive. The stabilizer of a pair of points is the identity. Therefore the design has a sharply 2-transitive group. The design contains the blocks [1 2 3 4 5] and [6 7 8 9 10] and the automorphism (1 4 10 5 11 9 3 6 7 8 2). If the points of the design are relabelled using the permutation (0 11)(2 6 3)(1 7 4 8 5 10 9) then these provide as supplementary difference sets the squares and non-squares modulo 11, i.e. {1,3,4,5,9} and {2,6,7,8,10} respectively.

Case II No repeated blocks; non-decomposable

(#A #B #C #R) = (0 22 0 0).

1 2 3 4 6	1 3 7 10 11	3 4 6 7 8	1 2 4 8 11	3 4 7 10 12
2 7 8 10 12	1 4 8 9 12	2 4 6 7 9	1 2 3 11 12	2 6 9 10 11
3 7 8 9 11	1 6 7 8 11	2 3 6 8 10	4 6 10 11 12	2 4 7 11 12
4 8 9 10 11	1 6 7 9 12	1 3 4 9 10	3 6 9 11 12	2 3 8 9 12
1 2 7 9 10	1 6 8 10 12			

Aut D has order 11, is cyclic and isomorphic to C_{11}. The design contains the blocks [1 2 3 4 6] and [1 3 4 9 10] and the automorphism (2 9 3 10 8 4 6 11 7 12 1). If the points are relabelled using the permutation (1)(2)(0 12)(6 8)(3 4 7 10 5 11 9) then the design can be represented by the supplementary difference sets {1,3,4,5,7} and {1,2,4,7,8}.

Case III Repeated blocks; decomposable.

(#A #B #C #R) = (0 0 0 22).

Restriction on the point 12 is:

1 2 3 4 5	1 3 6 9 10	2 3 7 10 11	2 5 6 9 11	3 5 7 8 9
1 2 3 4 5	1 4 7 9 11	2 3 7 10 11	2 5 6 9 11	3 5 7 8 9
1 2 6 7 8	1 4 7 9 11	2 4 8 9 10	3 4 6 8 11	4 5 6 7 10
1 2 6 7 8	1 5 8 10 11	2 4 8 9 10	3 4 6 8 11	4 5 6 7 10
1 3 6 9 10	1 5 8 10 11			

The design is a trivial doubling of the well-known 2-(11,5,2) design which has a 2-transitive group of order 660. Take {1,3,4,5,9} repeated to give the two supplementary difference sets.

PART II

A CATALOGUE OF THE 3-(12,6,4)

AND 2-(11,5,4) DESIGNS

Section I

A Catalogue of the Reducible 3-(12,6,4) and 2-(11,5,4) Designs

In the following Section a representative copy of each of the 26 non-isomorphic reducible 3-(12,6,4) designs created in Chapter 2 is presented. These 3-designs point orbits have been specified to provide an indirect listing of the 95 non-isomorphic reducible 2-(11,5,4) designs. In the case of repeated blocks, each of the 2-(11,5,4) designs available as point restrictions, is identified with the initially developed 2-design to which it is isomorphic.

As each of the reducible 3-designs has an isomorphic copy amongst the designs created in the latter census, the name of the corresponding designs and a permutation showing its equivalence have also been given. The information relevant to each design is presented using the format:

Design # (#AC #B #R)

[starter block], [starter block] under (permutation)

[starter block], [starter block] under (permutation)

Aut D (a brief discussion of the design's automorphism group)

Transitivity sets: {points in orbit} - (#A #B #C #R), {etc.}

Isomorphic to NAME under (permutation).

The Reducible Designs

Design 1 (24 20 0)
[1 3 4 5 9 11], [0 2 6 7 8 10] under (0 1 2 3 4 5 6 7 8 9 10)(11)
[0 1 2 6 8 11], [3 4 5 7 9 10] under (0 1 2 3 6 5 7 8 10 9 4)(11)
Aut D has order 40 and is generated by,
α = (4)(11)(1 6 5 9 10 2 0 7 8 3),
and β = (3 10)(4 11)(1 6 8 7)(0 9 5 2).
These satisfy $\alpha^{10} = \beta^4 = e$ with $\beta\alpha^5 = \alpha^5\beta$. If $S = \alpha^2$ and $T = \beta^2$ then $S^5 = T^2 = (ST)^2 = e$.
Transitivity sets: {4,11} - (2 10 10 0), {0,1,2,3,5,6,7,8,9,10} - (2 10 10 0).
Isomorphic to PIV-T1 N235 under (3 4)(6 2 11)(1 0 5 8 7 10).

Design 2 (32 12 0)
[1 3 4 5 9 11], [0 2 6 7 8 10] under (0 1 2 3 4 5 6 7 8 9 10)(11)
[0 1 2 8 10 11], [3 4 5 6 7 9] under (0 1 2 4 3 7 10 6 5 8 9)(11)

Aut D has order 4 and contains the elements

α = (1)(3)(6)(9)(0 5)(2 10)(4 8)(7 11),

and β = (0)(5)(7)(11)(1 3)(2 8)(4 10)(6 9).

so $\alpha\beta = \beta\alpha$.

Transitivity sets: {7,11} - (0 6 16 0), {6,9} - (6 6 10 0),
{0,5} - (4 6 12 0), {1,3} - (2 6 14 0),
{2,4,8,10} - (2 6 14 0).

Isomorphic to PIV-T1 N12 under (2 0 6 3 5 11 7 4 1 10 9).

Design 3 (36 8 0)

[1 3 4 5 9 11], [0 2 6 7 8 10] under (0 1 2 3 4 5 6 7 8 9 10)(11)
[0 1 6 8 10 11], [2 3 4 5 7 9] under (1 2 3 0 7 10 8 6 4 5 9)(11)

Aut D has order 4 and contains the elements

α = (2)(4)(6)(11)(1 3)(5 7)(0 9)(8 10),

and β = (0)(2)(4)(9)(1 3)(5 8)(6 11)(7 10).

so $\alpha\beta = \beta\alpha$ and the group is the same as for Design 2.

Transitivity sets: {2} - (4 4 14 0), {4} - (0 4 18 0), {0,9} - (0 4 18 0),
{1,3} - (4 4 14 0), {6,11} - (8 4 10 0),
{5,7,8,10} - (2 4 16 0).

Isomorphic to PIV-T1 N20 under (2 3 1)(6 11 0 9 7 8 5 4 10).

Design 4 (36 8 0)

[1 3 4 5 9 11], [0 2 6 7 8 10] under (0 1 2 3 4 5 6 7 8 9 10)(11)
[0 1 6 8 10 11], [2 3 4 5 7 9] under (1 2 3 0 7 6 8 10 4 5 9)(11)

Aut D has order 4 and contains the elements

α = (1)(2)(3)(5)(8)(9)(0 4)(6 11)(7 10),

and β = (5)(6)(9)(11)(2 3)(0 10)(1 8)(4 7).

This group is the same as that for Designs 2 and 3.

Transitivity sets: {5} - (4 4 14 0), {9} - (0 4 18 0), {2,3} - (4 4 14 0),
{1,8} - (0 4 18 0), {6,11} - (8 4 10 0),
{0,4,7,10} - (2 4 16 0).

Isomorphic to PIV-T1 N111 under (10 6 2 0 4 8 7 5 3 11 1 9).

Design 5 (36 8 0)

[1 3 4 5 9 11], [0 2 6 7 8 10] under (0 1 2 3 4 5 6 7 8 9 10)(11)
[0 1 7 8 10 11], [2 3 4 5 6 9] under (1 2 3 9 8 10 0 4 7 6 5)(11)

Aut D has order 2 and contains the element

α = (1)(3)(0 9)(2 11)(4 7)(5 6)(8 10).

Transitivity sets: {1} - (4 4 14 0), {3} - (0 4 18 0),
{2,11} - (0 4 18 0), {8,10} - (2 4 16 0),
{4,7} - (2 4 16 0), {5,6} - (6 4 12 0),
{0,9} - (6 4 12 0).

Isomorphic to PIV-T1 N5 under (2)(4)(7)(8 10 5 0 11 3 9 6 1).

Design 6 (40 4 0)

[1 3 4 5 9 11], [0 2 6 7 8 10] under (0 1 2 3 4 5 6 7 8 9 10)(11)
[1 6 7 8 10 11], [0 2 3 4 5 9] under (1 2 3 6 0 7 8 10 4 5 9)(11)

Aut D has order 2 and like Design 5 has a single involution given by

α = (1)(11)(0 6)(2 3)(4 9)(5 7)(8 10).

Transitivity sets: {1} - (2 2 18 0), {11} - (10 2 10 0),
{8,10} - (6 2 14 0), {2,3} - (4 2 16 0),
{5,7} - (0 2 20 0), {4,9} - (2 2 18 0),
{0,6} - (2 2 18 0).

Isomorphic to PIV-T1 N13 under (8)(11)(7 4 3 9 5 10 2 6 0 1).

Design 7 (40 4 0)

[1 3 4 5 9 11], [0 2 6 7 8 10] under (0 1 2 3 4 5 6 7 8 9 10)(11)
[0 1 2 3 4 11], [5 6 7 8 9 10] under (1 2 3 6 0 9 8 4 7 10 5)(11)

Aut D has order 2 and the only non-identity element is
α = (3)(6)(7)(11)(1 4)(2 5)(0 9)(8 10).
Transitivity sets: {7} - (0 2 20 0), {6} - (0 2 20 0), {3} - (4 2 16 0),
{11} - (12 2 8 0), {8,10} - (6 2 14 0),
{1,4} - (4 2 16 0), {0,9} - (2 2 18 0),
{2,5} - (0 2 20 0).
Isomorphic to PIV-T1 N4 under (1)(11)(2 3)(8 0 6 9 10 5 7 4).

Design 8 (40 4 0)
[1 3 4 5 9 11], [0 2 6 7 8 10] under (0 1 2 3 4 5 6 7 8 9 10)(11)
[0 1 6 8 10 11], [2 3 4 5 7 9] under (1 2 3 6 7 10 0 8 5 4 9)(11)
Aut D has order 16 and is generated by
α = (4 10 3 2)(8 5 0 11 6 1 7 9),
and β = (6)(8)(0 7)(1 11)(2 3)(4 10)(5 9).
These satisfy $\alpha^8 = \beta^2 = (\alpha\beta)^2 = e$ so Aut D $\cong D_8$.
Transitivity sets: {2,3,4,10} - (2 2 18 0), {0,1,5,6,7,8,9,11} -
(4 2 16 0).
Isomorphic to PIV-T1 N69 under (7)(11)(8 0 5 2)(1 9 4 3 10 6).

Design 9 (40 4 0)
[1 3 4 5 9 11], [0 2 6 7 8 10] under (0 1 2 3 4 5 6 7 8 9 10)(11)
[0 1 6 7 8 11], [2 3 4 5 9 10] under (1 2 3 9 6 8 0 5 7 10 4)(11)
Aut D has order 2 and the only non-identity element is
α = (5)(8)(9)(10)(0 11)(1 2)(3 7)(4 6).
Transitivity sets: {9} - (0 2 20 0), {10} - (0 2 20 0), {8} - (4 2 16 0),
{5} - (4 2 16 0), {3,7} - (2 2 18 0),
{1,2} - (2 2 18 0), {4,6} - (6 2 14 0),
{0,11} - (6 2 14 0).
Isomorphic to PIV-T1 N11 under (1)(6)(0 11)(4 8 7 5 9 10 3 2).

Design 10 (40 4 0)
[1 3 4 5 9 11], [0 2 6 7 8 10] under (0 1 2 3 4 5 6 7 8 9 10)(11)
[0 1 6 7 8 11], [2 3 4 5 9 10] under (1 2 3 6 10 0 8 7 4 5 9)(11)
Aut D has order 16 and is generated by
α = (4 6 11 0)(3 10 5 7 9 8 1 2),
and β = (4)(11)(0 6)(3 8 9 10)(5 7 1 2).
These satisfy $\alpha^8 = \beta^4 = e$ and $\alpha^4\beta = \beta\alpha^4$. If $S = \alpha$ and $T = \alpha\beta$ then
$T^2 = e$ and $TST = S^2$.
Transitivity sets: {0,4,6,11} - (10 2 10 0), {1,2,3,5,7,8,9,10} -
(0 2 20 0).
Isomorphic to PIV-T1 N21 under (9)(6 11 8 5 2 7 3 1 10 4 0).

Design 11 (44 0 0)
[1 3 4 5 9 11], [0 2 6 7 8 10] under (0 1 2 3 4 5 6 7 8 9 10)(11)
[1 2 3 4 5 11], [0 6 7 8 9 10] under (1 2 3 7 9 6 5 8 0 4 10)(11)
Aut D has order 10 and contains the elements
α = (0)(7)(9 11 5 2 8)(3 4 6 1 10),
and β = (9)(10)(0 7)(1 3)(2 5)(4 6)(8 11).
These satisfy $\alpha^5 = \beta^2 = (\beta\alpha)^2 = e$ so Aut D $\cong D_5$.
Transitivity sets: {0,7} - (2 0 20 0), {1,3,4,6,10} - (2 0 20 0),
{2,5,8,9,11} - (6 0 16 0).
Isomorphic to PIV-T1 N123 under (4 5)(8 10)(7 6 9 0)(11 1 3 2).

Design 12 (44 0 0)
[1 3 4 5 9 11], [0 2 6 7 8 10] under (0 1 2 3 4 5 6 7 8 9 10)(11)
[0 1 3 4 5 11], [2 6 7 8 9 10] under (0 1 2 5 10 8 4 6 7 9 3)(11)
Aut D has order 110. More importantly Aut $D_{(9)}$ has order 110 and is
2-transitive. This is the group of linear transformations
$x \to ax + b$, $a \neq 0 \pmod{11}$.

Transitivity sets: {9} - (22 0 0 0), {0,1,2,3,4,5,6,7,8,10,11} - (2 0 20 0).
Isomorphic to PIV-T5 N90 under (11 3 2 1 10 7 6 8 5 9 0 4).

Design 13 (36 0 8)
[1 2 3 4 5 12], [6 7 8 9 10 11] under (1 9 3 4 5 6 7 8 2 10 11)(12)
and (1 2 3 7 5 10 8 4 11 9 6)(12)
Aut D has order 12 and is generated by
α = (8)(10 11)(2 3 4)(1 7 5 6 12 9),
and β = (2)(8)(1 9)(3 4)(5 6)(7 12)(10 11).
These satisfy $\alpha^6 = \beta^2 = (\alpha\beta)^2 = e$ so Aut D $\cong D_6$.
Transitivity sets: {8} - (0 0 18 4) = D1 10, {10,11} - (0 0 18 4) = D1 4, {2,3,4} - (4 0 14 4) = D1 27, {1,5,6,7,9,12} - (4 0 14 4) = D1 1.
Isomorphic to E7 under (5)(6)(7)(11 1 3)(10 12 4 9 8 2).

Design 14 (32 0 12)
[1 2 3 4 5 12], [6 7 8 9 10 11] under (1 9 3 4 5 6 7 8 2 10 11)(12)
and (1 2 3 8 4 9 7 5 11 10 6)(12)
Aut D has order 32 and has the generators
α = (1 3 11 9)(2 5 6 8 4 12 7 10),
β = (1 3 11 9)(2 5 7 10 4 12 6 8),
and γ = (1 3)(2 5)(4 12)(6 10)(7 8)(9 11).
These satisfy $\alpha^8 = \beta^8 = \gamma^2 = e$, $\beta^2 = \alpha^6$ and $\beta\gamma\alpha = e$.
Transitivity sets: {1,3,9,11} - (0 0 16 6) = D1 29, {2,4,5,6,7,8,10,12} - (4 0 12 6) = D1 2.
Isomorphic to E3 under (3)(11)(10 8 7 9 6)(5 4 1 2 12).

Design 15 (32 8 4)
[1 2 3 4 5 12], [6 7 8 9 10 11] under (1 9 3 4 5 6 7 8 2 10 11)(12)
and (1 2 3 7 5 9 8 4 11 10 6)(12)
Aut D has order 16 and has the generators
α = (3 5)(8 11)(1 2 4 12)(6 9 10 7),
β = (3 8)(5 11)(1 6 4 10)(2 9 12 7),
and γ = (3 8)(5 11)(1 7 4 9)(2 10 12 6).
These satisfy $\alpha^4 = \beta^4 = \gamma^4 = e$, $\alpha^2 = \beta^2 = \gamma^2$, $\alpha\beta = \beta\alpha$ and $\gamma\beta = \beta\gamma$.
Transitivity sets: {3,5,8,11} - (0 4 16 2) = D1 16, {1,2,4,6,7,9,10,12} - (4 4 12 2) = D1 3.
Isomorphic to E57 under (9)(10)(1 3)(2 5 4 12)(11 6 8 7).

Design 16 (36 4 4)
[1 2 3 4 5 12], [6 7 8 9 10 11] under (1 9 3 4 5 6 7 8 2 10 11)(12)
and (1 2 3 7 4 9 8 5 11 10 6)(12)
Aut D has order 6 and has the generators
α = (7)(9)(11)(1 4 5)(2 12 3)(6 8 10),
and β = (6)(7)(1 2)(3 4)(5 12)(8 10)(9 11).
These satisfy $\alpha^3 = \beta^2 = e$ and $\beta\alpha\beta = \alpha^2$. Each permutation on {6,8,10} appears exactly once so Aut D $\cong S_3$.
Transitivity sets: {7} - (12 2 6 2) = D2 38, {9,11} - (0 2 18 2) = D1 11, {6,8,10} - (4 2 14 2) = D1 8, {1,2,3,4,5,12} - (2 2 16 2) = D1 5.
Isomorphic to E17 under (3 5)(11 6 9)(10 7 8)(12 2 1 4).

Design 17 (40 0 4)
[1 2 3 4 5 12], [6 7 8 9 10 11] under (1 9 3 4 5 6 7 8 2 10 11)(12)
and (1 2 3 8 5 10 7 4 11 9 6)(12)
Aut D has order 10 and contains the elements
α = (10)(12)(1 2 5 3 4)(6 9 7 11 8),
and β = (1)(7)(10)(12)(2 4)(3 5)(6 8)(9 11).

These satisfy $\alpha^5 = \beta^2 = (\alpha\beta)^2 = e$ so Aut D $\cong D_5$.
Transitivity sets: {10} - (0 0 20 2) = D1 41, {12} - (0 0 20 2) = D1 6,
{1,2,3,4,5} - (4 0 16 2) = D1 51,
{6,7,8,9,11} - (4 0 16 2) = D1 19.
Isomorphic to E35 under (5)(7)(3 4 12 2 1)(8 10 6 11 9).

Design 18 (40 0 4)
[1 2 3 4 5 12], [6 7 8 9 10 11] under (1 9 3 4 5 6 7 8 2 10 11)(12)
and (1 2 3 7 4 10 8 5 11 9 6)(12)
Aut D has order 10 and is generated by
α = (4 7)(1 6 12 10 2 9 3 8 5 11).
Therefore Aut D $\cong C_{10}$.
Transitivity sets: {4,7} - (10 0 10 2) = D2 21, {1,2,3,5,6,8,9,10,11,12} -
(2 0 18 2) = D1 7.
Isomorphic to E33 under (4)(6)(9 11)(7 8 10)(3 5 2 1 12).

Design 19 (36 0 8)
[1 2 3 4 5 12], [6 7 8 9 10 11] under (1 9 3 4 5 6 7 8 2 10 11)(12)
and (1 2 3 7 5 11 8 4 10 9 6)(12)
Aut D has order 36 and is generated by
α = (2)(3 4)(8 11 10)(1 6 12 9 5 7),
β = (3)(2 4)(8 10 11)(1 6 5 7 12 9),
and γ = (2 3 4)(11)(8 10)(1 6 5 9 12 7).
These satisfy $\alpha^6 = \beta^6 = \gamma^6 = e$, $\alpha^4 = \beta^2$, $\gamma\alpha = \beta\gamma$, and $\alpha\beta = \gamma^4$.
The group Aut D has three point orbits, two of which are {2,3,4} and {8,10,11}. Any permutation from the first of these appears just once with any permutation from the second. Therefore Aut D $\cong S_3 \times S_3$.
Transitivity sets: {2,3,4} - (12 0 6 4) = D1 22, {8,10,11} - (0 0 18 4) =
D2 43, {1,5,6,7,9,12} - (0 0 18 4) = D1 9.
Isomorphic to E6 under (5)(2 9 6 7 8 1 3 10 12 4 11).

Design 20 (40 0 4)
[1 2 3 4 5 12], [6 7 8 9 10 11] under (1 9 3 4 5 6 7 8 2 10 11)(12)
and (1 2 3 7 4 11 8 5 10 9 6)(12)
Aut D has order 8 and is generated by
α = (1 12)(7 11)(2 3 5 4)(6 9 10 8),
and β = (7)(11)(1 12)(2 3)(4 5)(6 9)(8 10).
These satisfy $\alpha^4 = \beta^2 = (\alpha\beta)^2 = e$ so Aut D $\cong D_4$.
Transitivity sets: {1,12} - (0 0 20 2) = D1 15, {7,11} - (4 0 16 2) = D2 19,
{2,3,4,5} - (8 0 12 2) = D1 20, {6,8,9,10} -
(0 0 20 2) = D1 43.
Isomorphic to E19 under (8 5 7 1 11 12 6 2 10 3 9 4).

Design 21 (36 4 4)
[1 2 3 4 5 12], [6 7 8 9 10 11] under (1 9 3 4 5 6 7 8 2 10 11)(12)
and (1 2 3 7 4 11 8 5 9 10 6)(12)
Aut D has order 18 and is generated by
α = (8)(6 10)(7 9 11)(1 2 4 3 5 12),
and β = (10)(6 8)(7 9 11)(1 3 4 12 5 2).
These satisfy $\alpha^6 = \beta^6 = e$ and $\alpha^2 = \beta^2$. Two of the three point orbits are {6,8,10} and {7,9,11}. On {7,9,11} Aut D acts as C_3; on {6,8,10} Aut D acts as S_3. Each permutation needed for the direct product of C_3 and S_3 is present so Aut D $\cong S_3 \times C_3$.
Transitivity sets: {6,8,10} - (0 2 18 2) = D2 42, {7,9,11} - (0 2 18 2) =
D2 10, {1,2,3,4,5,12} - (6 2 12 2) = D1 23.
Isomorphic to E36 under (4 9)(7 12)(2 8 1 6)(11 3 10 5).

Design 22 (24 0 20)
[1 2 3 4 5 12], [6 7 8 9 10 11] under (1 9 3 4 5 6 7 8 2 10 11)(12)
and (1 2 3 8 4 9 6 5 11 10 7)(12)

Aut D has order 240. The transposition (2 4) belongs to Aut D. Two elements fixing 2 and 4 are

R = (1 6 12 8 7)(5 11 10 9 3),

and T = (11)(1 3 9)(5 7 8 12 6 10).

These satisfy $T^6 = R^5 = (RT)^2 = e$. Therefore R and T generate A_5 (see Coxeter and Moser [3] p137). Consequently Aut D $\cong A_5 \times C_2$.

Transitivity sets: {2,4} - (12 0 0 10) = D3 4, {1,3,5,6,7,8,9,10,11,12} - (0 0 12 10) = D1 26.

Isomorphic to E2 under (10 11)(12 8 9 4)(2 1 7 5 6 3).

Design 23 (32 8 4)
[1 2 3 4 5 12], [6 7 8 9 10 11] under (1 9 3 4 5 6 7 8 2 10 11)(12)
and (1 2 3 8 4 11 6 5 9 10 7)(12)

Aut D has order 32 and is generated by

α = (2 9 4 11)(1 6 5 7 3 10 12 8),

β = (2 9 4 11)(1 7 12 6 3 8 5 10),

and γ = (2 9)(4 11)(1 6 3 10)(5 8 12 7).

These satisfy $\alpha^8 = \beta^8 = \gamma^4 = e$, $\alpha^4 = \gamma^2$, $\alpha^2 = \beta^6$, $\alpha\beta = \beta\alpha$ and $\alpha\beta\gamma = \gamma\alpha\beta$.

Transitivity sets: {2,4,9,11} - (8 4 8 2) = D2 68, {1,3,5,6,7,8,10,12} - (0 4 16 2) = D1 44.

Isomorphic to E16 under (9)(10)(8 6 7 11)(3 4 12 5 2 1).

Design 24 (24 16 4)
[1 2 3 4 5 12], [6 7 8 9 10 11] under (1 9 3 4 5 6 7 8 2 10 11)(12)
and (1 2 3 7 4 11 6 5 9 10 8)(12)

Aut D has order 48 and is generated by

α = (6)(10)(11)(7 9 8)(1 2 3 12 4 5),

and β = (1)(12)(6 10)(2 3 4 5)(7 9 11 8).

These satisfy $\alpha^6 = \beta^4 = e$ and $\alpha^3\beta = \beta\alpha^3$. If $T = \alpha^2\beta$ and $R = \beta$ then $R^4 = T^2 = (RT)^3 = e$ so S_4 is a subgroup (see Coxeter and Moser [3] p134). This subgroup does contain α^3 which commutes with all elements. Therefore Aut D $\cong S_4 \times C_2$.

Transitivity sets: {6,10} - (0 8 12 2) = D4 28, {7,8,9,11} - (0 8 12 2) = D2 16, {1,2,3,4,5,12} - (4 8 8 2) = D1 68.

Isomorphic to E55 under (3 8)(2 11 4 10)(5 9 12 7 1 6).

Design 25 (0 0 44)
[1 2 3 4 5 12], [6 7 8 9 10 11] under (1 9 3 4 5 6 7 8 2 10 11)(12)
all blocks repeated.

Aut D has order 7920. Every block in the design is repeated. The group is that of the well-known 3-transitive 3-(12,6,2) design which is the unique extension of the unique Hadamard 2-(11,5,2) design.

Transitivity sets: {1,2,3,4,5,6,7,8,9,10,11,12} - (0 0 0 22) = D5 16.

Isomorphic to E1 under (5)(9)(10)(7 8 6 11)(12 4 3 2 1).

Design 26 (0 40 4)
[1 2 3 4 5 12], [6 7 8 9 10 11] under (1 9 3 4 5 6 7 8 2 10 11)(12)
and (1 2 3 7 4 11 10 5 9 6 8)(12)

Aut D has order 1440 and is 1-transitive. See Chapter 5, Case VII (i.e. design E119).

Transitivity sets: {1,2,3,4,5,6,7,8,9,10,11,12} - (0 20 0 2) = D6 18.

Isomorphic to E119 under (6)(9)(12)(11 10 8 7)(5 4 3 2 1).

Section II

A Catalogue of the Designs Containing AC Type Blocks

The following section presents the designs retained as representatives from those created in Chapter 3: Section I. By construction none of these designs contains repeated blocks, so no reference to the number of such blocks will be made when listing their characteristic statistics. The division of these designs under each classification system will be given. This is followed by a key and data to be used to produce the design corresponding to each reference label.

Using the (q_0, q_3, q_4) classification system, the 392 non-isomorphic 3-(12,6,4) designs were partitioned into the following classes.

Design Variables q_0	q_3	q_4	# of Non-isomorphic Designs	Design Reference Labels
20	14	1	11	PI-T1 N1, PII-T4 N2, PII-T4 N19, PII-T4 N21, PII-T8 N2, PII-T8 N21, PII-T9 N23, PII-T9 N32, PII-T9 N58, PIII-T7 N169, PIII-T9 N1.
16	14	3	3	PI-T1 N2, PII-T4 N4, PII-T4 N7.
12	8	7	1	PI-T1 N3.
20	8	3	4	PI-T1 N4, PII-T1 N18, PIII-T3 N15, PV-T3 N52.
22	12	1	6	PI-T1 N5, PII-T7 N44, PII-T9 N2, PII-T9 N3, PIII-T3 N62, PV-T3 N21.
20	12	1	11	PI-T1 N6, PI-T5 N4, PII-T4 N18, PII-T8 N9, PII-T9 N29, PII-T9 N162, PIII-T7 N26, PIII-T7 N30, PIII-T7 N167, PIII-T7 N200, PIII-T7 N227.
14	14	3	1	PI-T1 N7.
16	18	1	2	PI-T1 N8, PI-T3 N2.
12	16	3	2	PI-T1 N9, PI-T3 N8.
18	10	3	3	PI-T2 N2, PI-T5 N6, PII-T9 N87.
14	16	3	1	PI-T3 N1.
23	11	1	5	PI-T3 N3, PII-T2 N1, PII-T4 N8, PII-T8 N3, PII-T9 N106.

18	14	1	8	PI-T3 N4, PI-T5 N26, PII-T9 N6, PII-T9 N19, PII-T9 N25, PII-T9 N36, PII-T9 N79, PII-T9 N145.
8	12	7	1	PI-T3 N6.
20	12	3	1	PI-T3 N7.
0	0	15	1	PI-T5 N1.
16	16	1	1	PI-T5 N12.
18	12	3	1	PI-T5 N13.
19	16	0	2	PI-T5 N20, PI-T5 N23.
15	20	0	1	PI-T5 N27.
37	4	0	2	PII-T1 N1, PIV-T1 N86.
35	6	0	3	PII-T1 N2, PII-T2 N3, PII-T8 N16.
25	7	1	9	PII-T1 N3, PII-T1 N7, PII-T7 N13, PII-T9 N45, PIII-T1 N3, PIII-T3 N25, PIII-T7 N100, PIII-T7 N146, PIV-T3 N67.
25	9	1	2	PII-T1 N4, PII-T1 N17.
27	7	1	6	PII-T1 N5, PII-T1 N11, PII-T3 N10, PII-T7 N6, PII-T8 N22, PIII-T2 N27.
24	10	1	6	PII-T1 N6, PII-T5 N8, PII-T7 N50, PII-T9 N105, PIII-T3 N17, PIII-T9 N4.
29	5	1	5	PII-T1 N8, PII-T1 N9, PII-T1 N10, PII-T7 N26, PIV-T1 N32.
38	5	0	3	PII-T1 N12, PIII-T1 N10, PIII-T6 N8.
35	8	0	2	PII-T1 N13, PIII-T6 N7.
26	8	1	11	PII-T1 N14, PII-T4 N11, PII-T5 N13, PII-T5 N17, PII-T7 N29, PII-T8 N17, PII-T9 N50, PIII-T3 N4, PIII-T3 N28, PIII-T3 N36, PIV-T3 N65.
25	10	0	6	PII-T1 N15, PII-T7 N9, PIII-T3 N39, PIV-T3 N61, PIV-T5 N28, PV-T3 N20.
30	7	0	3	PII-T1 N16, PIII-T7 N93, PIV-T3 N62.
26	6	1	11	PII-T1 N19, PII-T3 N14, PII-T5 N14, PII-T7 N33, PIII-T1 N5, PIII-T3 N7, PIII-T3 N34, PIII-T4 N61, PIII-T7 N54, PIII-T7 N60, PIII-T7 N114.
22	8	3	2	PII-T2 N2, PII-T5 N15.
33	5	1	1	PII-T3 N1.

24	4	3	3	PII-T3 N2, PII-T3 N4, PV-T4 N14.
32	6	1	2	PII-T3 N5, PIII-T9 N3.
36	2	1	2	PII-T3 N6, PIII-T10 N60.
28	0	3	2	PII-T3 N8, PIII-T10 N62.
22	6	3	3	PII-T3 N11, PII-T5 N9, PIII-T3 N29.
36	6	1	1	PII-T3 N12.
35	4	0	2	PII-T3 N13, PIII-T1 N31.
28	6	1	4	PII-T3 N16, PII-T9 N66, PIII-T1 N27, PIV-T1 N46.
26	12	1	2	PII-T3 N17, PII-T5 N20.
26	10	1	3	PII-T3 N18, PII-T7 N22, PIII-T1 N46.
31	7	1	2	PII-T3 N19, PIII-T3 N43.
24	12	1	3	PII-T4 N1, PIII-T3 N23, PIII-T3 N46.
16	12	3	4	PII-T4 N5, PII-T8 N25, PII-T9 N56, PII-T9 N75.
22	16	1	2	PII-T4 N6, PII-T7 N47.
20	10	3	2	PII-T4 N9, PII-T5 N7.
28	10	1	2	PII-T4 N12, PV-T3 N6.
30	8	1	4	PII-T4 N13, PIII-T9 N13, PIII-T10 N40, PV-T3 N28.
24	11	0	3	PII-T4 N17, PII-T9 N43, PIII-T7 N45.
23	12	0	6	PII-T4 N20, PII-T8 N10, PII-T9 N27, PIII-T7 N14, PIII-T7 N122, PV-T3 N1.
27	8	0	5	PII-T5 N2, PII-T9 N42, PIII-T1 N1, PIII-T3 N38, PV-T3 N23.
20	16	3	1	PII-T5 N4.
26	2	3	2	PII-T5 N10, PII-T7 N42.
30	4	1	5	PII-T5 N12, PII-T8 N24, PII-T9 N96, PIII-T1 N22, PIII-T1 N51.
32	3	0	1	PII-T5 N22.
23	9	1	9	PII-T7 N1, PIII-T7 N33, PIII-T7 N48, PIII-T7 N186, PV-T1 N12, PV-T2 N30, PV-T3 N14, PV-T3 N58, PV-T4 N80.
20	6	3	2	PII-T7 N2, PIII-T3 N11.

33	6	0	6	PII-T7 N7, PIII-T1 N11, PIII-T2 N25, PIII-T2 N30, PIII-T2 N32, PIII-T8 N29.
24	8	1	11	PII-T7 N28, PII-T7 N38, PII-T9 N62, PII-T9 N107, PII-T10 N3, PIII-T1 N16, PIII-T3 N3, PIII-T3 N8, PIII-T3 N27, PIII-T7 N43, PIII-T7 N234.
24	14	1	1	PII-T7 N36.
21	18	0	1	PII-T7 N37.
35	3	1	1	PII-T7 N43.
27	12	0	2	PII-T7 N46, PIII-T4 N45.
28	11	0	1	PII-T7 N48.
31	10	0	1	PII-T7 N52.
22	10	1	10	PII-T8 N6, PII-T9 N7, PII-T9 N33, PII-T9 N70, PII-T9 N108, PII-T9 N161, PIII-T3 N21, PIII-T7 N181, PIII-T7 N189, PIII-T8 N1.
27	10	0	1	PII-T8 N14.
26	9	0	5	PII-T8 N15, PII-T9 N112, PIII-T7 N27, PIII-T7 N74, PV-T4 N30.
21	16	0	1	PII-T8 N18.
14	12	3	1	PII-T9 N4.
22	13	0	2	PII-T9 N9, PV-T1 N14.
22	14	1	1	PII-T9 N13.
18	8	3	3	PII-T9 N14, PII-T9 N54, PIII-T3 N14.
21	14	0	2	PII-T9 N31, PII-T9 N41.
8	8	7	1	PII-T9 N51.
28	4	1	4	PII-T9 N134, PIII-T1 N2, PIII-T7 N55, PIV-T3 N68.
16	10	3	2	PII-T9 N148, PIII-T7 N180.
16	16	3	1	PII-T11 N1.
12	4	7	1	PII-T11 N3.
24	8	3	1	PII-T11 N10.
29	6	0	1	PIII-T1 N4.
29	10	0	3	PIII-T1 N6, PIII-T3 N5, PIII-T3 N37.
37	6	0	2	PIII-T1 N7, PIII-T3 N57.

31	5	1	3	PIII-T1 N9, PIII-T1 N12, PIII-T7 N113.
28	7	0	4	PIII-T1 N13, PIII-T1 N21, PIII-T3 N1, PIII-T3 N33.
31	4	0	1	PIII-T1 N14.
29	8	0	6	PIII-T1 N17, PIII-T3 N70, PIII-T4 N62, PIII-T7 N260, PIV-T5 N23, PV-T3 N8.
31	8	0	2	PIII-T1 N18, PIII-T3 N59.
31	3	1	1	PIII-T1 N24.
34	5	0	3	PIII-T1 N29, PIII-T2 N22, PIV-T1 N43.
42	5	0	1	PIII-T1 N30.
34	2	1	3	PIII-T1 N36, PIII-T4 N39, PIII-T6 N14.
41	2	0	2	PIII-T1 N39, PIII-T2 N39.
43	4	0	1	PIII-T1 N40.
43	0	0	1	PIII-T1 N41.
24	2	3	2	PIII-T1 N42, PIII-T1 N43.
29	7	1	7	PIII-T1 N45, PIII-T2 N2, PIII-T3 N54, PIII-T7 N2, PIII-T7 N4, PIII-T7 N72, PIII-T7 N134.
33	8	0	1	PIII-T1 N52.
39	6	0	1	PIII-T1 N53.
29	16	0	1	PIII-T1 N54.
32	4	1	2	PIII-T2 N4, PIII-T7 N86.
42	3	0	1	PIII-T2 N15.
27	5	1	3	PIII-T2 N33, PIII-T2 N34, PIII-T4 N20.
43	2	0	4	PIII-T2 N36, PIII-T2 N40, PIII-T4 N55, PIII-T6 N3.
39	4	0	5	PIII-T2 N41, PIII-T6 N1, PIII-T6 N2, PIII-T6 N11, PIV-T1 N9.
30	6	1	2	PIII-T2 N42, PIV-T1 N47.
25	12	0	6	PIII-T3 N24, PIII-T7 N110, PIII-T7 N179, PIII-T7 N203, PIII-T7 N243, PV-T2 N3.
31	6	0	6	PIII-T3 N32, PIII-T7 N109, PIII-T7 N131, PIV-T1 N49, PIV-T1 N51, PIV-T1 N71.
26	11	0	1	PIII-T3 N60.

24	13	0	1	PIII-T3 N68.
37	8	0	1	PIII-T4 N3.
34	3	0	1	PIII-T4 N44.
39	2	0	2	PIII-T4 N47, PIV-Tl N64.
32	5	0	2	PIII-T7 N35, PIII-T7 N39.
33	4	0	1	PIII-T7 N36.
41	0	0	1	PIII-T7 N38.
36	5	0	2	PIII-T7 N97, PIV-Tl N45.
19	13	1	1	PIII-T7 N119.
40	3	0	1	PIII-T7 N128.
40	5	0	1	PIII-T7 N129.
45	2	0	1	PIII-T7 N155.
23	16	0	1	PIII-T7 N198.
44	1	0	1	PIII-T8 N28.
31	12	0	1	PIII-T9 N14.
45	0	0	3	PIII-T10 N33, PIII-T10 N73, PIV-Tl N235.
37	0	0	2	PIII-T10 N55, PIV-T5 N53.
53	0	0	5	PIV-Tl N4, PIV-Tl N11, PIV-Tl N13, PIV-Tl N21, PIV-Tl N69.
51	0	0	3	PIV-Tl N5, PIV-Tl N20, PIV-Tl N111.
49	0	0	1	PIV-Tl N12.
32	7	0	1	PIV-Tl N73.
35	2	0	1	PIV-Tl N85.
35	1	1	1	PIV-Tl N90.
36	0	1	1	PIV-Tl N92.
55	0	0	2	PIV-Tl N123, PIV-T5 N90.
30	5	0	1	PIV-T3 N66.
33	10	0	1	PV-T3 N9.
23	13	1	1	PV-T3 N19

The following table presents the re-categorisation of these designs in accordance with the number of AC and B type blocks contained by each. The results are given using the following format:

#AC #B Type Blocks in the 3-Designs	Number of Non-isomorphic 3-Designs	#A #B #C Type Blocks in the Embedded 2-Designs	Corresponding Number of 2-Designs.
3-Designs' Reference Labels			
4 40	128	2 20 0 0 20 2	211 1044

PI-T1 N6,	PI-T3 N4,	PI-T5 N12,	PI-T5 N14,	PI-T5 N20,
PI-T5 N23,	PI-T5 N26,	PI-T5 N27,	PII-T7 N3,	PII-T1 N7,
PII-T1 N15,	PII-T1 N19,	PII-T3 N14,	PII-T4 N17,	PII-T4 N18,
PII-T4 N20,	PII-T5 N2,	PII-T5 N14,	PII-T5 N22,	PII-T7 N1,
PII-T7 N1,	PII-T7 N9,	PII-T7 N13,	PII-T7 N28,	PII-T5 N22,
PII-T7 N1,	PII-T7 N1,	PII-T7 N9,	PII-T7 N13,	PII-T7 N28,
PII-T7 N33,	PII-T7 N38,	PII-T8 N6,	PII-T8 N9,	PII-T8 N10,
PII-T8 N15,	PII-T8 N4,	PII-T9 N6,	PII-T9 N7,	PII-T9 N9,
PII-T9 N14,	PII-T9 N14,	PII-T9 N25,	PII-T9 N27,	PII-T9 N29,
PII-T9 N31,	PII-T9 N33,	PII-T9 N36,	PII-T9 N41,	PII-T9 N42,
PII-T9 N43,	PII-T9 N45,	PII-T9 N54,	PII-T9 N62,	PII-T9 N70,
PII-T9 N79,	PII-T9 N107,	PII-T9 N108,	PII-T9 N112,	PII-T9 N134,
PII-T9 N145,	PII-T9 N148,	PII-T9 N161,	PII-T9 N162,	PII-T10 N3,
PIII-T1 N1,	PIII-T1 N2,	PIII-T1 N3,	PIII-T1 N4,	PIII-T1 N5,
PIII-T1 N13,	PIII-T1 N14,	PIII-T1 N16,	PII-T1 N21,	PIII-T1 N42,
PIII-T1 N43,	PIII-T2 N33,	PIII-T2 N34,	PIII-T3 N1,	PIII-T3 N3,
PIII-T3 N7,	PIII-T3 N8,	PIII-T3 N11,	PIII-T3 N14,	PIII-T3 N21,
PIII-T3 N25,	PIII-T3 N27,	PIII-T3 N33,	PIII-T3 N34,	PIII-T3 N38,
PIII-T3 N39,	PIII-T4 N20,	PIII-T4 N61,	PIII-T7 N14,	PIII-T7 N26,
PIII-T7 N27,	PIII-T7 N30,	PIII-T7 N33,	PIII-T7 N43,	PIII-T7 N45,
PIII-T7 N48,	PIII-T7 N54,	PIII-T7 N55,	PIII-T7 N60,	PIII-T7 N74,
PIII-T7 N100,	PIII-T7 N114,	PIII-T7 N119,	PIII-T7 N122,	PIII-T7 n146,
PIII-T7 N167,	PIII-T7 N180,	PIII-T7 N181,	PIII-T7 N186,	PIII-T7 N189,
PIII-T7 N200,	PIII-T7 N227,	PIII-T7 N234,	PIII-T8 N1,	PIV-T3 N61,
PIV-T3 N66,	PIV-T3 N67,	PIV-T3 N68,	PIV-T5 N28,	PV-T7 N12,
PV-T1 N14,	PV-T2 N30,	PV-T3 N1,	PV-T3 N14,	PV-T3 N20,
PV-T3 N23,	PV-T3 N58,	PV-T4 N30,	PV-T4 N80.	

8 36	122	4 18 0 2 18 2 0 18 4	6 360 709

PI-T1 N1,	PI-T1 N4,	PI-T1 N5,	PI-T1 N7,	PI-T1 N8,
PI-T1 N9,	PI-T3 N2,	PI-T3 N3,	PI-T3 N8,	PI-T5 N6,
PII-T1 N4,	PII-T1 N5,	PII-T1 N6,	PII-T1 N8,	PII-T1 N9,
PII-T1 N10,	PII-T1 N11,	PII-T1 N14,	PII-T1 N16,	PII-T1 N17,
PII-T1 N18,	PII-T2 N1,	PII-T3 N2,	PII-T3 N4,	PII-T3 N8,

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PII-T4 N8, PII-T4 N11, PII-T4 N19, PII-T4 N21, PII-T5 N9,
PII-T5 N9, PII-T5 N10, PII-T5 N12, PII-T5 N13, PII-T5 N17,
PII-T7 N6, PII-T7 N26, PII-T7 N29, PII-T7 N42, PII-T7 N44,
PII-T7 N50, PII-T8 N2, PII-T8 N3, PII-T8 N14, PII-T8 N17,
PII-T8 N18, PII-T8 N21, PII-T8 N22, PII-T8 N24, PII-T8 N25,
PII-T9 N2, PII-T9 N3, PII-T9 N23, PII-T9 N32, PII-T9 N50,
PII-T9 N51, PII-T9 N56, PII-T9 N58, PII-T9 N66, PII-T9 N75,
PII-T9 N87, PII-T9 N96, PII-T9 N105, PII-T9 N106, PII-T11 N3,
PII-T1 N17, PIII-T1 N22, PIII-T1 N24, PIII-T1 N27, PIII-T1 N51,
PIII-T2 N27, PIII-T3 N4, PIII-T3 N15, PIII-T3 N17, PIII-T3 N24,
PIII-T3 N28, PIII-T3 N29, PIII-T3 N32, PIII-T3 N36, PIII-T3 N60,
PIII-T3 N62, PIII-T3 N68, PIII-T3 N70, PIII-T4 N44, PIII-T4 N62,
PIII-T7 N35, PIII-T7 N36, PIII-T7 N39, PIII-T7 N93, PIII-T7 N109,
PIII-T7 N110, PIII-T7 N131, PIII-T7 N169, PIII-T7 N179, PIII-T7 N203,
PIII-T7 N243, PIII-T7 N260, PIII-T9 N1, PIII-T9 N4, PIII-T10 N55,
PIII-T10 N62, PIV-T1 N32, PIV-T1 N46, PIV-T1 N49, PIV-T1 N51,
PIV-T1 N71, PIV-T1 N85, PIV-T3 N62, PIV-T3 N65, PIV-T5 N23,
PIV-T5 N53, PIV-T2 N3, PV-T3 N8, PB-T3 N21, PV-T3 N52,
PV-T4 N14.</td></tr>
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PII-T5 N7, PII-T5 N15, PII-T7 N7, PII-T7 N22, PII-T7 N37,
PII-T7 N46, PII-T7 N48, PII-T9 N13, PIII-T1 N6, PIII-T1 N9,
PIII-T1 N11, PIII-T1 N12, PIII-T1 N18, PIII-T1 N29, PII-T1 N31,
PIII-T1 N36, PIII-T1 N45, PIII-T1 N46, PIII-T2 N2, PIII-¢2 N4,
PIII-T2 N22, PIII-T2 N25, PIII-T2 N30, PIII-T2 N32, PIII-T2 N42,
PIII-T3 N5, PIII-T3 N23, PIII-T3 N37, PIII-T3 N46, PIII-T3 N54,
PIII-T3 N59, PIII-T4 N39, PIII-T4 N45, PIII-T6 N14, PIII-T7 N2,
PIII-T7 N4, PIII-T7 N72, PIII-T7 N86, PIII-T7 N113, PIII-T7 N134,
PIII-T7 N198, PIII-T8 N29, PIV-T1 N43, PIV-T1 N47, PIV-T1 N73,
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14
14
14</td><td>2
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123
128</td></tr>
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PII-T2 N3, PII-T3 N1, PII-T3 N5, PII-T3 N6, PII-T3 N17,
PII-T3 N19, PII-T4 N6, PII-T4 N12, PII-T4 N13, PII-T5 N20,
PII-T7 N36, PII-T7 N43, PII-T7 N47, PII-T7 N52, PII-T8 N16,
PII-T11 N1, PII-T11 N10, PII-T1 N52, PIII-T1 N52, PIII-T3 N43,
PIII-T4 N47, PIII-T7 N38, PIII-T7 N97, PIII-T9 N3, PII-T9 N13,
PIII-T10 N40, PIII-T10 N60, PIV-T1 N45, PIV-T1 N64, PIV-T1 N86,
PV-T3 N6, PV-T3 N28.</td></tr>
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PIV-Tl N5, PIV-Tl N20, PIV-Tl N111.					
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		10	2	10	2
		6	2	14	4
		4	2	16	6
		2	2	18	7
		0	2	20	7
PIV-Tl N4, PIV-Tl N11, PIV-Tl N13, PIV-Tl N21, PIV-Tl N69.					
44 0	2	22	0	0	1
		6	0	16	1
		2	0	20	3
PIV-Tl N123, PIV-T5 N90.					
NUMBER OF 3-(12,6,4) DESIGNS	392	NUMBER OF 2-(11,5,4) DESIGNS			3509

The following diagram demonstrates the relative positions of each of the P, T and N sections of the design with reference label PI-Tl N1. Using this key and the following data sets, a representative copy of any of the 392 non-isomorphic 3-(12,6,4) designs with AC type blocks can be obtained.

KEY FOR PRODUCING THE 3-(12,6,4) DESIGNS

PI-T1 N1

$$\left\{ \begin{matrix} 6 & 6 & 6 & 8 & 7 & 7 & 6 & 7 & 8 & 9 & 8 & 6 & 7 & 7 & 6 \\ 10 & 7 & 9 & 10 & 8 & 9 & 8 & 10 & 9 & 10 & 10 & 9 & 9 & 10 & 8 \end{matrix} \right\} = N1$$

[1	2	3	4	5	11]		[• • • • • •]
[6	7	8	9	10	11]		[ ]
[3	4	5	6	7	11]	PI	[Complementary]
[1	4	5	7	8	11]	(6x7)	[Blocks]
[1	2	5	8	9	11]	fixed	[ ]
[1	2	3	9	10	11]		[ ]
[2	3	4	6	10	11]		[• • • • • •]
[0	1	5	6	10	11]		[• • • • • •]
[0	1	3	6	7	11]	TI	[ ]
[0	1	4	6	9	11]	(2x15)	[ ]
[0	1	4	8	10	11]	fixed	[ ]
[0	2	3	7	8	11]		[Complementary]
[0	2	4	7	9	11]		[Blocks]
[0	2	5	6	8	11]	N1	[ ]
[0	2	5	7	10	11]	(2x15)	[ ]
[0	3	4	8	9	11]	variable	[ ]
[0	3	5	9	10	11]		[• • • • • •]

[1	2	•	•	•	11]	[0	•	•	•	8	10]
[1	3	Complements			11]	[0	Complements			6	9]
[2	4	w.r.t.			11]	[0	w.r.t.			7	9]
[3	5	{6,7,8,9,10}			11]	[0	{1,2,3,4,5}			7	10]
[4	5	•	•	•	11]	[0	•	•	•	6	8]

Below are listed the P and T sections to be used in conjunction with the key.

PI

1	2	3	4	5	11
6	7	8	9	10	11
3	4	5	6	7	11
1	4	5	7	8	11
1	2	5	8	9	11
1	2	3	9	10	11
2	3	4	6	10	11

-T1	1	1	1	1	2	2	2	2	3	3	1	1	2	3	4
	5	3	4	4	3	4	5	5	4	5	2	3	4	5	5
-T2	1	1	1	1	2	2	2	2	3	3	1	1	2	3	4
	3	4	4	5	3	4	4	5	5	5	2	3	5	4	5
-T3	1	1	1	1	2	2	2	2	3	4	1	1	2	3	3
	3	3	4	5	3	4	5	5	4	5	2	4	4	5	5
-T4	1	1	1	1	2	2	2	2	3	3	1	1	2	3	4
	3	3	4	4	4	4	5	5	5	5	2	5	3	4	5
-T5	1	1	1	1	2	2	2	3	3	4	1	1	2	2	3
	2	3	4	5	3	4	5	4	5	5	3	4	4	5	5

PII

1	2	3	4	5	11
6	7	8	9	10	11
3	4	5	6	7	11
1	4	5	7	8	11
1	2	5	8	9	11
1	2	3	6	10	11
2	3	4	9	10	11

-T1	1	1	1	1	2	2	2	2	3	3	1	1	2	3	4
	3	4	4	5	3	4	5	5	4	5	2	3	4	5	5
-T2	1	1	1	1	2	2	2	2	3	3	1	1	2	3	4
	3	4	4	5	3	4	4	5	5	5	2	3	5	4	5
-T3	1	1	1	1	2	2	2	2	3	3	1	1	2	3	4
	4	3	3	5	4	4	5	5	4	5	2	4	3	5	5
-T4	1	1	1	1	2	2	2	2	3	4	1	1	2	3	3
	4	3	3	5	3	4	5	5	4	5	2	4	4	5	5
-T5	1	1	1	1	2	2	2	2	3	4	1	1	2	3	3
	4	3	3	5	3	4	4	5	5	5	2	4	5	4	5
-T6	1	1	1	1	2	2	2	2	3	3	1	1	2	3	4
	4	4	3	3	4	4	5	5	5	5	2	5	3	4	5
-T7	1	1	1	1	2	2	2	3	3	3	1	1	2	2	4
	2	3	4	5	4	4	5	4	5	5	3	4	3	5	5
-T8	1	1	1	1	2	2	2	3	3	3	1	1	2	2	4
	2	4	4	5	3	4	5	4	5	5	3	3	4	5	5
-T9	1	1	1	1	2	2	2	3	3	4	1	1	2	2	3
	4	2	3	5	3	4	5	4	5	5	3	4	4	5	5
-T10	1	1	1	1	2	2	2	3	3	3	1	1	2	2	4
	4	4	2	3	4	5	5	4	5	5	3	5	3	4	5
-T11	1	1	1	1	2	2	2	3	3	4	1	1	2	2	3
	2	3	3	5	4	4	5	4	5	5	4	4	3	5	5

PIII

1	2	3	4	5	11
6	7	8	9	10	11
3	4	5	6	7	11
1	4	5	7	8	11
1	2	5	9	10	11
1	2	3	6	10	11
2	3	4	8	9	11

-T1	1	1	1	1	2	2	2	2	3	3	1	1	2	3	4
	5	3	4	4	3	4	5	5	4	5	2	3	4	5	5
-T2	1	1	1	1	2	2	2	2	3	3	1	1	2	3	4
	3	4	4	5	3	4	4	5	5	5	2	3	5	4	5
-T3	1	1	1	1	2	2	2	2	3	4	1	1	2	3	3
	4	3	3	5	3	4	5	5	4	5	2	4	4	5	5
-T4	1	1	1	1	2	2	2	2	3	4	1	1	2	3	3
	4	3	3	5	3	4	4	5	5	5	2	4	5	4	5
-T5	1	1	1	1	2	2	2	2	3	3	1	1	2	3	4
	4	4	3	3	4	4	5	5	5	5	2	5	3	4	5
-T6	2	1	1	1	1	2	2	2	3	4	1	1	2	3	3
	3	3	3	4	4	4	5	5	5	5	2	5	4	4	5
-T7	1	1	1	1	2	2	2	3	3	4	1	1	2	2	3
	5	2	3	4	3	4	5	4	5	5	3	4	4	5	5
-T8	1	1	1	1	2	2	2	3	3	4	1	1	2	2	3
	2	3	4	5	3	4	4	5	5	5	3	4	5	5	4
-T9	1	1	1	1	2	2	2	3	3	4	1	1	2	2	3
	2	3	4	4	3	5	5	4	5	5	3	5	4	4	5
-T10	1	1	1	1	2	2	2	3	3	4	1	1	2	2	3
	2	3	4	4	3	4	5	5	5	5	3	5	4	5	4
-T11	1	1	1	1	2	2	2	3	3	4	1	1	2	2	3
	2	3	3	4	4	4	5	5	5	5	4	5	3	5	4

PIV

1	2	3	4	5	11
6	7	8	9	10	11
3	4	5	6	7	11
1	4	5	8	9	11
1	2	5	7	10	11
1	2	3	6	8	11
2	3	4	9	10	11

-T1	1	1	1	1	2	2	2	2	3	3	1	1	2	3	4
	5	3	4	4	3	4	5	5	4	5	2	3	4	5	5
-T2	1	1	1	1	2	2	2	2	3	3	1	1	2	3	4
	3	4	4	5	3	4	4	5	5	5	2	3	5	4	5
-T3	1	1	1	1	2	2	2	2	3	4	1	1	2	3	3
	5	3	3	4	3	4	5	5	4	5	2	4	4	5	5
-T4	1	1	1	1	2	2	2	2	3	3	1	1	2	3	4
	3	3	4	4	4	4	5	5	5	5	2	5	3	4	5
-T5	1	1	1	1	2	2	2	3	3	4	1	1	2	2	3
	2	3	4	5	3	4	5	4	5	5	3	4	4	5	5

	1 2 3 4 5 11	-T1	2 1 1 1 1 2 2 2 3 3 1 1 2 2 4
			4 4 4 5 5 3 3 5 4 5 3 3 4 5 5
	6 7 8 9 10 11		
		-T2	1 1 1 1 2 2 2 2 3 3 1 1 2 2 4
	3 4 5 6 7 11		3 4 5 5 3 4 4 5 4 5 3 4 3 5 5
PV	3 4 5 8 9 11	-T3	1 1 1 1 2 2 2 2 3 4 1 1 2 2 3
			3 4 5 5 3 3 4 4 5 5 3 4 5 5 4
	1 2 5 6 8 11		
		-T4	1 1 1 1 2 2 2 2 3 4 1 1 2 2 3
	1 2 4 7 10 11		3 4 4 5 3 3 4 5 5 5 3 5 4 5 4
	1 2 3 9 10 11		

	1 2 3 4 5 11	-T1	2 1 1 1 1 2 2 2 3 3 1 1 2 2 4
			4 4 4 5 5 3 3 5 4 5 3 3 4 5 5
	6 7 8 9 10 11		
		-T2	1 1 1 1 2 2 2 2 3 3 1 1 2 2 4
PVI	3 4 5 6 7 11		3 4 5 5 3 4 4 5 4 5 3 4 3 5 5
	3 4 5 8 9 11	-T3	1 1 1 1 2 2 2 2 3 4 1 1 2 2 3
			3 4 5 5 3 3 4 4 5 5 3 4 5 5 4
	1 2 5 6 7 11		
		-T4	1 1 1 1 2 2 2 2 3 4 1 1 2 2 3
	1 2 4 8 10 11		3 4 4 5 3 3 4 5 5 5 3 5 4 5 4
	1 2 3 9 10 11		

Note that PVI has been given for completeness only, as all designs from this pattern are eliminated by earlier cases.

The N section, and other information concerning each of the 3-designs appears in the format:

<u>KEY CODE</u> (#AC #B) $(q_0 \quad q_3 \quad q_4)$

$$\begin{array}{ll} 6 & 6 \;\ldots\; \\ 10 & 7 \;\ldots\; \end{array} \left[\begin{array}{l} \text{The (2x15) array to be} \\ \text{substituted into the key} \end{array} \right]$$

<u>Aut D</u> (all non-trivial automorphisms of the design)

<u>Transitivity set</u>: {transitivity set} - (#A #B #C), etc.

<u>PI-T1 N1</u> (8 36) (20 14 1)

6	6	6	8	7	7	6	7	8	9	8	6	7	7	6
10	7	9	10	8	9	8	10	9	10	10	9	9	10	8

<u>Aut D</u> (1 5)(2 4)(6 10)(7 9)

<u>Transitivity sets</u>: {0}-(2 18 2), {3}-(2 18 2), {8}-(2 18 2), {11}-(2 18 2), {1,5}-(0 18 4), {2,4}-(0 18 4), {6,10}-(0 18 4), {7,9}-(0 18 4).

PI-T1 N2 (12 32) (16 14 3)

6	6	6	8	7	7	6	7	8	8	9	6	7	7	6
10	7	9	10	8	9	9	10	9	10	10	8	9	10	8

Aut D (0 2)(1 6)(3 8)(4 5)(7 11)(9 10)
Transitivity sets: {0,2}-(2 16 4), {3,8}-(2 16 4), {7,11}-(2 16 4), {1,6}-(0 16 6), {4,5}-(0 16 6), {9,10}-(0 16 6).

PI-T1 N3 (16 28) (12 8 7)

6	6	6	7	7	7	6	7	8	8	9	6	7	8	6
10	8	9	10	8	9	9	10	9	10	10	8	9	10	7

Aut D (1 5)(2 4)(6 10)(7 9) (0 11)(1 6)(2 7)(3 8)(4 9)(5 10)
(0 11)(1 10)(2 9)(3 8)(4 7)(5 6)
Transitivity sets: {0,11}-(2 14 6), {3,8}-(2 14 6), {2,4,7,9}-(2 14 6) {1,5,6,10}-(0 14 8).

PI-T1 N4 (8 36) (20 8 3)

6	6	6	7	7	8	6	7	8	7	9	6	8	7	6
10	8	9	10	8	10	9	10	9	9	10	8	10	9	7

Aut D (0 11)(1 6)(2 7)(3 8)(4 9)(5 10)
Transitivity sets: {0,11}-(2 18 2), {2,7}-(2 18 2), {1,6}-(0 18 4), {3,8}-(0 18 4), {4,9}-(0 18 4), {5,10}-(0 18 4).

PI-T1 N5 (8 36) (22 12 1)

6	7	6	8	6	7	6	7	8	7	8	9	6	6	7
10	10	9	10	8	9	8	10	9	9	10	10	9	7	8

Transitivity sets: {0}-(2 18 2), {3}-(2 18 2), {5}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4), {4}-(0 18 4), {6}-(0 18 4), {7}-(0 18 4), {8}-(0 18 4), {9}-(0 18 4), {10}-(0 18 4).

PI-T1 N6 (4 40) (20 12 1)

6	8	6	7	7	6	6	7	8	7	9	8	6	7	6
10	10	9	10	8	8	9	10	9	9	10	10	8	9	7

Aut D (0 11)(1 10)(2 9)(3 8)(4 7)(5 6)
Transitivity sets: {0,11}-(2 20 0), {1,10}-(0 20 2), {2,9}-(0 20 2), {3,8}-(0 20 2), {4,7}-(0 20 2), {5,6}-(0 20 2).

PI-T1 N7 (8 36) (14 14 3)

6	7	6	8	7	7	6	7	8	6	8	9	7	6	6
10	10	9	10	8	9	8	10	9	9	10	10	9	7	8

Aut D (1 5)(2 4)(6 10)(7 9) (0 11)(1 6)(2 7)(3 8)(4 9)(5 10)
(0 11)(1 10)(2 9)(3 8)(4 7)(5 6)
Transitivity sets: {0,11}-(2 18 2), {3,8}-(2 18 2), {1,5,6,10}-(0 18 4), {2,4,7,9}-(0 18 4).

PI-T1 N8 (8 36) (16 18 1)

6	7	6	8	7	7	6	7	8	6	9	8	7	6	6
10	10	9	10	8	9	9	10	9	8	10	10	9	7	8

Aut D (0 11)(1 10)(2 9)(3 8)(4 7)(5 6)
Transitivity sets: {0,11}-(2 18 2), {3,8}-(2 18 2), {1,10}-(0 18 4), {2,9}-(0 18 4), {4,7}-(0 18 4), {5,6}-(0 18 4).

PI-T1 N9 (8 36) (12 16 3)

6	8	6	7	7	7	6	7	8	6	9	8	7	6	6
10	10	9	10	8	9	9	10	9	8	10	10	9	8	7

Aut D (0 11)(1 2)(4 5)(6 9)(7 10) (0 11)(1 4)(2 5)(6 7)(9 10)
(1 5)(2 4)(6 10)(7 9) (0 11)(1 6)(2 7)(3 8)(4 9)(5 10)
(1 7 5 9)(2 6 4 10)(3 8) (1 9 5 7)(2 10 4 6)(3 8)
(0 11)(1 10)(2 9)(3 8)(4 7)(5 6)
Transitivity sets: {0,11}-(2 18 2), {3,8}-(2 18 2), {1,2,4,5,6,7,9,10}-(0 18 4).

<u>PI-T2 N2</u> (8 36) (18 10 3)

6 6 7 6 7 7 8 6 7 8 9 6 8 7 6
8 9 10 10 8 9 10 9 9 10 10 8 9 10 7

<u>Aut D</u> (0 11)(1 6)(2 7)(3 8)(4 9)(5 10)

<u>Transitivity sets:</u> {0,11}-(2 18 2), {2,7}-(2 18 2), {1,6}-(0 18 4), {3,8}-(0 18 4), {4,9}-(0 18 4), {5,10}-(0 18 4).

<u>PI-T3 N1</u> (12 32) (14 16 3)

6 8 6 6 7 7 6 7 8 9 8 6 7 6 7
7 10 9 10 8 9 8 10 9 10 10 9 9 8 10

<u>Aut D</u> (0 10)(1 4)(2 7)(3 8)(5 11)(6 9)

<u>Transitivity sets:</u> {0,10}-(2 16 4), {5,11}-(2 16 4), {3,8}-(2 16 4), {1,4}-(0 16 6), {2,7}-(0 16 6), {6,9}-(0 16 6).

<u>PI-T3 N2</u> (8 36) (16 18 1)

6 8 6 6 7 7 6 7 8 8 9 6 7 6 7
7 10 9 10 8 9 9 10 9 10 10 8 9 8 10

<u>Aut D</u> (0 11)(1 10)(2 9)(3 8)(4 7)(5 6)

<u>Transitivity sets:</u> {0,11}-(2 18 2), {3,8}-(2 18 2), {1,10}-(0 18 4), {2,9}-(0 18 4), {4,7}-(0 18 4), {5,6}-(0 18 4).

<u>PI-T3 N3</u> (8 36) (23 11 1)

6 8 6 7 7 7 6 6 8 9 8 7 6 6 7
7 10 9 10 8 9 8 10 9 10 10 9 9 8 10

<u>Transitivity sets:</u> {0}-(2 18 2), {8}-(2 18 2), {10}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4), {3}-(0 18 4), {4}-(0 18 4), {5}-(0 18 4), {6}-(0 18 4), {7}-(0 18 4), {9}-(0 18 4).

<u>PI-T3 N4</u> (4 40) (18 14 1)

6 8 7 6 7 6 6 7 8 9 8 7 6 6 7
7 10 9 10 8 9 8 10 9 10 10 9 9 8 10

<u>Transitivity sets:</u> {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

<u>PI-T3 N6</u> (16 28) (8 12 7)

6 7 6 6 7 7 6 7 8 9 8 6 7 6 8
8 10 9 10 8 9 8 10 9 10 10 9 9 7 10

<u>Aut D</u> (1 2)(3 5)(6 7)(8 10)
(0 9)(1 7)(2 6)(4 11)
(0 9)(1 6)(2 7)(3 5)(4 11)(8 10)
(0 11)(1 7)(2 6)(3 10)(4 9)(5 8)
(0 4)(3 10)(5 8)(9 11)
(0 4)(1 2)(3 8)(5 10)(6 7)(9 11)
(0 11)(1 6)(2 7)(3 8)(4 9)(5 10)

<u>Transitivity sets:</u> {0,4,9,11}-(2 14 6), {3,5,8,10}-(2 14 6), {1,2,6,7}-(0 14 8).

<u>PI-T3 N7</u> (16 28) (20 12 3)

6 7 6 7 7 7 6 6 8 9 8 7 6 6 8
8 10 9 10 8 9 8 10 9 10 10 9 9 7 10

<u>Aut D</u> (0 4)(3 10)(5 8)(9 11)
(0 11)(1 6)(2 7)(3 8)(4 9)(5 10)
(0 9)(1 6)(2 7)(3 5)(4 11)(8 10)

<u>Transitivity sets:</u> {0,4,9,11}-(2 14 6), {3,5,8,10}-(2 14 6), {1,6}-(0 14 8), {2,7}-(0 14 8).

PI-T3 N8 (8 36) (12 16 3)

6	7	7	6	7	6	6	7	8	9	8	7	6	6	8
8	10	9	10	8	9	8	10	9	10	10	9	9	7	10

Aut D (0 4)(3 10)(5 8)(9 11) (1 2)(3 5)(6 7)(8 10)
(0 9)(1 7)(2 6)(4 11) (0 4)(1 2)(3 8)(5 10)(6 7)(9 11)
(0 9)(1 6)(2 7)(3 5)(4 11)(8 10) (0 11)(1 6)(2 7)(3 8)(4 9)(5 10)
(0 11)(1 7)(2 6)(3 10)(4 9)(5 8)

Transitivity sets: {0,4,9,11}-(2 18 2), {1,2,6,7}-(0 18 4),
{3,5,8,10}-(0 18 4).

PI-T5 N1 (24 20) (0 0 15)

6	6	6	6	7	7	7	8	8	9	6	6	7	7	8
7	8	9	10	8	9	10	9	10	10	8	9	9	10	10

Aut D (design as 1-transitive, see Chapter 5)

Transitivity sets: {0,1,2,3,4,5,6,7,8,9,10,11}-(2 10 10).

PI-T5 N6 (8 36) (18 10 3)

6	6	6	6	7	7	7	8	8	9	6	6	7	7	8
7	8	10	9	9	8	10	9	10	10	9	8	10	9	10

Aut D (1 2)(3 5)(6 7)(8 10) (0 11)(1 6)(2 7)(3 8)(4 9)(5 10)
(0 11)(1 7)(2 6)(3 10)(4 9)(5 8)

Transitivity sets: {0,11}-(2 18 2), {4,9}-(2 18 2), {1,2,6,7}-(0 18 4),
{3,5,8,10}-(0 18 4)

PI-T5 N12 (4 40) (16 16 1)

6	6	6	6	7	7	8	8	7	9	6	6	7	8	7
7	9	8	10	8	10	10	9	9	10	9	8	10	10	9

Aut D (0 11)(1 6)(2 10)(3 9)(4 8)(5 7)

Transitivity sets: {0,11}-(2 20 0), {1,6}-(0 20 2), {2,10}-(0 20 2),
{3,9}-(0 20 2), {4,8}-(0 20 2), {5,7}-(0 20 2).

PI-T5 N13 (12 32) (18 12 3)

6	6	6	6	7	8	7	7	8	9	6	6	7	8	7
7	8	9	10	8	10	10	9	9	10	8	9	10	10	9

Aut D (0 2 5)(1 8 9)(3 4 6)(7 10 11) (2 0 5)(1 9 8)(3 6 4)(7 11 10)
(0 7)(1 3)(2 11)(4 9)(5 10)(6 8) (0 10)(1 4)(2 7)(3 8)(5 11)(6 9)
(0 11)(1 6)(2 10)(3 9)(4 8)(5 7)

Transitivity sets: {0,2,5,7,10,11}-(2 16 4), {1,3,4,6,8,9}-(0 16 6).

PI-T5 N14 (4 40) (20 12 1)

6	6	6	6	7	8	7	7	8	9	6	6	7	8	7
7	9	8	10	8	10	10	9	9	10	9	8	10	10	9

Aut D (0 11)(1 6)(2 10)(3 9)(4 8)(5 7)

Transitivity sets: {0,11}-(2 20 0), {1,6}-(0 20 2), {2,10}-(0 20 2),
{3,9}-(0 20 2), {4,8}-(0 20 2), {5,7}-(0 20 2).

PI-T5 N20 (4 40) (19 16 0)

6	6	7	6	7	6	8	8	7	9	6	7	6	8	7
7	9	10	10	8	8	10	9	9	10	9	10	8	10	9

Aut D (0 11)(1 10)(2 9)(3 8)(4 7)(5 6)

Transitivity sets: {0,11}-(2 20 0), {1,10}-(0 20 2), {2,9}-(0 20 2),
{3,8}-(0 20 2), {4,7}-(0 20 2), {5,6}-(0 20 2).

PI-T5 N23 (4 40) (19 16 0)

6	6	8	6	7	6	7	8	7	9	6	8	6	7	7
7	9	10	10	8	8	9	9	10	10	9	10	8	9	10

Aut D (0 11)(1 6)(2 10)(3 9)(4 8)(5 7)

Transitivity sets: {0,11}-(2 20 0), {1,6}-(0 20 2), {2,10}-(0 20 2),
{3,9}-(0 20 2), {4,8}-(0 20 2), {5,7}-(0 20 2).

PI-T5 N26 (4 40) (18 14 1)
6 6 7 6 7 8 6 8 7 9 6 7 8 6 7
7 9 10 10 8 10 8 9 9 10 9 10 10 8 9
Aut D (0 11) (1 6) (2 10) (3 9) (4 8) (5 7)
Transitivity sets: {0,11}-(2 20 0), {1,6}-(0 20 2), {2,10}-(0 20 2),
{3,9}-(0 20 2), {4,8}-(0 20 2), {5,7}-(0 20 2).

PI-T5 N27 (4 40) (15 20 0)
6 6 7 6 7 7 8 8 6 9 6 7 7 8 6
7 9 9 10 8 10 10 9 8 10 9 9 10 10 8
Aut D (1 2 3 4 5) (6 7 8 9 10) (1 3 5 2 4) (6 8 10 7 9)
(1 4 2 5 3) (6 9 7 10 8) (1 5 4 3 2) (6 10 9 8 7)
(0 11) (1 6) (2 10) (3 9) (4 8) (5 7) (0 11) (1 7) (2 6) (3 10) (4 9) (5 8)
(0 11) (1 8) (2 7) (3 6) (4 10) (5 9) (0 11) (1 9) (2 8) (3 7) (4 6) (5 10)
(0 11) (1 10) (2 9) (3 8) (4 7) (5 6)
Transitivity sets: {0,11}-(2 20 0), {1,2,3,4,5,6,7,8,9,10}-(0 20 2).

PII-T1 N1 (16 28) (37 4 0)
6 7 9 6 7 6 6 7 8 8 6 7 8 6 7
7 9 10 10 8 8 9 10 10 9 9 10 10 8 9
Transitivity sets: {11}-(6 14 2), {0}-(2 14 6), {1}-(2 14 6),
{4}-(2 14 6), {7}-(2 14 6), {10}-(2 14 6),
{2}-(0 14 8), {3}-(0 14 8), {5}-(0 14 8),
{6}-(0 14 8), {8}-(0 14 8), {9}-(0 14 8).

PII-T1 N2 (16 28) (35 6 0)
6 7 9 6 7 6 6 7 8 8 6 7 8 6 7
7 10 10 9 8 8 10 9 9 10 9 10 10 8 9
Transitivity sets: {11}-(6 14 2), {1}-(4 14 4), {0}-(2 14 6),
{7}-(2 14 6), {9}-(2 14 6), {2}-(0 14 8),
{3}-(0 14 8), {4}-(0 14 8), {5}-(0 14 8),
{6}-(0 14 8), {8}-(0 14 8), {10}-(0 14 8).

PII-T1 N3 (4 40) (25 7 1)
6 6 7 9 7 6 6 8 8 7 8 6 6 7 7
8 10 9 10 9 8 7 10 9 10 10 9 9 10 8
Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2),
{2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2),
{5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2),
{8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PII-T1 N4 (8 36) (25 9 1)
6 6 7 9 7 7 6 6 8 8 6 8 7 6 7
8 9 10 10 8 9 7 10 10 9 9 10 10 8 9
Transitivity sets: {0}-(2 18 2), {7}-(2 18 2), {10}-(2 18 2),
{11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4),
{3}-(0 18 4), {4}-(0 18 4), {5}-(0 18 4),
{6}-(0 18 4), {8}-(0 18 4), {9}-(0 18 4).

PII-T1 N5 (8 36) (27 7 1)
6 6 7 9 7 7 6 6 8 8 6 8 7 6 7
8 10 9 10 8 10 7 9 9 10 9 10 10 8 9
Transitivity sets: {0}-(2 18 2), {7}-(2 18 2), {9}-(2 18 2),
{11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4),
{3}-(0 18 4), {4}-(0 18 4), {5}-(0 18 4),
{6}-(0 18 4), {8}-(0 18 4), {10}-(0 18 4).

PII-T1 N6 (8 36) (24 10 1)
6 6 7 9 7 7 6 8 8 6 8 6 7 6 7
8 9 10 10 8 9 7 10 10 9 9 10 10 8 9
Aut D (0 11) (1 9) (2 8) (3 7) (4 6) (5 10)
Transitivity sets: {0,11}-(2 18 2), {3,7}-(2 18 2); {1,9}-(0 18 4),
{2,8}-(0 18 4), {4,6}-(0 18 4), {5,10}-(0 18 4).

PII-T1 N7 (4 40) (25 7 1)

6	6	7	9	7	7	6	8	8	6	8	6	7	6	7
8	10	9	10	8	10	7	10	9	9	10	9	10	8	9

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PII-T1 N8 (8 36) (29 5 1)

6	7	9	6	7	6	6	7	8	8	6	8	7	6	7
8	9	10	10	8	7	9	10	10	9	9	10	10	8	9

Transitivity sets: {0}-(2 18 2), {7}-(2 18 2), {10}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4), {3}-(0 18 4), {4}-(0 18 4), {5}-(0 18 4), {6}-(0 18 4), {8}-(0 18 4), {9}-(0 18 4).

PII-T1 N9 (8 36) (29 5 1)

6	7	9	6	7	6	6	7	8	8	6	8	7	6	7
8	10	10	9	8	7	10	9	9	10	9	10	10	8	9

Transitivity sets: {0}-(2 18 2), {7}-(2 18 2), {9}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4), {3}-(0 18 4), {4}-(0 18 4), {5}-(0 18 4), {6}-(0 18 4), {8}-(0 18 4), {10}-(0 18 4).

PII-T1 N10 (8 36) (29 5 1)

6	7	9	6	7	6	6	7	8	7	6	8	8	6	7
8	10	10	9	8	8	10	9	9	10	9	10	10	7	9

Transitivity sets: {0}-(2 18 2), {7}-(2 18 2), {9}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4), {3}-(0 18 4), {4}-(0 18 4), {5}-(0 18 4), {6}-(0 18 4), {8}-(0 18 4), {10}-(0 18 4).

PII-T1 N11 (8 36) (27 7 1)

7	6	6	9	6	7	6	8	8	7	8	6	6	7	7
9	8	10	10	8	9	7	10	9	10	10	9	9	10	8

Transitivity sets: {0}-(2 18 2), {8}-(2 18 2), {10}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4), {3}-(0 18 4), {4}-(0 18 4), {5}-(0 18 4), {6}-(0 18 4), {7}-(0 18 4), {9}-(0 18 4).

PII-T1 N12 (20 24) (38 5 0)

7	6	6	9	6	8	6	7	7	8	8	6	6	7	7
9	8	10	10	8	9	7	10	9	10	10	9	9	10	8

Transitivity sets: {4}-(4 12 6), {10}-(4 12 6), {11}-(4 12 6), {0}-(2 12 8), {1}-(2 12 8), {6}-(2 12 8), {8}-(2 12 8), {2}-(0 12 10), {3}-(0 12 10), {5}-(0 12 10), {7}-(0 12 10), {9}-(0 12 10).

PII-T1 N13 (20 24) (35 8 0)

7	6	6	9	7	7	6	6	8	8	6	8	7	7	6
9	8	10	10	8	10	7	9	9	10	9	10	10	9	8

Transitivity sets: {0}-(6 12 4), {6}-(4 12 6), {7}-(2 12 8), {8}-(2 12 8), {9}-(2 12 8), {10}- (2 12 8), {11}-(2 12 8), {1}-(0 12 10), {2}-(0 12 10), {3}-(0 12 10), {4}-(0 12 10), {5}-(0 12 10).

PII-T1 N14 (8 36) (26 8 1)

7	6	6	9	7	7	6	8	8	6	8	6	7	7	6
9	8	10	10	8	10	7	10	9	9	10	9	10	9	8

Transitivity sets: {0}-(2 18 2), {8}-(2 18 2), {10}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4), {3}-(0 18 4), {4}-(0 18 4), {5}-(0 18 4), {6}-(0 18 4), {7}-(0 18 4), {9}-(0 18 4).

PII-T1 N15 (4 40) (25 10 0)

7	6	9	6	6	6	7	8	8	7	8	6	6	7	7
9	8	10	10	8	7	9	10	9	10	10	9	9	10	8

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PII-T1 N16 (8 36) (30 7 0)

7	6	9	6	7	6	6	7	8	8	6	8	7	7	6
9	8	10	10	8	7	9	10	10	9	9	10	10	9	8

Transitivity sets: {0}-(4 18 0), {6}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4), {3}-(0 18 4), {4}-(0 18 4), {5}-(0 18 4), {7}-(0 18 4), {8}-(0 18 4), {9}-(0 18 4), {10}-(0 18 4).

PII-T1 N17 (8 36) (25 9 1)

8	6	7	6	7	7	6	7	6	9	8	6	7	8	6
9	9	10	10	8	9	8	10	8	10	10	9	9	10	7

Transitivity sets: {0}-(2 18 2), {4}-(2 18 2), {9}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4), {3}-(0 18 4), {5}-(0 18 4), {6}-(0 18 4), {7}-(0 18 4), {8}-(0 18 4), {10}-(0 18 4).

PII-T1 N18 (8 36) (20 8 3)

8	6	7	6	7	7	6	7	6	8	9	6	7	8	6
9	9	10	10	8	9	9	10	8	10	10	8	9	10	7

Aut D (0 11)(1 6)(2 7)(3 8)(4 9)(5 10)

Transitivity sets: {0,11}-(2 18 2), {4,9}-(2 18 2), {1,6}-(0 18 4), {2,7}-(0 18 4), {3,8}-(0 18 4), {5,10}-(0 18 4).

PII-T1 N19 (4 40) (26 6 1)

8	6	7	6	7	8	6	7	6	7	9	6	8	7	6
9	9	10	10	8	10	9	10	8	9	10	8	10	9	7

Aut D (0 11)(1 6)(2 7)(3 8)(4 9)(5 10)

Transitivity sets: {0,11}-(2 20 0), {1,6}-(0 20 2), {2,7}-(0 20 2), {3,8}-(0 20 2), {4,9}-(0 20 2), {5,10}-(0 20 2).

PII-T2 N1 (8 36) (23 11 1)

6	6	7	9	7	6	7	6	7	8	6	8	8	6	7
8	9	10	10	8	8	9	10	10	9	9	10	10	7	9

Transitivity sets: {0}-(2 18 2), {2}-(2 18 2), {8}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {3}-(0 18 4), {4}-(0 18 4), {5}-(0 18 4), {6}-(0 18 4), {7}-(0 18 4), {9}-(0 18 4), {10}-(0 18 4).

PII-T2 N2 (12 32) (22 8 3)

6	6	7	9	7	6	7	8	6	7	8	6	8	6	7
8	9	10	10	8	8	9	10	9	10	9	10	10	7	9

Aut D (0 11)(1 9)(2 8)(3 7)(4 6)(5 10)

Transitivity sets: {0,11}-(2 16 4), {2,8}-(2 16 4), {3,7}-(2 16 4), {1,9}-(0 16 6), {4,6}-(0 16 6), {5,10}-(0 16 6).

PII-T2 N3 (16 28) (35 6 0)

7	6	6	9	7	7	8	6	6	8	8	6	7	7	6
9	8	10	10	8	10	9	7	9	10	10	9	9	10	8

Transitivity sets: {11}-(6 14 2), {4}-(4 14 4), {0}-(2 14 6), {8}-(2 14 6), {10}-(2 14 6), {1}-(0 14 8), {2}-(0 14 8), {3}-(0 14 8), {5}-(0 14 8), {6}-(0 14 8), {7}-(0 14 8), {9}-(0 14 8).

PII-T3 N1 (16 28) (33 5 1)

6 6 8 9 6 7 6 7 8 7 8 6 6 7 7
9 7 10 10 8 10 8 10 9 9 10 9 10 9 8

Transitivity sets: {5}-(6 14 2), {4}-(4 14 4), {0}-(2 14 6), {10}-(2 14 6), {11}-(2 14 6), {1}-(0 14 8), {2}-(0 14 8), {3}-(0 14 8), {6}-(0 14 8), {7}-(0 14 8), {8}-(0 14 8), {9}-(0 14 8).

PII-T3 N2 (8 36) (24 4 3)

6 6 7 9 6 8 6 7 8 7 8 6 6 7 7
9 8 10 10 7 10 8 10 9 9 10 9 10 9 8

Transitivity sets: {0}-(2 18 2), {5}-(2 18 2), {10}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4), {3}-(0 18 4), {4}-(0 18 4), {6}-(0 18 4), {7}-(0 18 4), {8}-(0 18 4), {9}-(0 18 4).

PII-T3 N4 (8 36) (24 4 3)

6 6 7 9 6 7 6 8 8 7 8 6 6 7 7
9 8 10 10 8 10 7 10 9 9 10 9 10 9 8

Transitivity sets: {0}-(2 18 2), {5}-(2 18 2), {10}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4), {3}-(0 18 4), {4}-(0 18 4), {6}-(0 18 4), {7}-(0 18 4), {8}-(0 18 4), {9}-(0 18 4).

PII-T3 N5 (16 28) (32 6 1)

6 6 7 9 6 8 6 7 7 8 8 6 6 7 7
9 8 10 10 8 10 7 10 9 9 10 9 10 9 8

Transitivity sets: {10}-(6 14 2), {6}-(4 14 4), {0}-(2 14 6), {5}-(2 14 6), {11}-(2 14 6), {1}-(0 14 8), {2}-(0 14 8), {3}-(0 14 8), {4}-(0 14 8), {7}-(0 14 8), {8}-(0 14 8), {9}-(0 14 8).

PII-T3 N6 (16 28) (36 2 1)

6 7 8 6 6 8 7 7 7 6 8 6 9 6 7
9 10 9 10 8 10 9 10 9 8 10 9 10 7 8

Transitivity sets: {10}-(6 14 2), {0}-(2 14 6), {5}-(2 14 6), {6}-(2 14 6), {9}-(2 14 6), {11}-(2 14 6), {1}-(0 14 8), {2}-(0 14 8), {3}-(0 14 8), {4}-(0 14 8), {7}-(0 14 8), {8}-(0 14 8).

PII-T3 N8 (8 36) (28 0 3)

6 7 8 6 7 8 6 7 6 7 8 6 9 6 7
9 10 9 10 9 10 8 10 8 9 10 9 10 7 8

Aut D (0 11)(1 6)(2 7)(3 8)(4 9)(5 10)

Transitivity sets: {0,11}-(2 18 2), {5,10}-(2 18 2), {1,6}-(0 18 4), {2,7}-(0 18 4), {3,8}-(0 18 4), {4,9}-(0 18 4).

PII-T3 N10 (8 36) (27 7 1)

6 7 7 9 6 7 6 8 8 6 8 7 6 7 6
9 8 10 10 8 10 7 10 9 9 10 9 10 9 8

Transitivity sets: {0}-(2 18 2), {8}-(2 18 2), {10}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4), {3}-(0 18 4), {4}-(0 18 4), {5}-(0 18 4), {6}-(0 18 4), {7}-(0 18 4), {9}-(0 18 4).

PII-T3 N11 (8 36) (22 6 3)

6 6 7 9 6 7 6 8 8 7 8 6 6 7 7
10 8 9 10 8 9 7 10 9 10 10 9 9 10 8

Transitivity sets: {0}-(2 18 2), {5}-(2 18 2), {10}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4), {3}-(0 18 4), {4}-(0 18 4), {6}-(0 18 4), {7}-(0 18 4), {8}-(0 18 4), {9}-(0 18 4).

PII-T3 N12 (24 20) (36 6 1)

6	6	7	9	6	8	6	7	7	8	8	6	6	7	7
10	8	9	10	8	9	7	10	9	10	10	9	9	10	8

Aut D (0 9) (1 5) (2 7) (4 10) (6 11)

Transitivity sets: {4,10}-(4 10 8), {6,11}-(4 10 8), {0,9}-(2 10 10), {1,5}-(2 10 10), {2,7}-(0 10 12), {3}-(0 10 12), {8}-(0 10 12).

PII-T3 N13 (12 32) (35 4 0)

7	6	8	9	6	7	6	7	8	6	8	7	6	6	7
9	7	10	10	8	10	8	10	9	9	10	9	10	9	8

Aut D (0 11) (4 5) (6 8) (7 10)

Transitivity sets: {4,5}-(4 16 2), {0,11}-(2 16 4), {1}-(0 16 6), {2}-(0 16 6), {3}-(0 16 6), {6,8}-(0 16 6), {7,10}-(0 16 6), {9}-(0 16 6).

PII-T3 N14 (4 40) (26 6 1)

7	6	7	9	6	8	6	7	8	6	8	7	6	6	7
9	8	10	10	7	10	8	10	9	9	10	9	10	9	8

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PII-T3 N16 (8 36) (28 6 1)

7	6	7	9	6	7	6	8	8	6	8	7	6	6	7
10	8	9	10	8	9	7	10	10	9	9	10	10	9	8

Aut D (0 11) (1 9) (2 8) (3 7) (4 6) (5 10)

Transitivity sets: {0,11}-(2 18 2), {3,7}-(2 18 2), {1,9}-(0 18 4), {2,8}-(0 18 4), {4,6}-(0 18 4), {5,10}-(0 18 4).

PII-T3 N17 (16 28) (26 12 1)

7	7	8	6	6	7	6	7	6	8	9	7	6	8	6
10	9	9	10	8	9	9	10	8	10	10	8	9	10	7

Aut D (0 11) (1 6) (2 7) (3 8) (4 9) (5 10)

Transitivity sets: {0,11}-(2 14 6), {2,7}-(2 14 6), {3,8}-(2 14 6), {4,9}-(2 14 6), {1,6}-(0 14 8), {5,10}-(0 14 8).

PII-T3 N18 (12 32) (26 10 1)

9	6	7	6	6	6	7	8	8	7	8	6	6	7	7
10	8	9	10	7	8	9	10	9	10	10	9	9	10	8

Aut D (0 5) (1 6) (2 3) (4 9) (7 8) (10 11)

Transitivity sets: {0,5}-(2 16 4), {7,8}-(2 16 4), {10,11}-(2 16 4), {1,6}-(0 16 6), {2,3}-(0 16 6), {4,9}-(0 16 6).

PII-T3 N19 (16 28) (31 7 1)

9	6	7	6	6	6	7	8	8	7	8	6	6	7	7
10	8	10	9	7	8	10	10	9	9	10	9	10	9	8

Transitivity sets: {7}-(4 14 4), {8}-(4 14 4), {0}-(2 14 6), {5}-(2 14 6), {10}-(2 14 6), {11}-(2 14 6), {1}-(0 14 8), {2}-(0 14 8), {3}-(0 14 8), {4}-(0 14 8), {6}-(0 14 8), {9}-(0 14 8).

PII-T4 N1 (12 32) (24 12 1)

6	6	7	9	7	7	6	6	8	8	6	8	7	6	7
9	8	9	10	8	9	7	10	10	10	9	9	10	8	10

Aut D (0 11) (1 9) (2 8) (3 7) (4 6) (5 10)

Transitivity sets: {0,11}-(2 16 4), {3,7}-(2 16 4), {5,10}-(2 16 4), {1,9}-(0 16 6), {2,8}-(0 16 6), {4,6}-(0 16 6).

PII-T4 N2 (8 36) (20 14 1)

6	6	8	9	7	7	6	7	8	6	8	6	7	6	7
9	7	10	10	8	9	8	10	9	10	10	9	9	8	10

Aut D (0 10)(1 4)(2 7)(3 8)(5 11)(6 9)

Transitivity sets: {0,10}-(2 18 2), {5,11}-(2 18 2), {1,4}-(0 18 4), {2,7}-(0 18 4), {3,8}-(0 18 4), {6,9}-(0 18 4).

PII-T4 N4 (12 32) (16 14 3)

6	6	7	9	7	7	6	8	8	6	8	6	7	6	7
9	8	9	10	8	9	7	10	10	10	9	9	10	8	10

Aut D (0 11)(1 9)(2 8)(3 7)(4 6)(5 10)

Transitivity sets: {0,11}-(2 16 4), {3,7}-(2 16 4), {5,10}-(2 16 4), {1,9}-(0 16 6), {2,8}-(0 16 6), {4,6}-(0 16 6).

PII-T4 N5 (8 36) (16 12 3)

6	6	7	9	7	7	6	8	8	6	8	6	7	6	7
9	8	10	10	8	9	7	10	9	10	10	9	9	8	10

Transitivity sets: {0}-(2 18 2), {5}-(2 18 2), {10}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4), {3}-(0 18 4), {4}-(0 18 4), {6}-(0 18 4), {7}-(0 18 4), {8}-(0 18 4), {9}-(0 18 4).

PII-T4 N6 (16 28) (22 16 1)

6	6	7	9	7	7	6	7	8	6	8	6	7	6	8
9	8	9	10	8	9	8	10	10	10	9	9	10	7	10

Transitivity sets: {0}-(2 14 6), {3}-(2 14 6), {4}-(2 14 6), {5}-(2 14 6), {7}-(2 14 6), {9}-(2 14 6), {10}-(2 14 6), {11}-(2 14 6), {1}-(0 14 8), {2}-(0 14 8), {6}-(0 14 8), {8}-(0 14 8).

PII-T4 N7 (12 32) (16 14 3)

6	6	7	9	7	7	6	7	8	6	8	6	7	6	8
9	8	10	10	8	9	8	10	9	10	10	9	9	7	10

Aut D (0 11)(1 6)(2 7)(3 8)(4 9)(5 10)

Transitivity sets: {0,11}-(2 16 4), {4,9}-(2 16 4), {5,10}-(2 16 4), {1,6}-(0 16 6), {2,7}-(0 16 6), {3,8}-(0 16 6).

PII-T4 N8 (8 36) (23 11 1)

6	7	8	6	7	7	6	8	6	9	8	6	7	6	7
9	10	9	10	8	9	7	10	8	10	10	9	9	8	10

Transitivity sets: {0}-(2 18 2), {5}-(2 18 2), {10}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4), {3}-(0 18 4), {4}-(0 18 4), {6}-(0 18 4), {7}-(0 18 4), {8}-(0 18 4), {9}-(0 18 4).

PII-T4 N9 (12 32) (20 10 3)

6	7	8	6	7	7	6	7	6	9	8	6	7	6	8
9	10	9	10	8	9	8	10	8	10	10	9	9	7	10

Aut D (0 11)(1 6)(2 7)(3 8)(4 9)(5 10)

Transitivity sets: {0,11}-(2 16 4), {4,9}-(2 16 4), {5,10}-(2 16 4), {1,6}-(0 16 6), {2,7}-(0 16 6), {3,8}-(0 16 6).

PII-T4 N11 (8 36) (26 8 1)

6	7	7	9	6	7	6	8	8	6	8	7	6	6	7
9	8	10	10	8	9	7	10	9	10	10	9	9	8	10

Transitivity sets: {0}-(2 18 2), {8}-(2 18 2), {10}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4), {3}-(0 18 4), {4}-(0 18 4), {5}-(0 18 4), {6}-(0 18 4), {7}-(0 18 4), {9}-(0 18 4).

PII-T4 N12 (16 28) (28 10 1)

6	7	7	9	6	7	6	7	8	6	8	7	6	6	8
9	8	10	10	8	9	8	10	9	10	10	9	9	7	10

Aut D (0 11)(1 6)(2 7)(3 8)(4 9)(5 10)
Transitivity sets: {0,11}-(2 14 6), {3,8}-(2 14 6), {4,9}-(2 14 6), {5,10}-(2 14 6), {1,6}-(0 14 8), {2,7}-(0 14 8).

PII-T4 N13 (16 28) (30 8 1)

6	7	8	7	7	7	6	6	6	9	8	7	6	6	8
9	10	9	10	8	9	8	10	8	10	10	9	9	7	10

Aut D (0 11)(1 6)(2 7)(3 8)(4 9)(5 10)
Transitivity sets: {0,11}-(2 14 6), {3,8}-(2 14 6), {4,9}-(2 14 6), {5,10}-(2 14 6), {1,6}-(0 14 8), {2,7}-(0 14 8).

PII-T4 N17 (4 40) (24 11 0)

7	6	8	9	7	6	6	7	8	6	8	7	6	6	7
9	7	10	10	8	9	8	10	9	10	10	9	9	8	10

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PII-T4 N18 (4 40) (20 12 1)

7	6	7	9	7	6	6	8	8	6	8	7	6	6	7
9	8	10	10	8	9	7	10	9	10	10	9	9	8	10

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PII-T4 N19 (8 36) (20 14 1)

7	6	7	9	7	6	6	7	8	6	8	7	6	6	8
9	8	10	10	8	9	8	10	9	10	10	9	9	7	10

Aut D (0 11)(1 6)(2 7)(3 8)(4 9)(5 10)
Transitivity sets: {0,11}-(2 18 2), {4,9}-(2 18 2), {1,6}-(0 18 4), {2,7}-(0 18 4), {3,8}-(0 18 4), {5,10}-(0 18 4).

PII-T4 N20 (4 40) (23 12 0)

7	7	8	6	7	6	6	8	6	9	8	7	6	6	7
9	10	9	10	8	9	7	10	8	10	10	9	9	8	10

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PII-T4 N21 (8 36) (20 14 1)

7	7	8	6	7	6	6	7	6	9	8	7	6	6	8
9	10	9	10	8	9	8	10	8	10	10	9	9	7	10

Aut D (0 11)(1 6)(2 7)(3 8)(4 9)(5 10)
Transitivity sets: {0,11}-(2 18 2), {4,9}-(2 18 2), {1,6}-(0 18 4), {2,7}-(0 18 4), {3,8}-(0 18 4), {5,10}-(0 18 4).

PII-T5 N2 (4 40) (27 8 0)

6	7	8	6	7	6	8	7	6	9	8	6	7	7	6
8	10	9	10	8	7	10	9	9	10	10	8	9	10	9

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PII-T5 N4 (24 20) (20 16 3)

6 6 7 9 7 6 7 7 8 6 8 6 7 6 8
9 8 9 10 8 8 9 10 10 10 9 9 10 7 10

Aut D (1 4)(2 3)(6 9)(7 8) (0 11)(1 2)(3 4)(6 8)(7 9)
(0 11)(1 3)(2 4)(6 7)(8 9) (0 11)(1 6)(2 7)(3 8)(4 9)(5 10)
(0 11)(1 9)(2 8)(3 7)(4 6)(5 10) (5 10)(1 7 4 8)(2 6 3 9)
(5 10)(1 8 4 7)(2 9 3 6)

Transitivity sets: {0,11}-(2 10 10), {5,10}-(2 10 10),
{1,2,3,4,6,7,8,9}-(2 10 10).

PII-T5 N7 (12 32) (20 10 3)

6 6 7 9 7 6 7 8 7 6 8 6 8 6 7
9 8 10 10 8 8 9 10 9 10 10 9 9 7 10

Aut D (0 11)(1 9)(2 8)(3 7)(4 6)(5 10)

Transitivity sets: {0,11}-(2 16 4), {2,8}-(2 16 4), {5,10}-(2 16 4),
{1,9}-(0 16 6), {3,7}-(0 16 6), {4,6}-(0 16 6).

PII-T5 N8 (8 36) (24 10 1)

6 6 7 9 7 7 8 6 8 6 8 6 7 7 6
9 8 10 10 8 9 10 7 9 10 10 9 9 10 8

Aut D (1 4)(2 3)(6 9)(7 8)

Transitivity sets: {0}-(2 18 2), {5}-(2 18 2), {10}-(2 18 2),
{11}-(2 18 2), {1,4}-(0 18 4), {2,3}-(0 18 4),
{6,9}-(0 18 4), {7,8}-(0 18 4).

PII-T5 N9 (8 36) (22 6 3)

6 6 7 9 7 7 8 6 7 6 8 6 8 7 6
9 8 10 10 8 9 10 8 9 10 10 9 9 10 7

Aut D (1 4)(2 3)(6 9)(7 8) (0 11)(1 6)(2 7)(3 8)(4 9)(5 10)
(0 11)(1 9)(2 8)(3 7)(4 6)(5 10)

Transitivity sets: {0,11}-(2 18 2), {5,10}-(2 18 2), {1,4,6,9}-(0 18 4)
{2,3,7,8}-(0 18 4).

PII-T5 N10 (8 36) (26 2 3)

6 7 8 6 7 6 8 6 7 9 8 6 6 7 7
9 10 9 10 8 7 10 8 9 10 10 9 8 10 9

Aut D (1 4)(2 3)(6 9)(7 8)

Transitivity sets: {0}-(2 18 2), {5}-(2 18 2), {10}-(2 18 2),
{11}-(2 18 2), {1,4}-(0 18 4), {2,3}-(0 18 4),
{6,9}-(0 18 4), {7,8}-(0 18 4).

PII-T5 N12 (8 36) (30 4 1)

6 7 8 6 7 6 8 7 6 9 8 6 7 7 6
9 10 9 10 8 7 10 9 8 10 10 9 9 10 8

Aut D (1 4)(2 3)(6 9)(7 8) (0 5)(1 6)(2 3)(4 9)(7 8)(10 11)
(0 5)(1 9)(4 6)(10 11)

Transitivity sets: {0,5}-(2 18 2), {10,11}-(2 18 2), {2,3}-(0 18 4),
{7,8}-(0 18 4), {1,4,6,9}-(0 18 4).

PII-T5 N13 (8 36) (26 8 1)

6 6 7 9 7 6 7 6 7 8 6 8 8 6 7
10 8 9 10 8 8 9 9 10 10 9 10 9 7 10

Transitivity sets: {0}-(2 18 2), {2}-(2 18 2), {8}-(2 18 2),
{11}-(2 18 2), {1}-(0 18 4), {3}-(0 18 4),
{4}-(0 18 4), {5}-(0 18 4), {6}-(0 18 4),
{7}-(0 18 4), {9}-(0 18 4), {10}-(0 18 4).

PII-T5 N14 (4 40) (26 6 1)
6 6 7 7 7 6 7 8 9 6 8 6 8 6 7
10 8 9 10 8 8 9 10 10 9 10 7 9 9 10
Aut D (0 11)(1 9)(2 8)(3 7)(4 6)(5 10)
Transitivity sets: {0,11}-(2 20 0), {1,9}-(0 20 2), {2,8}-(0 20 2),
{3,7} -(0 20 2), {4,6}-(0 20 2), {5,10}-(0 20 2).

PII-T5 N15 (12 32) (22 8 3)
6 6 7 9 7 6 7 8 7 6 8 6 8 6 7
10 8 9 10 8 8 9 10 10 9 10 9 9 7 10
Aut D (0 11)(1 9)(2 8)(3 7)(4 6)(5 10)
Transitivity sets: {0,11}-(2 16 4), {2,8}-(2 16 4), {5,10}-(2 16 4),
{1,9}-(0 16 6), {3,7}-(0 16 6), {4,6}-(0 16 6).

PII-T5 N17 (8 36) (26 8 1)
6 6 7 9 7 7 8 6 7 6 8 6 8 7 6
10 8 9 10 8 9 10 8 10 9 10 9 9 10 7
Aut D (0 11)(1 9)(2 8)(3 7)(4 6)(5 10)
Transitivity sets: {0,11}-(2 18 2), {5,10}-(2 18 2), {1,9}-(0 18 4),
{2,8}-(0 18 4), {3,7}-(0 18 4), {4,6}-(0 18 4).

PII-T5 N20 (16 28) (26 12 1)
7 6 7 9 7 6 7 6 8 6 8 7 6 6 8
10 8 9 10 8 8 9 9 10 10 9 10 9 7 10
Aut D (0 11)(1 6)(2 7)(3 8)(4 9)(5 10)
Transitivity sets: {0,11}-(2 14 6), {2,7}-(2 14 6), {3,8}-(2 14 6),
{4,9}-(2 14 6), {1,6}-(0 14 8), {5,10}-(0 14 8).

PII-T5 N22 (4 40) (32 3 0)
7 7 8 6 7 6 8 6 6 9 8 7 6 7 6
10 9 10 9 8 7 9 10 8 10 10 9 9 10 8
Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2),
{2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2),
{5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2),
{8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PII-T7 N1 (4 40) (23 9 1)
6 6 7 9 6 7 8 8 6 7 6 7 6 8 7
7 8 9 10 8 10 10 9 9 10 9 8 10 10 9
Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2),
{2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2),
{5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2),
{8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PII-T7 N2 (4 40) (20 6 3)
6 6 7 9 6 7 8 8 6 7 6 7 6 8 7
7 8 10 10 8 9 10 9 9 10 10 8 9 10 9
Aut D (0 11)(1 9)(2 8)(3 7)(4 6)(5 10)
Transitivity sets: {0,11}-(2 20 0), {1,9}-(0 20 2), {2,8}-(0 20 2),
{3,7}-(0 20 2), {4,6}-(0 20 2), {5,10}-(0 20 2).

PII-T7 N6 (8 36) (27 7 1)
6 6 7 9 6 8 7 7 6 8 6 7 6 8 7
7 8 10 10 8 9 10 9 9 10 10 8 9 10 9
Transitivity sets: {0}-(2 18 2), {6}-(2 18 2), {9}-(2 18 2),
{11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4),
{3}-(0 18 4), {4}-(0 18 4), {5}-(0 18 4),
{7}-(0 18 4), {8}-(0 18 4), {10}-(0 18 4).

PII-T7 N7 (12 32) (33 6 0)

6 6 7 9 6 8 7 7 6 8 6 7 6 8 7

7 9 10 10 8 9 10 9 8 10 10 9 9 10 8

Transitivity sets: {6}-(4 16 2), {0}-(2 16 4), {9}-(2 16 4), {10}-(2 16 4), {11}-(2 16 4), {1}-(0 16 6), {2}-(0 16 6), {3}-(0 16 6), {4}-(0 16 6), {5}-(0 16 6), {7}-(0 16 6), {8}-(0 16 6).

PII-T7 N9 (4 40) (25 10 0)

6 6 8 9 6 7 7 8 6 7 6 8 6 7 7

7 9 10 10 8 10 9 9 8 10 9 10 10 9 8

Aut D (0 11)(2 4)(3 5)(6 10)(7 8)

Transitivity sets: {0,11}-(2 20 0), {1}-(0 20 2), {2,4}-(0 20 2), {3,5}-(0 20 2), {6,10}-(0 20 2), {7,8}-(0 20 2), {9}-(0 20 2).

PII-T7 N13 (4 40) (25 7 1)

6 7 6 9 6 7 8 8 6 7 7 6 6 8 7

7 8 10 10 8 9 10 9 9 10 10 8 9 10 9

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PII-T7 N22 (12 32) (26 10 1)

6 7 9 6 6 7 8 6 7 8 7 6 6 8 7

7 9 10 10 8 9 10 8 10 9 10 9 9 10 8

Aut D (0 5)(1 6)(2 3)(4 9)(7 8)(10 11)

Transitivity sets: {0,5}-(2 16 4), {2,3}-(2 16 4), {10,11}-(2 16 4), {1,6}-(0 16 6), {4,9}-(0 16 6), {7,8}-(0 16 6).

PII-T7 N26 (8 36) (29 5 1)

6 8 7 6 6 7 8 8 6 7 6 7 9 6 7

7 10 10 9 8 9 10 9 9 10 10 8 10 8 9

Transitivity sets: {0}-(2 18 2), {1}-(2 18 2), {3}-(2 18 2), {11}-(2 18 2), {2}-(0 18 4), {4}-(0 18 4), {5}-(0 18 4), {6}-(0 18 4), {7}-(0 18 4), {8}-(0 18 4), {9}-(0 18 4), {10}-(0 18 4).

PII-T7 N28 (4 40) (24 8 1)

6 8 7 6 7 8 6 6 7 8 6 7 9 6 7

7 9 10 10 9 10 9 8 9 10 8 10 10 9 8

Aut D (0 11)(1 6)(2 7)(3 8)(4 9)(5 10)

Transitivity sets: {0,11}-(2 20 0), {1,6}-(0 20 2), {2,7}-(0 20 2), {3,8}-(0 20 2), {4,9}-(0 20 2), {5,10}-(0 20 2).

PII-T7 N29 (8 36) (26 8 1)

6 7 8 9 6 7 6 8 6 7 7 8 6 6 7

7 9 10 10 8 10 9 9 8 10 9 10 10 9 8

Aut D (0 11)(1 9)(2 8)(3 7)(4 6)(5 10)

Transitivity sets: {0,11}-(2 18 2), {2,8}-(2 18 2), {1,9}-(0 18 4), {3,7}-(0 18 4), {4,6}-(0 18 4), {5,10}-(0 18 4).

PII-T7 N33 (4 40) (26 6 1)

7 7 6 6 6 8 6 8 7 8 7 6 9 6 7

8 10 9 10 7 10 9 9 9 10 10 8 10 8 9

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PII-T7 N36 (16 28) (24 14 1)

7	6	9	6	6	6	7	8	7	8	8	6	6	7	7
9	8	10	10	7	8	10	9	9	10	10	9	9	10	8

Aut D (0 5)(1 6)(2 3)(4 9)(7 8)(10 11)

Transitivity sets: {0,5}-(2 14 6), {1,6}-(2 14 6), {7,8}-(2 14 6), {10,11}-(2 14 6), {2,3}-(0 14 8), {4,9}-(0 14 8).

PII-T7 N37 (12 32) (21 18 0)

7	6	8	6	6	6	7	8	7	8	9	6	6	7	7
9	9	10	10	7	8	10	9	9	10	10	8	9	10	8

Aut D (0 11)(2 4)(3 5)(7 8)(9 10)

Transitivity sets: {0,11}-(2 16 4), {1}-(2 16 4), {6}-(2 16 4), {7,8}-(2 16 4), {2,4}-(0 16 6), {3,5}-(0 16 6), {9,10}-(0 16 6).

PII-T7 N38 (4 40) (24 8 1)

7	6	7	6	6	8	6	8	7	8	6	7	9	6	7
9	8	10	10	7	10	9	9	9	10	8	10	10	9	8

Aut D (0 11)(1 6)(2 7)(3 8)(4 9)(5 10)

Transitivity sets: {0,11}-(2 20 0), {1,6}-(0 20 2), {2,7}-(0 20 2), {3,8}-(0 20 2), {4,9}-(0 20 2), {5,10}-(0 20 2).

PII-T7 N42 (8 36) (26 2 3)

7	7	6	6	6	8	6	8	7	8	7	6	9	6	7
9	10	9	10	7	10	8	9	9	10	10	9	10	8	8

Aut D (0 11)(1 6)(2 7)(3 8)(4 9)(5 10)

Transitivity sets: {0,11}-(2 18 2), {5,10}-(2 18 2), {1,6}-(0 18 4), {2,7}-(0 18 4), {3,8}-(0 18 4), {4,9}-(0 18 4).

PII-T7-N43 (16 28) (35 3 1)

7	7	6	6	6	8	6	7	8	8	7	6	9	6	7
9	10	9	10	8	10	7	9	9	10	10	9	10	8	8

Transitivity sets: {10}-(6 14 2), {0}-(2 14 6), {5}-(2 14 6), {6}-(2 14 6), {9}-(2 14 6), {11}-(2 14 6), {1}-(0 14 8), {2}-(0 14 8), {3}-(0 14 8), {4}-(0 14 8), {7}-(0 14 8), {8}-(0 14 8).

PII-T7 N44 (8 36) (22 12 1)

7	7	6	6	6	8	7	8	6	8	6	7	9	7	6
9	8	9	10	7	10	10	9	9	10	8	9	10	10	8

Transitivity sets: {0}-(2 18 2), {1}-(2 18 2), {6}-(2 18 2), {11}-(2 18 2), {2}-(0 18 4), {3}-(0 18 4), {4}-(0 18 4), {5}-(0 18 4), {7}-(0 18 4), {8}-(0 18 4), {9}-(0 18 4), {10}-(0 18 4).

PII-T7 N46 (12 32) (27 12 0)

7	7	9	6	6	6	7	8	6	8	8	7	6	7	6
9	8	10	10	7	8	10	9	9	10	10	9	9	10	8

Transitivity sets: {0}-(2 16 4), {1}-(2 16 4), {6}-(2 16 4), {7}-(2 16 4), {8}-(2 16 4), {11}-(2 16 4), {2}-(0 16 6), {3}-(0 16 6), {4}-(0 16 6), {5}-(0 16 6), {9}-(0 16 6), {10}-(0 16 6).

PII-T7 N47 (16 28) (22 16 1)

7	7	8	6	6	6	7	8	6	8	9	7	6	7	6
9	9	10	10	7	8	10	9	9	10	10	8	9	10	8

Aut D (0 11)(1 6)(2 7)(3 8)(4 9)(5 10)

Transitivity sets: {0,11}-(2 14 6), {1,6}-(2 14 6), {2,7}-(2 14 6), {3,8}-(2 14 6), {4,6}-(0 14 8), {5,10}-(0 14 8).

PII-T7 N48 (12 32) (28 11 0)

7	7	8	6	6	6	7	8	6	8	9	7	6	7	6
9	10	10	9	7	8	10	9	9	10	10	8	10	9	8

Transitivity sets: {0}-(2 16 4), {1}-(2 16 4), {3}-(2 16 4), {7}-(2 16 4), {8}-(2 16 4), {11}-(2 16 4), {2}-(0 16 6), {4}-(0 16 6), {5}-(0 16 6), {6}-(0 16 6), {9}-(0 16 6), {10}-(0 16 6).

PII-T7 N50 (8 36) (24 10 1)

7	6	6	9	6	7	6	8	7	8	8	6	6	7	7
10	8	9	10	8	9	7	10	10	9	10	9	10	9	8

Aut D (0 11)(1 9)(2 8)(3 7)(4 6)(5 10)

Transitivity sets: {0,11}-(2 18 2), {5,10}-(2 18 2), {1,9}-(0 18 4), {2,8}-(0 18 4), {3,7}-(0 18 4), {4,6}-(0 18 4).

PII-T7 N52 (16 28) (31 10 0)

7	6	8	6	6	6	7	8	7	8	9	6	6	7	7
10	9	10	9	7	8	9	9	10	10	10	8	10	9	8

Aut D (0 11)(2 4)(3 5)(7 8)

Transitivity sets: {7,8}-(4 14 4), {0,11}-(2 14 6), {3,5}-(2 14 6), {1}-(0 14 8), {2,4}-(0 14 8), {6}-(0 14 8), {9}-(0 14 8), {10}-(0 14 8).

PII-T8 N2 (8 36) (20 14 1)

6	6	7	9	7	6	8	8	6	7	6	7	6	8	7
7	9	10	10	8	8	10	9	10	9	9	10	8	10	9

Transitivity sets: {0}-(2 18 2), {3}-(2 18 2), {7}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4), {4}-(0 18 4), {5}-(0 18 4), {6}-(0 18 4), {8}-(0 18 4), {9}-(0 18 4), {10}-(0 18 4).

PII-T8 N3 (8 36) (23 11 1)

6	6	7	9	7	6	8	8	6	7	6	7	6	8	7
7	10	9	10	8	8	10	9	9	10	9	10	8	10	9

Transitivity sets: {0}-(2 18 2), {3}-(2 18 2), {7}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4), {4}-(0 18 4), {5}-(0 18 4), {6}-(0 18 4), {8}-(0 18 4), {9}-(0 18 4), {10}-(0 18 4).

PII-T8 N6 (4 40) (22 10 1)

6	6	7	9	7	6	8	8	6	7	6	7	6	8	7
7	10	9	10	9	8	10	9	8	10	9	10	9	10	8

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PII-T8 N9 (4 40) (20 12 1)

6	6	7	9	7	8	6	8	6	7	6	7	8	6	7
7	9	10	10	8	10	8	9	10	9	9	10	10	8	9

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PII-T8 N10 (4 40) (23 12 0)

6	6	7	9	7	8	6	8	6	7	6	7	8	6	7
7	10	9	10	8	10	8	9	9	10	9	10	10	8	9

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PII-T8 N14 (8 36) (27 10 0)

6	7	9	6	7	6	8	8	6	7	6	7	8	6	7
7	10	10	9	8	8	10	9	10	9	9	10	10	8	9

Transitivity sets: {0}-(2 18 2), {7}-(2 18 2), {9}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4), {3}-(0 18 4), {4}-(0 18 4), {5}-(0 18 4), {6}-(0 18 4), {8}-(0 18 4), {10}-(0 18 4).

PII-T8 N15 (4 40) (26 9 0)

6	6	7	9	7	7	6	8	6	8	6	8	7	6	7
8	10	9	10	8	10	7	9	9	10	9	10	10	8	9

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PII-T8 N16 (16 28) (35 6 0)

6	7	9	6	7	6	7	8	6	8	6	8	7	6	7
8	9	10	10	8	7	10	10	9	9	9	10	10	8	9

Transitivity sets: {0}-(4 14 4), {6}-(2 14 6), {7}-(2 14 6), {8}-(2 14 6), {9}-(2 14 6), {10}-(2 14 6), {11}-(2 14 6), {1}-(0 14 8), {2}-(0 14 8), {3}-(0 14 8), {4}-(0 14 8), {5}-(0 14 8).

PII-T8 N17 (8 36) (26 8 1)

7	6	6	9	7	7	6	8	6	8	6	8	7	7	6
9	8	10	10	8	10	7	9	9	10	9	10	10	9	8

Transitivity sets: {0}-(2 18 2), {8}-(2 18 2), {10}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4), {3}-(0 18 4), {4}-(0 18 4), {5}-(0 18 4), {6}-(0 18 4), {7}-(0 18 4), {9}-(0 18 4).

PII-T8 N18 (8 36) (21 16 0)

7	6	9	6	6	6	7	8	7	8	6	8	6	7	7
9	8	10	10	8	7	10	9	9	10	9	10	9	10	8

Transitivity sets: {0}-(2 18 2), {1}-(2 18 2), {6}-(2 18 2), {11}-(2 18 2), {2}-(0 18 4), {3}-(0 18 4), {4}-(0 18 4), {5}-(0 18 4), {7}-(0 18 4), {8}-(0 18 4), {9}-(0 18 4), {10}-(0 18 4).

PII-T8 N21 (8 36) (20 14 1)

7	6	9	6	7	6	7	8	6	8	6	8	7	7	6
9	8	10	10	8	7	10	9	9	10	9	10	9	10	8

Transitivity sets: {0}-(2 18 2), {1}-(2 18 2), {6}-(2 18 2), {11}-(2 18 2), {2}-(0 18 4), {3}-(0 18 4), {4}-(0 18 4), {5}-(0 18 4), {7}-(0 18 4), {8}-(0 18 4), {9}-(0 18 4), {10}-(0 18 4).

PII-T8 N22 (8 36) (27 7 1)

7	6	9	6	7	6	7	8	6	8	6	8	7	7	6
9	8	10	10	8	7	10	10	9	9	9	10	10	9	8

Transitivity sets: {0}-(2 18 2), {6}-(2 18 2), {8}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4), {3}-(0 18 4), {4}-(0 18 4), {5}-(0 18 4), {7}-(0 18 4), {9}-(0 18 4), {10}-(0 18 4).

PII-T8 N24 (8 36) (30 4 1)

7	6	8	6	7	6	7	8	6	9	6	8	7	7	6
9	10	10	9	8	7	10	9	8	10	9	10	10	9	8

Transitivity sets: {0}-(2 18 2), {6}-(2 18 2), {8}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4), {3}-(0 18 4) {4}-(0 18 4), {5}-(0 18 4), {7}-(0 18 4), {9}-(0 18 4), {10}-(0 18 4).

PII-T8 N25 (8 36) (16 12 3)
7 6 8 6 7 6 7 8 6 8 6 9 7 7 6
9 9 10 10 8 7 10 9 9 10 8 10 9 10 8
Aut D (0 11)(1 6)(2 7)(3 8)(4 9)(5 10)
Transitivity sets: {0,11}-(2 18 2), {1,6}-(2 18 2), {2,7}-(0 18 4),
{3,8}-(0 18 4), {4,9}-(0 18 4), {5,10}-(0 18 4).

PII-T9 N2 (8 36) (22 12 1)
6 6 6 9 7 7 7 8 8 6 6 6 7 7 8
8 7 9 10 8 9 10 10 9 10 9 8 10 9 10
Aut D (0 11)(1 6)(2 7)(3 8)(4 9)(5 10)
Transitivity sets: {0,11}-(2 18 2), {4,9}-(2 18 2), {1,6}-(0 18 4),
{2,7}-(0 18 4), {3,8}-(0 18 4), {5,10}-(0 18 4).

PII-T9 N3 (8 36) (22 12 1)
6 6 6 9 7 7 7 8 8 6 6 6 7 7 8
8 7 9 10 8 10 9 9 10 10 9 8 10 9 10
Aut D (0 11)(1 6)(2 7)(3 8)(4 9)(5 10)
Transitivity sets: {0,11}-(2 18 2), {4,9}-(2 18 2), {1,6}-(0 18 4),
{2,7}-(0 18 4), {3,8}-(0 18 4), {5,10}-(0 18 4).

PII-T9 N4 (4 40) (14 12 3)
6 6 6 9 7 7 8 8 7 6 6 6 7 8 7
8 7 9 10 8 9 10 9 10 10 9 8 9 10 10
Aut D (0 11)(1 6)(2 7)(3 8)(4 9)(5 10)
Transitivity sets: {0,11}-(2 20 0), {1,6}-(0 20 2), {2,7}-(0 20 2),
{3,8}-(0 20 2), {4,9}-(0 20 2), {5,10}-(0 20 2).

PII-T9 N6 (4 40) (18 14 1)
6 6 6 9 7 7 8 8 7 6 6 6 7 8 7
8 7 9 10 8 10 10 9 9 10 9 8 10 10 9
Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2),
{2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2),
{5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2),
{8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PII-T9 N7 (4 40) (22 10 1)
6 6 6 9 7 8 7 7 8 6 6 6 7 8 7
8 7 9 10 8 9 10 9 10 10 9 8 9 10 10
Aut D (0 11)(1 6)(2 7)(3 8)(4 9)(5 10)
Transitivity sets: {0,11}-(2 20 0), {1,6}-(0 20 2), {2,7}-(0 20 2),
{3,8}-(0 20 2), {4,9}-(0 20 2), {5,10}-(0 20 2).

PII-T9 N9 (4 40) (22 13 0)
6 6 6 9 7 8 7 7 8 6 6 6 7 8 7
8 7 9 10 8 10 10 9 9 10 9 8 10 10 9
Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2),
{2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2),
{5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2),
{8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PII-T9 N13 (12 32) (22 14 1)
6 6 7 9 6 7 7 8 8 6 6 7 6 7 8
8 7 9 10 8 9 10 9 10 10 9 8 9 10 10
Transitivity sets: {0}-(2 16 4), {1}-(2 16 4), {4}-(2 16 4),
{6}-(2 16 4), {9}-(2 16 4), {11}-(2 16 4),
{2}-(0 16 6), {3}-(0 16 6), {5}-(0 16 6),
{7}-(0 16 6), {8}-(0 16 6), {10}-(0 16 6).

PII-T9 N14 (4 40) (18 8 3)

6	6	7	9	6	7	8	8	7	6	6	7	6	8	7
8	7	9	10	8	9	10	9	10	10	9	8	9	10	10

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PII-T9 N19 (4 40) (18 14 1)

6	6	7	9	7	6	8	8	7	6	7	6	6	8	7
8	7	10	10	8	9	10	9	9	10	10	8	9	10	9

Aut D (0 11)(1 9)(2 8)(3 7)(4 6)(5 10)

Transitivity sets: {0,11}-(2 20 0), {1,9}-(0 20 2), {2,8}-(0 20 2), {3,7}-(0 20 2), {4,6}-(0 20 2), {5,10}-(0 20 2).

PII-T9 N23 (8 36) (20 14 1)

6	6	7	9	7	7	6	8	8	6	7	6	7	6	8
8	7	10	10	8	9	9	9	10	10	10	8	9	9	10

Transitivity sets: {0}-(2 18 2), {4}-(2 18 2), {9}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4), {3}-(0 18 4), {5}-(0 18 4), {6}-(0 18 4), {7}-(0 18 4), {8}-(0 18 4), {10}-(0 18 4).

PII-T9 N25 (4 40) (18 14 1)

6	6	7	9	7	8	6	8	7	6	7	6	8	6	7
8	7	10	10	8	10	9	9	9	10	10	8	10	9	9

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PII-T9 N27 (4 40) (23 12 0)

6	6	8	9	7	7	6	8	7	6	8	6	7	6	7
8	7	10	10	8	10	9	9	9	10	10	8	10	9	9

Transitivity sets: {0}-(2 20 2), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PII-T9 N29 (4 40) (20 12 1)

6	6	7	9	7	7	8	8	6	6	7	6	7	8	6
8	7	10	10	8	9	10	9	9	10	10	8	9	10	9

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PII-T9 N31 (4 40) (21 14 0)

6	6	7	9	7	8	7	8	6	6	7	6	8	7	6
8	7	10	10	8	10	9	9	9	10	10	8	10	9	9

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PII-T9 N32 (8 36) (20 14 1)

6	6	8	9	7	7	7	8	6	6	8	6	7	7	6
8	7	10	10	8	9	10	9	9	10	10	8	9	10	9

Transitivity sets: {0}-(2 18 2), {1}-(2 18 2), {6}-(2 18 2), {11}-(2 18 2), {2}-(0 18 4), {3}-(0 18 4), {4}-(0 18 4), {5}-(0 18 4), {7}-(0 18 4), {8}-(0 18 4), {9}-(0 18 4), {10}-(0 18 4).

PII-T9 N33 (4 40) (22 10 1)

6	6	8	9	7	7	7	8	6	6	8	6	7	7	6
8	7	10	10	8	10	9	9	9	10	10	8	10	9	9

Aut D (1 10)(2 7)(3 9)(4 6)(5 8)

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1,10}-(0 20 2),
{2,7}-(0 20 2), {3,9}-(0 20 2), {4,6}-(0 20 2),
{5,8}-(0 20 2).

PII-T9 N36 (4 40) (18 14 1)

6	7	6	6	7	6	8	8	7	9	6	6	7	8	7
8	9	9	10	8	7	10	9	10	10	9	8	9	10	10

Aut D (0 11)(1 6)(2 7)(3 8)(4 9)(5 10)

Transitivity sets: {0,11}-(2 20 0), {1,6}-(0 20 2), {2,7}-(0 20 2),
{3,8}-(0 20 2), {4,9}-(0 20 2), {5,10}-(0 20 2).

PII-T9 N41 (4 40) (21 14 0)

6	7	8	6	7	6	6	8	7	9	8	6	7	6	7
8	9	10	10	8	7	9	9	10	10	10	8	9	9	10

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2),
{2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2),
{5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2),
{8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PII-T9 N42 (4 40) (27 8 0)

6	7	8	6	7	6	6	8	7	9	8	6	7	6	7
8	10	9	10	8	7	9	10	9	10	10	8	10	9	9

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2),
{2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2),
{5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2),
{8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PII-T9 N43 (4 40) (24 11 0)

6	7	7	6	7	6	8	8	6	9	7	6	7	8	6
8	9	10	10	8	7	10	9	9	10	10	8	9	10	9

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2),
{2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2),
{5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2),
{8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PII-T9 N45 (4 40) (25 7 1)

6	7	8	6	7	6	7	8	6	9	8	6	7	7	6
8	10	9	10	8	7	9	10	9	10	10	8	10	9	9

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2),
{2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2),
{5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2),
{8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PII-T9 N50 (8 36) (26 8 1)

6	6	6	9	7	7	7	8	8	6	6	6	7	7	8
9	7	8	10	9	8	10	10	9	10	9	8	10	9	10

Aut D (0 11)(1 6)(2 7)(3 8)(4 9)(5 10)

Transitivity sets: {0,11}-(2 18 2), {4,9}-(2 18 2), {1,6}-(0 18 4),
{2,7}-(0 18 4), {3,8}-(0 18 4), {5,10}-(0 18 4).

PII-T9 N51 (8 36) (8 8 7)

6 6 6 9 7 7 8 8 7 6 6 6 7 8 7
9 7 8 10 8 9 10 9 10 10 8 9 9 10 10

Aut D
(0 10)(2 8)(3 7)(5 11)
(1 4)(2 3)(6 9)(7 8)
(0 5)(1 9)(4 6)(10 11)
(0 11)(1 2 4 3)(6 7 9 8)
(0 11)(1 3 4 2)(6 8 9 7)
(5 10)(1 7 4 8)(2 9 3 6)
(5 10)(1 8 4 7)(2 6 3 9)
(0 10)(1 4)(2 7)(3 8)(5 11)(6 9)
(0 5)(1 6)(2 3)(4 9)(7 8)(10 11)
(0 11)(1 6)(2 7)(3 8)(4 9)(5 10)
(0 11)(1 9)(2 8)(3 7)(4 6)(5 10)
(0 10 11 5)(1 2 6 7)(3 9 8 4)
(0 10 11 5)(1 3 6 8)(2 9 7 4)
(0 5 11 10)(1 7 6 2)(3 4 8 9)
(0 5 11 10)(1 8 6 3)(2 4 7 9)

Transitivity sets: {0,5,10,11}-(2 18 2), {1,2,3,4,6,7,8,9}-(0 18 4)

PII-T9 N54 (4 40) (18 8 3)

6 6 6 9 7 7 8 8 7 6 6 6 7 8 7
9 7 8 10 9 8 10 9 10 10 9 8 9 10 10

Aut D (0 11)(1 6)(2 7)(3 8)(4 9)(5 10)

Transitivity sets: {0,11}-(2 20 0), {1,6}-(0 20 2), {2,7}-(0 20 2),
{3,8}-(0 20 2), {4,9}-(0 20 2), {5,10}-(0 20 2).

PII-T9 N56 (8 36) (16 12 3)

6 6 6 9 7 8 7 7 8 6 6 6 7 8 7
9 7 8 10 8 9 10 9 10 10 8 9 9 10 10

Aut D (0 5)(1 9)(4 6)(10 11)
(0 11)(1 6)(2 7)(3 8)(4 9)(5 10)
(0 10)(1 4)(2 7)(3 8)(5 11)(6 9)

Transitivity sets: {0,5,10 11}-(2 18 2), {1,4,6,9}-(0 18 4),
{2,7}-(0 18 4), {3,8}-(0 18 4).

PII-T9 N58 (8 36) (20 14 1)

6 6 6 9 7 8 7 7 8 6 6 6 7 8 7
9 7 8 10 8 10 10 9 9 10 8 9 10 10 9

Aut D (0 10)(1 4)(2 7)(3 8)(5 11)(6 9)

Transitivity sets: {0,10}-(2 18 2), {5,11}-(2 18 2), {1,4}-(0 18 4),
{2,7}-(0 18 4), {3,8}-(0 18 4), {6,9}-(0 18 4).

PII-T9 N62 (4 40) (24 8 1)

6 6 6 9 7 7 7 8 8 6 6 6 7 8 7
9 8 7 10 9 8 10 9 10 10 9 8 9 10 10

Aut D (0 11)(1 6)(2 7)(3 8)(4 9)(5 10)

Transitivity sets: {0,11}-(2 20 0), {1,6}-(0 20 2), {2,7}-(0 20 2),
{3,8}-(0 20 2), {4,9}-(0 20 2), {5,10}-(0 20 2),

PII-T9 N66 (8 36) (28 6 1)

6 6 8 6 7 7 7 6 9 8 6 6 7 7 8
9 7 9 10 8 10 9 8 10 10 9 8 10 9 10

Aut D (0 11)(1 6)(2 7)(3 8)(4 9)(5 10)

Transitivity sets: {0,11}-(2 18 2), {4,9}-(2 18 2), {1,6}-(0 18 4),
{2,7}-(0 18 4), {3,8}-(0 18 4), {5,10}-(0 18 4).

PII-T9 N70 (4 40) (22 10 1)

6 6 8 6 7 7 8 6 7 9 6 6 7 8 7
9 7 9 10 9 8 10 8 10 10 9 8 9 10 10

Aut D (0 11)(1 6)(2 7)(3 8)(4 9)(5 10)

Transitivity sets: {0,11}-(2 20 0), {1,6}-(0 20 2), {2,7}-(0 20 2),
{3,8}-(0 20 2), {4,9}-(0 20 2), {5,10}-(0 20 2).

PII-T9 N75 (8 36) (16 12 3)

6 6 7 9 7 6 8 8 6 7 6 7 6 8 7
9 7 9 10 8 8 10 9 10 10 9 9 8 10 10

Aut D (0 11)(1 9)(2 8)(3 7)(4 6)(5 10)
(0 7 11 3)(1 8 9 2)(4 5 6 10)
(0 3 11 7)(1 2 9 8)(4 10 6 5)

Transitivity sets: {0,3,7,11}-(2 18 2), {1,2,8,9}-(0 18 4),
{4,5,6,10}-(0 18 4).

PII-T9 N79 (4 40) (18 14 1)

6	6	7	8	7	6	9	8	7	6	7	6	6	8	7
9	7	10	10	8	8	10	9	9	10	10	8	9	10	9

Aut D (0 11) (1 9) (2 8) (3 7) (4 6) (5 10)

Transitivity sets: {0,11}-(2 20 0), {1,9}-(0 20 2), {2,8}-(0 20 2), {3,7}-(0 20 2), {4,6}-(0 20 2), {5,10}-(0 20 2).

PII-T9 N87 (8 36) (18 10 3)

6	6	7	9	7	8	6	8	7	6	7	6	8	6	7
9	7	10	10	8	10	8	9	9	10	10	9	10	8	9

Aut D (1 4) (2 3) (6 9) (7 8) (0 11) (5 10) (1 6) (2 7) (3 8) (4 9)
(0 11) (5 10) (1 9) (2 8) (3 7) (4 6)

Transitivity sets: {0,11}-(2 18 2), {5,10}-(2 18 2), {1,4,6,9}-(0 18 4), {2,3,7,8}-(0 18 4).

PII-T9 N96 (8 36) (30 4 1)

6	7	8	6	7	6	6	8	7	9	8	6	7	6	7
9	10	9	10	8	7	8	10	9	10	10	9	10	8	9

Aut D (1 4) (2 3) (6 9) (7 8) (0 5) (10 11) (1 6) (2 3) (4 9) (7 8)
(0 5) (10 11) (1 9) (4 6)

Transitivity sets: {0,5}-(2 18 2), {10 11}-(2 18 2), {2,3}-(0 18 4), {7,8}-(0 18 4), {1,4,6,9}-(0 18 4).

PII-T9 N105 (8 36) (24 10 1)

6	6	8	6	7	7	7	6	8	9	6	6	7	7	8
10	7	9	9	9	8	10	8	10	10	9	8	10	9	10

Aut D (0 11) (1 6) (2 7) (3 8) (4 9) (5 10)

Transitivity sets: {0,11}-(2 18 2), {4,9}-(2 18 2), {1,6}-(0 18 4), {2,7}-(0 18 4), {3,8}-(0 18 4), {5,10}-(0 18 4).

PII-T9 N106 (8 36) (23 11 1)

6	6	7	9	7	6	8	8	6	7	6	7	6	8	7
10	7	9	10	8	8	10	9	9	10	9	10	8	10	9

Transitivity sets: {0}-(2 18 2), {3}-(2 18 2), {7}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4), {4}-(0 18 4), {5}-(0 18 4), {6}-(0 18 4), {8}-(0 18 4), {9}-(0 18 4), {10}-(0 18 4).

PII-T9 N107 (4 40) (24 8 1)

6	6	7	9	7	6	8	8	7	6	7	6	6	8	7
10	7	8	10	9	8	10	9	10	9	10	8	9	10	9

Aut D (0 11) (1 9) (2 8) (3 7) (4 6) (5 10)

Transitivity sets: {0,11}-(2 20 0), {1,9}-(0 20 2), {2,8}-(0 20 2), {3,7}-(0 20 2), {4,6}-(0 20 2), {5,10}-(0 20 2).

PII-T9 N108 (4 40) (22 10 1)

6	6	7	8	7	6	9	8	7	6	7	6	6	8	7
10	7	9	10	8	8	10	9	10	9	10	8	9	10	9

Aut D (0 11) (1 9) (2 8) (3 7) (4 6) (5 10)

Transitivity sets: {0,11}-(2 20 0), {1,9}-(0 20 2), {2,8}-(0 20 2), {3,7}-(0 20 2), {4,6}-(0 20 2), {5,10}-(0 20 2).

PII-T9 N112 (4 40) (26 9 0)

6	6	7	9	7	7	6	8	6	8	6	8	7	6	7
10	8	9	10	8	10	7	9	9	10	9	10	10	8	9

Transitivity sets: {0}-(2 20 2), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PII-T9 N134 (4 40) (28 4 1)

6	7	8	7	7	6	6	8	6	9	8	7	6	7	6
10	9	10	10	8	7	9	9	8	10	10	10	9	9	8

Aut D (1 7)(2 10)(3 8)(4 6)(5 9)

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1,7}-(0 20 2), {2,10}-(0 20 2), {3,8}-(0 20 2), {4,6}-(0 20 2), {5,9}-(0 20 2).

PII-T9 N145 (4 40) (18 14 1)

7	6	6	9	7	8	6	8	7	6	6	7	8	6	7
10	7	8	10	8	10	9	9	9	10	8	10	10	9	9

Aut D (0 11)(1 6)(2 7)(3 8)(4 9)(5 10)

Transitivity sets: {0,11}-(2 20 0), {1,6}-(0 20 2), {2,7}-(0 20 2), {3,8}-(0 20 2), {4,9}-(0 20 2), {5,10}-(0 20 2).

PII-T9 N148 (4 40) (16 10 3)

7	6	6	9	7	7	8	8	6	6	6	7	7	8	6
10	7	8	10	8	9	10	9	9	10	8	10	9	10	9

Aut D (0 11)(1 9)(2 8)(3 7)(4 6)(5 10)

Transitivity sets: {0,11}-(2 20 0), {1,9}-(0 20 2), {2,8}-(0 20 2), {3,7}-(0 20 2), {4,6}-(0 20 2), {5,10}-(0 20 2).

PII-T9 N161 (4 40) (22 10 1)

7	6	8	6	7	8	6	6	7	9	6	7	8	6	7
10	7	9	10	8	10	9	8	9	10	8	10	10	9	9

Aut D (0 11)(1 6)(2 7)(3 8)(4 9)(5 10)

Transitivity sets: {0,11}-(2 20 0), {1,6}-(0 20 2), {2,7}-(0 20 2), {3,8}-(0 20 2), {4,9}-(0 20 2), {5,10}-(0 20 2).

PII-T9 N162 (4 40) (20 12 1)

7	6	8	6	7	7	8	6	6	9	6	7	7	8	6
10	7	9	10	8	9	10	8	9	10	8	10	9	10	9

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PII-T10 N3 (4 40) (24 8 1)

6	7	6	8	8	6	7	6	7	8	6	7	9	6	7
9	10	7	9	10	9	10	8	9	10	8	9	10	10	8

Aut D (0 11)(1 6)(2 7)(3 8)(4 9)(5 10)

Transitivity sets: {0,11}-(2 20 0), {1,6}-(0 20 2), {2,7}-(0 20 2), {3,8}-(0 20 2), {4,9}-(0 20 2), {5,10}-(0 20 2).

PII-T11 N1 (16 28) (16 16 3)

6	6	7	9	6	7	7	8	8	6	6	7	6	7	8
7	8	9	10	8	9	10	9	10	10	9	8	9	10	10

Aut D

(1 4)(2 3)(6 9)(7 8)	(0 5)(1 9)(4 6)(10 11)
(0 10)(2 8)(3 7)(5 11)	(3 8)(0 1 5 9)(4 11 6 10)
(3 8)(0 9 5 1)(4 10 6 11)	(2 7)(0 4 5 6)(1 11 9 10)
(2 7)(0 6 5 4)(1 10 9 11)	(0 5)(1 6)(2 3)(4 9)(7 8)(10 11)
(0 10)(1 4)(2 7)(3 8)(5 11)(6 9)	(0 11)(1 6)(2 7)(3 8)(4 9)(5 10)
(0 11)(1 9)(2 8)(3 7)(4 6)(5 10)	(0 1 11 6)(2 8 7 3)(4 5 9 10)
(0 4 11 9)(1 5 6 10)(2 3 7 8)	(0 6 11 1)(2 3 7 8)(4 10 9 5)
(0 9 11 4)(1 10 6 5)(2 8 7 3)	

Transitivity sets: {0,1,4,5,6,9,10,11}-(2 14 6), {2,3,7,8}-(0 14 8).

PII-T11 N3 (8 36) (12 4 7)

6	6	7	9	6	7	8	8	7	6	6	7	6	8	7
7	8	9	10	8	9	10	9	10	10	9	8	9	10	10

Aut D (1 4)(2 3)(6 9)(7 8)
(0 10)(2 8)(3 7)(5 11)
(0 10)(1 4)(2 7)(3 8)(5 11)(6 9)
(0 11)(1 9)(2 8)(3 7)(4 6)(5 10)
(0 5)(1 9)(4 6)(10 11)
(0 5)(1 6)(2 3)(4 9)(7 8)(10 11)
(0 11)(1 6)(2 7)(3 8)(4 9)(5 10)

Transitivity sets: {0,5,10 11}-(2 18 2), {1,4,6,9}-(0 18 4).
{2,3,7,8}-(0 18 4).

PII-T11 N10 (16 28) (24 8 3)

6	6	7	9	6	8	7	7	8	6	6	7	6	8	7
7	8	9	10	8	9	10	9	10	10	9	8	9	10	10

Aut D (0 5)(1 9)(4 6)(10 11)
(0 9)(1 5)(2 7)(4 10)(6 11)
(3 8)(0 6 5 4)(1 11 9 10)
(0 11)(1 6)(2 7)(3 8)(4 9)(5 10)
(0 1)(2 7)(4 11)(5 9)(6 10)
(3 8)(0 4 5 6)(1 10 9 11)
(0 10)(1 4)(2 7)(3 8)(5 11)(6 9)

Transitivity sets: {0,1,4,5,6,9,10,11}-(2 14 6), {2,7}-(0 14 8),
{3,8}-(0 14 8).

PIII-T1 N1 (4 40) (27 8 0)

6	7	6	8	6	6	7	8	9	7	8	6	6	7	7
8	9	10	9	8	7	9	10	10	10	10	9	9	10	8

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2),
{2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2),
{5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2),
{8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PIII-T1 N2 (4 40) (28 4 1)

6	7	6	8	6	6	7	8	9	7	8	6	6	7	7
8	10	9	10	8	7	10	9	10	9	10	10	9	9	8

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2),
{2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2),
{5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2),
{8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PIII-T1 N3 (4 40) (25 7 1)

6	8	6	7	6	6	7	8	9	7	8	6	6	7	7
8	9	10	9	7	8	9	10	10	10	10	9	9	10	8

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2),
{2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2),
{5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2),
{8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PIII-T1 N4 (4 40) (29 6 0)

6	8	6	7	6	6	7	8	9	7	8	6	6	7	7
8	10	9	10	7	8	10	9	10	9	10	10	9	9	8

Aut D (0 11)(1 2)(4 5)(6 8)(7 10)

Transitivity sets: {0,11}-(2 20 0), {1,2}-(0 20 2), {3}-(0 20 2),
{4,5}-(0 20 2), {6,8}-(0 20 2), {7,10}-(0 20 2),
{9}-(0 20 2).

PIII-T1 N5 (4 40) (26 6 1)

6	8	6	7	6	6	7	8	8	7	9	6	6	7	7
8	9	10	9	7	9	9	10	10	10	10	8	9	10	8

Aut D (0 11)(1 8)(2 7)(3 6)(4 10)(5 9)

Transitivity sets: {0,11}-(2 20 0), {1,8}-(0 20 2), {2,7}-(0 20 2),
{3,6}-(0 20 2), {4,10}-(0 20 2), {5,9}-(0 20 2).

PIII-T1 N6 (12 32) (29 10 0)

6	8	6	7	6	7	6	7	8	9	6	8	7	6	7
8	9	10	9	7	10	8	9	10	10	9	10	10	9	8

Aut D (0 1)(2 11)(3 6)(4 8)(7 10)

Transivitity sets: {0,1}-(2 16 4), {2,11}-(2 16 4), {3,6}-(2 16 4), {4,8}-(0 16 6), {5}-(0 16 6), {7,10}-(0 16 6), {9}-(0 16 6).

PIII-T1 N7 (20 24) (37 6 0)

6	8	6	7	6	7	6	7	8	9	6	8	7	6	7
8	10	9	10	7	10	8	9	9	10	10	10	9	9	8

Transitivity sets: {2}-(8 12 2), {0}-(4 12 6), {1}-(4 12 6), {11}-(4 12 6), {3}-(0 12 10), {4}-(0 12 10), {5}-(0 12 10), {6}-(0 12 10), {7}-(0 12 10), {8}-(0 12 10), {9}-(0 12 10), {10}-(0 12 10).

PIII-T1 N9 (12 32) (31 5 1)

6	7	6	7	7	6	6	7	9	8	6	7	8	8	6
8	9	9	10	8	8	9	10	10	10	10	9	9	10	7

Transitivity sets: {0}-(2 16 4), {2}-(2 16 4), {4}-(2 16 4), {8}-(2 16 4), {9}-(2 16 4), {11}-(2 16 4), {1}-(0 16 6), {3}-(0 16 6), {5}-(0 16 6), {6}-(0 16 6), {7}-(0 16 6), {10}-(0 16 6).

PIII-T1 N10 (20 24) (38 5 0)

6	7	6	7	7	6	6	7	8	9	6	7	8	8	6
8	9	9	10	8	9	8	10	10	10	10	9	9	10	7

Transitivity sets: {2}-(6 12 4), {1}-(4 12 6), {0}-(2 12 8), {4}-(2 12 8), {8}-(2 12 8), {9}-(2 12 8), {11}-(2 12 8), {3}-(0 12 10), {5}-(0 12 10), {6}-(0 12 10), {7}-(0 12 10), {10}-(0 12 10).

PIII-T1 N11 (12 32) (33 6 0)

6	7	6	7	7	6	6	7	8	9	6	7	8	8	6
8	9	9	10	8	10	8	9	10	10	9	10	10	9	7

Aut D (0 4)(1 2)(3 8)(5 9)(6 7)(10 11)

Transitivity sets: {0,4}-(2 16 4), {1,2}-(2 16 4), {10,11}-(2 16 4), {3,8}-(0 16 6), {5,9}-(0 16 6), {6,7}-(0 16 6).

PIII-T1 N12 (12 32) (31 5 1)

6	7	6	8	7	6	6	7	9	8	6	8	7	7	6
8	9	9	10	8	7	9	10	10	10	10	9	9	10	8

Transitivity sets: {11}-(4 16 2), {0}-(2 16 4), {2}-(2 16 4), {5}-(2 16 4), {7}-(2 16 4), {1}-(0 16 6), {3}-(0 16 6), {4}-(0 16 6), {6}-(0 16 6), {8}-(0 16 6), {9}-(0 16 6), {10}-(0 16 6).

PIII-T1 N13 (4 40) (28 7 0)

6	7	6	8	7	6	6	7	9	8	6	8	7	7	6
8	10	9	10	8	7	9	10	10	9	10	10	9	9	8

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PIII-T1 N14 (4 40) (31 4 0)

6	8	6	7	7	6	6	7	9	7	6	8	8	7	6
8	10	9	10	8	8	9	10	10	9	10	10	9	9	7

Aut D (0 11)(1 2)(4 5)(7 10)

Transitivity sets: {0,11}-(2 20 0), {1,2}-(0 20 2), {3}-(0 20 2), {4,5}-(0 20 2), {6}-(0 20 2), {7,10}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2).

PIII-T1 N16 (4 40) (24 8 1)

6	7	6	8	7	6	7	8	9	6	8	6	7	7	6
8	10	9	10	8	7	10	9	10	9	10	10	9	9	8

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PIII-T1 N17 (8 36) (29 8 0)

6	7	6	8	7	7	6	8	6	9	8	6	7	7	6
8	10	9	10	8	10	7	9	9	10	10	10	9	9	8

Transitivity sets: {0}-(2 18 2), {1}-(2 18 2), {3}-(2 18 2), {11}-(2 18 2), {2}-(0 18 4), {4}-(0 18 4), {5}-(0 18 4), {6}-(0 18 4), {7}-(0 18 4), {8}-(0 18 4), {9}-(0 18 4), {10}-(0 18 4).

PIII-T1 N18 (12 32) (31 8 0)

6	8	6	7	7	7	6	7	6	9	8	6	7	8	6
8	9	9	10	8	9	8	10	10	10	10	9	9	10	7

Aut D (0 4)(1 2)(3 8)(5 10)(6 7)(9 11)

Transitivity sets: {0,4}-(2 16 4), {1,2}-(2 16 4), {9,11}-(2 16 4), {3,8}-(0 16 6), {5,10}-(0 16 6), {6,7}-(0 16 6).

PIII-T1 N21 (4 40) (28 7 0)

6	8	6	7	7	8	6	7	6	7	9	6	8	7	6
8	9	9	10	8	10	9	10	10	9	10	8	10	9	7

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PIII-T1 N22 (8 36) (30 4 1)

7	6	6	8	7	6	6	8	9	7	8	6	6	7	7
9	8	10	9	9	7	8	10	10	10	10	9	9	10	8

Transitivity sets: {0}-(2 18 2), {6}-(2 18 2), {10}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4), {3}-(0 18 4), {4}-(0 18 4), {5}-(0 18 4), {7}-(0 18 4), {8}-(0 18 4), {9}-(0 18 4).

PIII-T1 N24 (8 36) (31 3 1)

7	6	6	8	7	6	6	8	7	8	9	6	6	7	7
9	8	10	9	9	9	7	10	10	10	10	8	9	10	8

Transitivity sets: {0}-(2 18 2), {2}-(2 18 2), {3}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {4}-(0 18 4), {5}-(0 18 4), {6}-(0 18 4), {7}-(0 18 4), {8}-(0 18 4), {9}-(0 18 4), {10}-(0 18 4).

PIII-T1 N27 (8 36) (28 6 1)

7	6	6	8	7	7	6	6	9	7	6	8	8	6	7
9	8	10	9	8	10	8	9	10	10	9	10	10	7	9

Transitivity sets: {0}-(2 18 2), {8}-(2 18 2), {9}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4), {3}-(0 18 4), {4}-(0 18 4), {5}-(0 18 4), {6}-(0 18 4), {7}-(0 18 4), {10}-(0 18 4).

PIII-T1 N29 (12 32) (34 5 0)

7	8	6	6	6	7	6	8	9	7	8	6	6	7	7
9	9	8	10	7	9	8	10	10	10	10	9	9	10	8

Transitivity sets: {10}-(4 16 2), {0}-(2 16 4), {6}-(2 16 4), {8}-(2 16 4), {11}-(2 16 4), {1}-(0 16 6), {2}-(0 16 6), {3}-(0 16 6), {4}-(0 16 6), {5}-(0 16 6), {7}-(0 16 6), {9}-(0 16 6).

PIII-T1 N30 (28 16) (42 5 0)
7 7 6 6 7 7 6 6 9 8 6 7 8 8 6
9 9 8 10 8 10 8 9 10 10 9 10 10 9 7
Transitivity sets: {11}-(6 8 8), {5}-(4 8 10), {8}-(4 8 10), {10}-(4 8 10), {0}-(2 8 12), {1}-(2 8 12), {3}-(2 8 12), {4}-(2 8 12), {9}-(2 8 12), {2}-(0 8 14), {6}-(0 8 14), {7}-(0 8 14).

PIII-T1 N31 (12 32) (35 4 0)
7 8 6 6 7 7 6 6 9 7 6 8 8 7 6
9 9 8 10 8 10 8 9 10 10 9 10 10 9 7
Transitivity sets: {8}-(4 16 2), {0}-(2 16 4), {9}-(2 16 4), {10}-(2 16 4), {11}-(2 16 4), {1}-(0 16 6), {2}-(0 16 6), {3}-(0 16 6), {4}-(0 16 6), {5}-(0 16 6), {6}-(0 16 6), {7}-(0 16 6).

PIII-T1 N36 (12 32) (34 2 1)
7 8 6 8 7 7 6 6 6 7 9 8 6 6 7
9 10 10 9 8 10 8 9 9 10 10 10 8 7 9
Transitivity sets: {0}-(4 16 2), {9}-(4 16 2), {8}-(2 16 4), {11}-(2 16 4), {1}-(0 16 6), {2}-(0 16 6), {3}-(0 16 6), {4}-(0 16 6), {5}-(0 16 6), {6}-(0 16 6), {7}-(0 16 6), {10}-(0 16 6).

PIII-T1 N39 (20 24) (41 2 0)
7 8 6 6 6 7 6 8 9 7 8 6 6 7 7
10 10 8 9 7 10 8 9 10 9 10 10 9 9 8
Aut D (0 11) (1 2) (4 5) (6 8) (7 10)
Transitivity sets: {0,11}-(4 12 6), {6,8}-(4 12 6), {9}-(4 12 6), {1,2}-(0 12 10), {3}-(0 12 10), {4,5}-(0 12 10), {7,10}-(0 12 10).

PIII-T1 N40 (28 16) (43 4 0)
7 7 6 6 7 7 6 6 9 8 6 7 8 8 6
10 9 8 9 8 10 8 9 10 10 10 9 9 10 7
Aut D (1 5) (2 4) (7 10) (8 9)
Transitivity sets: {8,9}-(6 8 8), {0}-(4 8 10), {6}-(4 8 10), {11}-(4 8 10), {2,4}-(2 8 12), {1,5}-(0 8 14), {3}-(0 8 14), {7,10}-(0 8 14).

PIII-T1 N41 (20 24) (43 0 0)
7 8 6 6 7 7 6 6 9 7 6 8 8 7 6
10 10 8 9 8 10 8 9 10 9 10 10 9 9 7
Aut D (0 11) (1 2) (4 5) (7 10) (1 5) (2 4) (7 10) (8 9)
(0 11) (1 4) (2 5) (8 9)
Transitivity sets: {0,11}-(4 12 6), {6}-(4 12 6), {8,9}-(4 12 6), {3}-(0 12 10), {1,2,4,5}-(0 12 10), {7,10}-(0 12 10).

PIII-T1 N42 (4 40) (24 2 3)
8 6 6 7 7 6 6 8 9 7 8 6 6 7 7
9 8 10 9 9 8 7 10 10 10 10 9 9 10 8
Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PIII-T1 N43 (4 40) (24 2 3)
8 6 6 7 7 6 6 8 8 7 9 6 6 7 7
9 8 10 9 9 9 7 10 10 10 10 8 9 10 8
Aut D (0 11) (1 8) (2 7) (3 6) (4 10) (5 9)
Transitivity sets: {0,11}-(2 20 0), {1,8}-(0 20 2), {2,7}-(0 20 2), {3,6}-(0 20 2), {4,10}-(0 20 2), {5,9}-(0 20 2).

PIII-T1 N45 (12 32) (29 7 1)

8	6	6	7	7	7	6	6	9	8	6	8	7	6	7
9	8	10	9	8	10	7	9	10	10	9	10	10	8	9

Transitivity sets: {0}-(2 16 4), {3}-(2 16 4), {6}-(2 16 4),
{7}-(2 16 4), {9}-(2 16 4), {11}-(2 16 4),
{1}-(0 16 6), {2}-(0 16 6), {4}-(0 16 6),
{5}-(0 16 6), {8}-(0 16 6), {10}-(0 16 6).

PIII-T1 N46 (12 32) (26 10 1)

8	6	6	7	7	7	6	6	8	9	6	8	7	6	7
9	8	10	9	9	10	7	8	10	10	9	10	10	9	8

Aut D (0 8)(1 4)(2 10)(5 9)(7 11)
Transitivity sets: {0,8}-(2 16 4), {3}-(2 16 4), {6}-(2 16 4),
{7,11}-(2 16 4), {1,4}-(0 16 6), {2,10}-(0 16 6),
{5,9}-(0 16 6).

PIII-T1 N51 (8 36) (30 4 1)

8	7	6	6	6	7	6	8	9	7	8	6	6	7	7
9	9	8	10	8	9	7	10	10	10	10	9	9	10	8

Transitivity sets: {0}-(2 18 2), {8}-(2 18 2), {10}-(2 18 2),
{11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4),
{3}-(0 18 4), {4}-(0 18 4), {5}-(0 18 4),
{6}-(0 18 4), {7}-(0 18 4), {9}-(0 18 4).

PIII-T1 N52 (16 28) (33 8 0)

8	7	6	6	7	7	6	6	8	9	6	8	7	7	6
9	8	9	10	9	10	7	8	10	10	9	10	10	9	8

Aut D (1 5)(2 4)(7 10)(8 9)
Transitivity sets: {0}-(2 14 6), {3}-(2 14 6), {6}-(2 14 6),
{7,10}-(2 14 6), {8,9}-(2 14 6), {11}-(2 14 6),
{1,5}-(0 14 8), {2,4}-(0 14 8).

PIII-T1 N53 (24 20) (39 6 0)

8	7	6	6	7	7	6	6	9	8	6	8	7	7	6
9	9	8	10	8	10	7	9	10	10	9	10	10	9	8

Aut D (1 5)(2 4)(7 10)(8 9)
Transitivity sets: {0}-(6 10 6), {6}-(6 10 6), {3}-(2 10 10),
{7,10}-(2 10 10), {8,9}-(2 10 10), {11}-(2 10 10),
{1,5}-(0 10 12), {2,4}-(0 10 12).

PIII-T1 N54 (24 20) (29 16 0)

8	7	6	6	7	7	6	6	8	8	6	9	7	7	6
9	9	9	10	9	10	7	8	10	10	9	10	10	8	8

Aut D (1 5)(2 4)(7 10)(8 9)
(0 11)(1 4)(2 5)(7 8)(9 10)
(3 6)(8 2 9 4)(5 7 1 10)
(0 11)(1 9)(2 10)(3 6)(4 7)(5 8)
(0 11)(1 2)(4 5)(7 9)(8 10)
(3 6)(8 4 9 2)(5 10 1 7)
(0 11)(1 8)(2 7)(3 6)(4 10)(5 9)
Transitivity sets: {0,11}-(2 10 10), {3,6}-(2 10 10),
{1,2,4,5,7,8,9,10}-(2 10 10).

PIII-T2 N2 (12 32) (29 7 1)

6	6	7	8	7	6	7	6	7	9	6	8	8	6	7
8	9	10	9	8	10	9	8	10	10	9	10	10	7	9

Transitivity sets: {0}-(2 16 4), {2}-(2 16 4), {7}-(2 16 4),
{9}-(2 16 4), {10}-(2 16 4), {11}-(2 16 4),
{1}-(0 16 6), {3}-(0 16 6), {4}-(0 16 6),
{5}-(0 16 6), {6}-(0 16 6), {8}-(0 16 6).

PIII-T2 N4 (12 32) (32 4 1)

6	6	7	8	7	6	7	6	7	8	6	8	9	6	7
8	10	9	9	9	8	10	9	10	10	9	10	10	7	8

Aut D (0 8)(1 4)(3 6)(5 9)(7 11)

Transitivity sets: {0,8}-(2 16 4), {2}-(2 16 4), {7,11}-(2 16 4), {10}-(2 16 4), {1,4}-(0 16 6), {3,6}-(0 16 6), {5,9}-(0 16 6).

PIII-T2 N15 (24 20) (42 3 0)

7	6	6	7	6	7	8	6	8	9	6	7	8	6	7
8	9	10	9	7	10	9	8	10	10	9	10	10	8	9

Transitivity sets: {11}-(6 10 6), {0}-(4 10 8), {9}-(4 10 8), {1}-(2 10 10), {3}-(2 10 10), {5}-(2 10 10), {8}-(2 10 10), {10}-(2 10 10), {2}-(0 10 12), {4}-(0 10 12), {6}-(0 10 12), {7}-(0 10 12).

PIII-T2 N22 (12 32) (34 5 0)

7	6	8	6	7	6	7	6	8	9	6	8	7	7	6
9	10	9	8	8	9	10	7	10	10	9	10	10	9	8

Transitivity sets: {0}-(4 16 2), {6}-(2 16 4), {8}-(2 16 4), {9}-(2 16 4), {11}-(2 16 4), {1}-(0 16 6), {2}-(0 16 6), {3}-(0 16 6), {4}-(0 16 6), {5}-(0 16 6), {7}-(0 16 6), {10}-(0 16 6).

PIII-T2 N25 (12 32) (33 6 0)

7	6	8	6	7	6	7	6	8	9	6	8	7	7	6
10	9	10	8	8	9	10	7	9	10	10	10	9	9	8

Transitivity sets: {0}-(2 16 4), {1}-(2 16 4), {2}-(2 16 4), {8}-(2 16 4), {10}-(2 16 4), {11}-(2 16 4), {3}-(0 16 6), {4}-(0 16 6), {5}-(0 16 6), {6}-(0 16 6), {7}-(0 16 6), {9}-(0 16 6).

PIII-T2 N27 (8 36) (27 7 1)

7	6	7	8	7	6	8	6	6	8	9	7	6	6	7
10	10	9	9	8	9	10	7	9	10	10	10	8	8	9

Transitivity sets: {0}-(2 18 2), {1}-(2 18 2), {7}-(2 18 2), {11}-(2 18 2), {2}-(0 18 4), {3}-(0 18 4), {4}-(0 18 4), {5}-(0 18 4), {6}-(0 18 4), {8}-(0 18 4), {9}-(0 18 4), {10}-(0 18 4).

PIII-T2 N30 (12 32) (33 6 0)

8	6	7	6	7	6	7	6	7	9	6	8	8	7	6
9	10	9	8	8	9	10	8	10	10	9	10	10	9	7

Transitivity sets: {0}-(2 16 4), {1}-(2 16 4), {2}-(2 16 4), {7}-(2 16 4), {9}-(2 16 4), {11}-(2 16 4), {3}-(0 16 6), {4}-(0 16 6), {5}-(0 16 6), {6}-(0 16 6), {8}-(0 16 6), {10}-(0 16 6).

PIII-T2 N32 (12 32) (33 6 0)

8	6	7	6	7	6	7	8	6	7	9	6	8	7	6
9	9	10	8	8	9	10	10	10	9	10	8	10	9	7

Aut D (0 11)(3 5)(6 9)(7 10)

Transitivity sets: {1}-(4 16 2), {2}-(4 16 2), {0,11}-(2 16 4), {3,5}-(0 16 6), {4}-(0 16 6), {6,9}-(0 16 6), {7,10}-(0 16 6), {8}-(0 16 6).

PIII-T2 N33 (4 40) (27 5 1)

8	6	7	6	7	6	7	8	6	7	9	6	8	7	6
9	9	10	8	8	10	9	10	9	10	10	8	10	9	7

Aut D (0 11)(3 5)(6 9)(7 10)

Transitivity sets: {0,11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3,5}-(0 20 2), {4}-(0 20 2), {6,9}-(0 20 2), {7,10}-(0 20 2), {8}-(0 20 2).

<u>PIII-T2 N34</u> (4 40) (27 5 1)

8	6	7	6	7	6	7	8	6	7	9	6	8	7	6
9	10	9	8	8	9	10	10	9	10	10	8	10	9	7

<u>Aut D</u> (0 11)(3 5)(6 9)(7 10)

<u>Transitivity sets:</u> {0,11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3,5}-(0 20 2), {4}-(0 20 2), {6,9}-(0 20 2), {7,10}-(0 20 2), {8}-(0 20 2).

<u>PIII-T2 N36</u> (24 20) (43 2 0)

8	6	7	6	6	7	8	6	7	9	8	6	6	7	7
10	9	10	8	7	10	9	8	9	10	10	10	9	9	8

<u>Aut D</u> (0 11)(1 2)(6 8)(7 10)

<u>Transitivity sets:</u> {0,11}-(4 10 8), {3}-(4 10 8), {1,2}-(4 10 8), {7,10}-(2 10 10), {4}-(0 10 12), {5}-(0 10 12), {6,8}-(0 10 12), {9}-(0 10 12).

<u>PIII-T2 N39</u> (20 24) (41 2 0)

8	6	7	6	7	6	7	6	7	9	6	8	8	7	6
10	9	10	8	8	9	10	8	9	10	10	10	9	9	7

<u>Aut D</u> (0 11)(1 2)(7 10)

<u>Transitivity sets:</u> {1,2}-(6 12 4), {0,11}-(2 12 8), {7,10}-(2 12 8), {3}-(0 12 10), {4}-(0 12 10), {5}-(0 12 10), {6}-(0 12 10), {8}-(0 12 10), {9}-(0 12 10).

<u>PIII-T2 N40</u> (24 20) (43 2 0)

8	6	7	6	7	6	7	6	7	8	6	9	8	7	6
10	9	10	9	9	8	10	8	9	10	10	10	9	8	7

<u>Aut D</u> (0 11)(1 2)(7 10)(8 9)

<u>Transitivity sets:</u> {5}-(8 10 4), {1,2}-(4 10 8), {4}-(4 10 8), {0,11}-(2 10 10), {3}-(0 10 12), {6}-(0 10 12), {7,10}-(0 10 12), {8,9}-(0 10 12).

<u>PIII-T2 N41</u> (20 24) (39 4 0)

8	6	7	8	7	6	7	6	6	7	9	8	6	6	7
10	9	10	9	8	9	10	8	10	9	10	10	8	7	9

<u>Aut D</u> (0 11)(1 2)(6 9)(7 10)

<u>Transitivity sets:</u> {1,2}-(6 12 4), {0,11}-(2 12 8), {7,10}-(2 12 8), {3}-(0 12 10), {4}-(0 12 10), {5}-(0 12 10), {6,9}-(0 12 10), {8}-(0 12 10).

<u>PIII-T2 N42</u> (12 32) (30 6 1)

8	6	7	8	7	6	7	6	6	7	9	8	6	6	7
10	9	10	9	8	10	9	8	9	10	10	10	8	7	9

<u>Transitivity sets:</u> {0}-(2 16 4), {1}-(2 16 4), {2}-(2 16 4), {7}-(2 16 4), {10}-(2 16 4), {11}-(2 16 4), {3}-(0 16 6), {4}-(0 16 6), {5}-(0 16 6), {6}-(0 16 6), {8}-(0 16 6), {9}-(0 16 6).

<u>PIII-T3 N1</u> (4 40) (28 7 0)

7	6	8	6	7	6	6	7	9	8	6	7	8	6	7
9	7	10	9	8	9	8	10	10	10	10	9	9	8	10

<u>Transivity sets:</u> {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

<u>PIII-T3 N3</u> (4 40) (24 8 1)

7	6	7	6	7	6	6	8	8	9	6	7	8	6	7
9	9	10	8	8	9	7	10	10	10	10	9	9	8	10

<u>Transivity sets:</u> {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PIII-T3 N4 (8 36) (26 8 1)

7 6 7 6 7 6 6 7 9 8 6 7 8 6 8
9 8 10 9 8 9 8 10 10 10 10 9 9 7 10

Transitivity sets: {0}-(2 18 2), {4}-(2 18 2), {9}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4), {3}-(0 18 4), {5}-(0 18 4), {6}-(0 18 4), {7}-(0 18 4), {8}-(0 18 4), {10}-(0 18 4).

PIII-T3 N5 (12 32) (29 10 0)

7 6 7 6 7 6 6 7 8 9 6 7 8 6 8
9 9 10 8 8 9 8 10 10 10 10 9 9 7 10

Transitivity sets: {0}-(2 16 4), {1}-(2 16 4), {2}-(2 16 4), {4}-(2 16 4), {9}-(2 16 4), {11}-(2 16 4), {3}-(0 16 6), {5}-(0 16 6), {6}-(0 16 6), {7}-(0 16 6), {8}-(0 16 6), {10}-(0 16 6).

PIII-T3 N7 (4 40) (26 6 1)

7 6 7 6 7 6 7 8 8 6 9 6 8 6 7
9 9 10 8 8 9 9 10 10 10 10 7 9 8 10

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PIII-T3 N8 (4 40) (24 8 1)

7 6 8 8 7 6 6 7 9 6 8 7 6 6 7
9 7 10 9 8 9 8 10 10 10 10 9 9 8 10

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PIII-T3 N11 (4 40) (20 6 3)

7 6 7 8 7 6 6 8 9 6 8 7 6 6 7
9 8 10 9 8 9 7 10 10 10 10 9 9 8 10

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PIII-T3 N14 (4 40) (18 8 3)

7 6 7 8 7 6 6 8 8 6 9 7 6 6 7
9 9 10 9 8 9 7 10 10 10 10 9 8 8 10

Aut D (0 11)(1 9)(2 10)(3 6)(4 7)(5 8)

Transitivity sets: {0,11}-(2 20 0), {1,9}-(0 20 2), {2,10}-(0 20 2), {3,6}-(0 20 2), {4,7}-(0 20 2), {5,8}-(0 20 2).

PIII-T3 N15 (8 36) (20 8 3)

7 6 7 8 7 6 6 7 9 6 8 7 6 6 8
9 8 10 9 8 9 8 10 10 10 10 9 9 7 10

Transitivity sets: {0}-(2 18 2), {4}-(2 18 2), {9}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4), {3}-(0 18 4), {5}-(0 18 4), {6}-(0 18 4), {7}-(0 18 4), {8}-(0 18 4), {10}-(0 18 4).

PIII-T3 N17 (8 36) (24 10 1)

7 6 7 8 7 6 6 7 8 6 9 7 6 6 8
9 9 10 9 8 9 8 10 10 10 10 9 8 7 10

Transitivity sets: {0}-(2 18 2), {4}-(2 18 2), {9}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4), {3}-(0 18 4), {5}-(0 18 4), {6}-(0 18 4), {7}-(0 18 4), {8}-(0 18 4), {10}-(0 18 4).

PIII-T3 N21 (4 40) (22 10 1)

7 7 8 6 7 6 6 8 6 9 8 7 6 6 7
9 10 9 8 8 9 7 10 10 10 10 9 9 8 10

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2),
{2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2),
{5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2),
{8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PIII-T3 N23 (12 32) (24 12 1)

7 7 8 6 7 6 6 7 6 9 8 7 6 6 8
9 10 9 8 8 9 8 10 10 10 10 9 9 7 10

Aut D (0 4)(1 2)(3 8)(5 10)(6 7)(9 11)

Transitivity sets: {0,4}-(2 16 4), {1,2}-(2 16 4), {9,11}-(2 16 4),
{3,8}-(0 16 6), {5,10}-(0 16 6), {6,7}-(0 16 6).

PIII-T3 N24 (8 36) (25 12 0)

7 7 8 6 7 6 6 7 6 8 9 7 6 6 8
9 10 9 9 8 9 8 10 10 10 10 9 8 7 10

Aut D (0 4)(1 2)(3 8)(5 10)(6 7)(9 11)

Transitivity sets: {0,4}-(2 18 2), {9,11}-(2 18 2), {1,2}-(0 18 4),
{3,8}-(0 18 4), {5,10}-(0 18 4), {6,7}-(0 18 4).

PIII-T3 N25 (4 40) (25 7 1)

7 6 8 6 7 6 6 7 9 8 6 7 8 6 7
10 7 9 9 8 10 8 9 10 10 9 10 10 8 9

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2),
{2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2),
{5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2),
{8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PIII-T3 N27 (4 40) (24 8 1)

7 6 7 6 7 6 6 8 9 8 6 7 8 6 7
10 8 9 9 8 10 7 9 10 10 9 10 10 8 9

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2),
{2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2),
{5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2),
{8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PIII-T3 N28 (8 36) (26 8 1)

7 6 7 6 7 6 6 8 8 9 6 7 8 6 7
10 9 9 8 8 10 7 9 10 10 9 10 10 8 9

Transitivity sets: {0}-(2 18 2), {6}-(2 18 2), {7}-(2 18 2),
{11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4),
{3}-(0 18 4), {4}-(0 18 4), {5}-(0 18 4),
{8}-(0 18 4), {9}-(0 18 4), {10}-(0 18 4).

PIII-T3 N29 (8 36) (22 6 3)

7 6 7 6 7 6 6 7 9 8 6 7 8 6 8
10 8 9 9 8 10 8 9 10 10 9 10 10 7 9

Aut D (0 4)(1 2)(3 8)(5 9)(6 7)(10 11)

Transitivity sets: {0,4}-(2 18 2), {10,11}-(2 18 2), {1,2}-(0 18 4),
{3,8}-(0 18 4), {5,9}-(0 18 4), {6,7}-(0 18 4).

PIII-T3 N32 (8 36) (31 6 0)

7 6 7 6 7 6 7 8 8 6 9 6 8 6 7
10 9 9 8 8 10 10 9 10 9 10 7 10 8 9

Aut D (0 7)(1 8)(2 5)(3 10)(6 11)

Transitivity sets: {0,7}-(2 18 2), {6,11}-(2 18 2), {1,8}-(0 18 4),
{2,5}-(0 18 4), {3,10}-(0 18 4), {4}-(0 18 4),
{9}-(0 18 4).

PIII-T3 N33 (4 40) (28 7 0)

7 6 8 8 7 6 6 7 9 6 8 7 6 6 7
10 7 9 10 8 10 8 9 10 9 10 10 9 8 9

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PIII-T3 N34 (4 40) (26 6 1)

7 6 7 8 7 6 6 8 9 6 8 7 6 6 7
10 8 9 10 8 10 7 9 10 9 10 10 9 8 9

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PIII-T3 N36 (8 36) (26 8 1)

7 6 7 8 7 6 6 7 9 6 8 7 6 6 8
10 8 9 10 8 10 8 9 10 9 10 10 9 7 9

Transitivity sets: {0}-(2 18 2), {4}-(2 18 2), {10}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4), {3}-(0 18 4), {5}-(0 18 4), {6}-(0 18 4), {7}-(0 18 4), {8}-(0 18 4), {9}-(0 18 4).

PIII-T3 N37 (12 32) (29 10 0)

7 6 7 8 7 6 6 7 8 6 9 7 6 6 8
10 9 9 10 8 10 8 9 10 9 10 10 8 7 9

Transitivity sets: {0}-(2 16 4), {4}-(2 16 4), {6}-(2 16 4), {7}-(2 16 4), {10}-(2 16 4), {11}-(2 16 4), {1}-(0 16 6), {2}-(0 16 6), {3}-(0 16 6), {5}-(0 16 6), {8}-(0 16 6), {9}-(0 16 6).

PIII-T3 N38 (4 40) (27 8 0)

7 7 8 6 7 6 6 8 6 9 8 7 6 6 7
10 9 10 8 8 10 7 9 9 10 10 10 9 8 9

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PIII-T3 N39 (4 40) (25 10 0)

7 7 8 6 7 6 6 8 6 8 9 7 6 6 7
10 9 10 9 8 10 7 9 9 10 10 10 8 8 9

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PIII-T3 N43 (16 28) (31 7 1)

6 6 7 7 7 7 6 6 9 8 6 7 8 6 8
10 8 9 9 8 10 8 9 10 10 9 10 10 7 9

Transitivity sets: {0}-(2 14 6), {3}-(2 14 6), {4}-(2 14 6), {5}-(2 14 6), {8}-(2 14 6), {9}-(2 14 6), {10}-(2 14 6), {11}-(2 14 6), {1}-(0 14 8), {2}-(0 14 8), {6}-(0 14 8), {7}-(0 14 8).

PIII-T3 N46 (12 32) (24 12 1)

6 6 7 8 7 7 6 6 8 9 6 8 7 6 7
10 9 8 9 9 10 7 8 10 10 9 10 10 8 9

Transitivity sets: {0}-(2 16 4), {3}-(2 16 4), {6}-(2 16 4), {7}-(2 16 4), {8}-(2 16 4), {11}-(2 16 4), {1}-(0 16 6), {2}-(0 16 6), {4}-(0 16 6), {5}-(0 16 6), {9}-(0 16 6), {10}-(0 16 6).

PIII-T3 N54 (12 32) (29 7 1)

6	7	8	6	6	7	6	7	8	9	6	8	7	6	7
10	9	9	8	7	10	9	8	10	10	9	10	10	8	9

Transitivity sets: {0}-(2 16 4), {3}-(2 16 4), {6}-(2 16 4), {7}-(2 16 4), {8}-(2 16 4), {11}-(2 16 4), {1}-(0 16 6), {2}-(0 16 6), {4}-(0 16 6), {5}-(0 16 6), {9}-(0 16 6), {10}-(0 16 6).

PIII-T3 N57 (20 24) (37 6 0)

6	7	7	8	6	7	6	7	9	6	8	7	6	6	8
10	8	9	10	8	10	8	9	10	9	10	10	9	7	9

Transitivity sets: {0}-(4 12 6), {8}-(4 12 6), {3}-(2 12 8), {4}-(2 12 8), {5}-(2 12 8), {9}-(2 12 8), {10}-(2 12 8), {11}-(2 12 8), {1}-(0 12 10), {2}-(0 12 10), {6}-(0 12 10), {7}-(0 12 10).

PIII-T3 N59 (12 32) (31 8 0)

6	6	7	8	7	7	6	6	8	9	6	8	7	6	7
9	9	8	10	10	9	7	8	10	10	10	9	9	8	10

Transitivity sets: {0}-(2 16 4), {3}-(2 16 4), {4}-(2 16 4), {7}-(2 16 4), {8}-(2 16 4), {11}-(2 16 4), {1}-(0 16 6), {2}-(0 16 6), {5}-(0 16 6), {6}-(0 16 6), {9}-(0 16 6), {10}-(0 16 6).

PIII-T3 N60 (8 36) (26 11 0)

6	6	8	8	7	7	6	7	9	6	8	6	7	6	7
9	7	10	9	8	9	8	10	10	10	10	9	9	8	10

Transitivity sets: {0}-(2 18 2), {5}-(2 18 2), {10}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4), {3}-(0 18 4), {4}-(0 18 4), {6}-(0 18 4), {7}-(0 18 4), {8}-(0 18 4), {9}-(0 18 4).

PIII-T3 N62 (8 36) (22 12 1)

6	6	7	8	7	7	6	8	9	6	8	6	7	6	7
9	8	10	9	8	9	7	10	10	10	10	9	9	8	10

Transitivity sets: {0}-(2 18 2), {5}-(2 18 2), {10}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4), {3}-(0 18 4), {4}-(0 18 4), {6}-(0 18 4), {7}-(0 18 4), {8}-(0 18 4), {9}-(0 18 4).

PIII-T3 N68 (8 36) (24 13 0)

6	7	8	6	7	7	6	8	6	9	8	6	7	6	7
9	10	9	8	8	9	7	10	10	10	10	9	9	8	10

Transitivity sets: {0}-(2 18 2), {5}-(2 18 2), {10}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4), {3}-(0 18 4), {4}-(0 18 4), {6}-(0 18 4), {7}-(0 18 4), {8}-(0 18 4), {9}-(0 18 4).

PIII-T3 N70 (8 36) (29 8 0)

6	7	8	6	7	7	6	7	6	8	9	6	7	6	8
9	10	9	8	8	9	9	10	10	10	10	8	9	7	10

Aut D (0 4)(1 2)(3 8)(5 10)(6 7)(9 11)

Transitivity sets: {0,4}-(2 18 2), {9,11}-(2 18 2), {1,2}-(0 18 4), {3,8}-(0 18 4), {5,10}-(0 18 4), {6,7}-(0 18 4).

PIII-T4 N3 (24 20) (37 8 0)

6	6	7	8	7	6	7	7	8	6	9	6	7	6	8
8	9	9	10	8	10	10	9	10	9	10	8	10	7	9

Aut D (0 11)(1 3)(2 4)(6 7)

Transitivity sets: {6,7}-(4 10 8), {10}-(4 10 8), {0,11}-(2 10 10), {1,3}-(2 10 10), {2,4}-(2 10 10), {5}-(0 10 12), {8}-(0 10 12), {9}-(0 10 12).

PIII-T4 N20 (4 40) (27 5 1)
7 6 7 8 7 6 8 6 9 6 8 7 6 6 7
9 8 10 9 8 9 10 7 10 10 10 9 9 8 10
Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2),
{2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2),
{5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2),
{8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PIII-T4 N39 (12 32) (34 2 1)
8 6 8 7 7 6 7 6 9 6 8 7 6 6 7
9 7 10 9 8 9 10 8 10 10 10 9 9 8 10
Transitivity sets: {11}-(6 16 0), {1}-(4 16 2), {0}-(2 16 4),
{2}-(0 16 6), {3}-(0 16 6), {4}-(0 16 6),
{5}-(0 16 6), {6}-(0 16 6), {7}-(0 16 6),
{8}-(0 16 6), {9}-(0 16 6), {10}-(0 16 6).

PIII-T4 N44 (8 36) (34 3 0)
6 6 8 7 7 6 7 6 9 8 6 7 8 6 7
10 7 9 9 8 9 10 8 10 10 9 10 10 8 9
Transitivity sets: {11}-(4 18 0), {0}-(2 18 2), {1}-(2 18 2),
{2}-(0 18 4), {3}-(0 18 4), {4}-(0 18 4),
{5}-(0 18 4), {6}-(0 18 4), {7}-(0 18 4),
{8}-(0 18 4), {9}-(0 18 4), {10}-(0 18 4).

PIII-T4 N45 (12 32) (27 12 0)
6 6 7 8 7 6 7 6 7 9 6 8 8 6 7
10 9 8 9 9 8 10 8 10 10 9 10 10 7 9
Transitivity sets: {0}-(2 16 4), {2}-(2 16 4), {4}-(2 16 4),
{5}-(2 16 4), {10}-(2 16 4), {11}-(2 16 4),
{1}-(0 16 6), {3}-(0 16 6), {6}-(0 16 6),
{7}-(0 16 6), {8}-(0 16 6), {9}-(0 16 6).

PIII-T4 N47 (16 28) (39 2 0)
6 6 8 7 7 6 7 6 9 8 6 7 8 6 7
9 7 10 9 8 9 10 8 10 10 10 9 9 8 10
Aut D (0 2)(1 11)(3 9)(5 6)(7 10)
Transitivity sets: {1,11}-(6 14 2), {0,2}-(2 14 6), {3,9}-(0 14 8),
{4}-(0 14 8), {5,6}-(0 14 8), {7,10}-(0 14 8),
{8}-(0 14 8).

PIII-T4 N55 (24 20) (43 2 0)
6 7 8 7 6 7 8 6 9 6 8 7 6 6 7
9 8 9 10 7 10 10 8 10 9 10 9 10 8 9
Transitivity sets: {11}-(6 10 6), {1}-(4 10 8), {7}-(4 10 8),
{0}-(2 10 10), {5}-(2 10 10), {6}-(2 10 10),
{8}-(2 10 10), {9}-(2 10 10), {2}-(0 10 12),
{3}-(0 10 12), {4}-(0 10 12), {10}-(0 10 12).

PIII-T4 N61 (4 40) (26 6 1)
8 6 7 6 7 6 7 6 8 9 6 8 7 7 6
10 9 10 8 8 9 10 7 9 10 10 10 9 9 8
Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2),
{2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2),
{5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2),
{8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PIII-T4 N62 (8 36) (29 8 0)
8 6 7 6 7 6 7 6 7 9 6 8 8 7 6
10 9 10 8 8 9 10 8 9 10 10 10 9 9 7
Aut D (0 11)(1 2)(3 4)(7 10)
Transitivity sets: {0,11}-(2 18 2), {1,2}-(2 18 2), {3,4}-(0 18 4),
{5}-(0 18 4), {6}-(0 18 4), {7,10}-(0 18 4),
{8}-(0 18 4), {9}-(0 18 4).

PIII-T6 N1 (20 24) (39 4 0)

6	7	8	6	6	7	6	7	9	8	6	8	7	6	7
7	8	9	9	10	10	8	9	10	10	9	10	10	8	9

Aut D (0 1)(3 6)(4 8)(7 10)

Transitivity sets: {9}-(4 12 6), {11}-(4 12 6), {0,1}-(2 12 8), {3,6}-(2 12 8), {7,10}-(2 12 8), {2}-(0 12 10), {4,8}-(0 12 10), {5}-(0 12 10).

PIII-T6 N2 (20 24) (39 4 0)

6	7	8	6	6	7	6	7	8	9	6	8	7	6	7
7	9	9	8	10	10	9	8	10	10	9	10	10	8	9

Transitivity sets: {0}-(4 12 6), {6}-(4 12 6), {8}-(4 12 6), {3}-(2 12 8), {7}-(2 12 8), {10}-(2 12 8), {11}-(2 12 8), {1}-(0 12 10), {2}-(0 12 10), {4}-(0 12 10), {5}-(0 12 10), {9}-(0 12 10).

PIII-T6 N3 (24 20) (43 2 0)

6	7	8	6	6	7	6	7	8	9	6	8	7	6	7
7	9	10	8	9	10	9	8	10	10	10	9	9	8	10

Transitivity sets: {11}-(8 10 4), {0}-(4 10 8), {1}-(4 10 8), {5}-(4 10 8), {8}-(2 10 10), {9}-(2 10 10), {2}-(0 10 12), {3}-(0 10 12), {4}-(0 10 12), {6}-(0 10 12), {7}-(0 10 12), {10}-(0 10 12).

PIII-T6 N7 (20 24) (35 8 0)

7	6	7	6	7	6	6	7	8	9	6	7	8	6	8
8	9	9	8	10	10	8	9	10	10	9	10	10	7	9

Aut D (0 4)(1 2)(3 8)(5 9)(6 7)(10 11)

Transitivity sets: {10,11}-(4 12 6), {0,4}-(2 12 8), {1,2}-(2 12 8), {6,7}-(2 12 8), {3,8}-(0 12 10), {5,9}-(0 12 10).

PIII-T6 N8 (20 24) (38 5 0)

7	6	8	6	8	7	6	7	9	6	8	6	7	6	7
8	7	10	9	9	10	8	9	10	10	10	9	9	8	10

Transitivity sets: {11}-(6 12 4), {1}-(4 12 6), {0}-(2 12 8), {4}-(2 12 8), {6}-(2 12 8), {8}-(2 12 8), {10}-(2 12 8), {2}-(0 12 10), {3}-(0 12 10), {5}-(0 12 10), {7}-(0 12 10), {9}-(0 12 10).

PIII-T6 N11 (20 24) (39 4 0)

7	7	8	6	6	7	6	8	6	9	8	6	7	6	7
8	9	10	8	9	10	7	9	10	10	10	9	9	8	10

Transitivity sets: {11}-(6 12 4), {0}-(2 12 8), {1}-(2 12 8), {4}-(2 12 8), {5}-(2 12 8), {8}-(2 12 8), {9}-(2 12 8), {10}-(2 12 8), {2}-(0 12 10), {3}-(0 12 10), {6}-(0 12 10), {7}-(0 12 10).

PIII-T6 N14 (12 32) (34 2 1)

7	6	7	6	7	6	6	7	8	9	6	7	8	6	8
10	8	9	9	10	8	9	8	10	10	10	9	9	7	10

Transitivity sets: {0}-(2 16 4), {2}-(2 16 4), {4}-(2 16 4), {9}-(2 16 4), {10}-(2 16 4), {11}-(2 16 4), {1}-(0 16 6), {3}-(0 16 6), {5}-(0 16 6), {6}-(0 16 6), {7}-(0 16 6), {8}-(0 16 6).

PIII-T7 N2 (12 32) (29 7 1)

6	6	7	8	7	7	6	6	8	9	6	7	8	6	7
7	8	9	10	8	9	9	10	10	10	8	10	9	9	10

Aut D (0 10 11)(1 6 9)(8 3 5) (10 0 11)(6 1 9)(5 3 8)

Transitivity sets: {0,10,11}-(4 16 2), {1,6,9}-(0 16 6), {2}-(0 16 6), {4}-(0 16 6), {3,5,8}-(0 16 6), {7}-(0 16 6).

PIII-T7 N4 (12 32) (29 7 1)

6	6	7	8	7	7	6	6	8	9	6	7	8	6	7
7	8	9	10	8	10	9	10	9	10	8	10	10	9	9

Aut D (11 0 10)(6 9 1)(3 5 8) (0 11 10)(9 6 1)(5 3 8)

Transitivity sets: {0,10,11}-(4 16 2), {1,6,9}-(0 16 6), {2}-(0 16 6), {4}-(0 16 6), {3,5,8}-(0 16 6), {7}-(0 16 6).

PIII-T7 N14 (4 40) (23 12 0)

6	6	8	6	7	7	8	6	7	9	6	6	7	8	7
8	7	9	9	9	8	10	10	10	10	9	8	9	10	10

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PIII-T7 N26 (4 40) (20 12 1)

6	6	8	7	7	6	8	6	7	9	6	7	6	8	7
8	7	9	9	8	9	10	10	10	10	8	9	9	10	10

Aut D (0 11)(1 8)(2 7)(3 6)(4 10)(5 9)

Transitivity sets: {0,11}-(2 20 0), {1,8}-(0 20 2), {2,7}-(0 20 2), {3,6}-(0 20 2), {4,10}-(0 20 2), {5,9}-(0 20 2).

PIII-T7 N27 (4 40) (26 9 0)

6	6	8	7	7	6	8	6	7	9	6	7	6	8	7
8	7	9	10	8	9	10	10	9	10	8	10	9	10	9

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PIII-T7 N30 (4 40) (20 12 1)

6	6	8	8	7	6	7	6	7	9	6	8	6	7	7
8	7	9	10	8	9	10	10	9	10	8	10	9	10	9

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PIII-T7 N33 (4 40) (23 9 1)

6	6	8	8	7	6	7	7	6	9	6	8	7	6	7
8	7	9	10	8	9	10	10	9	10	8	10	9	10	9

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PIII-T7 N35 (8 36) (32 5 0)

6	6	8	8	7	6	7	7	9	6	8	6	7	6	7
8	7	9	10	8	9	10	10	10	9	10	8	9	10	9

Transitivity sets: {0}-(2 18 2), {2}-(2 18 2), {5}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {3}-(0 18 4), {4}-(0 18 4), {6}-(0 18 4), {7}-(0 18 4), {8}-(0 18 4), {9}-(0 18 4), {10}-(0 18 4).

PIII-T7 N36 (8 36) (33 4 0)

6	6	8	8	7	6	7	7	9	6	8	6	7	6	7
8	7	9	10	8	10	9	10	10	9	10	8	10	9	9

Aut D (0 11)(1 8)(2 7)(3 6)(4 10)(5 9)

Transitivity sets: {0,11}-(2 18 2), {3,6}-(2 18 2), {1,8}-(0 18 4), {2,7}-(0 18 4), {4,10}-(0 18 4), {5,9}-(0 18 4).

PIII-T7 N38 (16 28) (41 0 0)

6	6	7	7	7	6	7	8	9	6	8	6	8	6	7
8	8	9	10	8	9	10	10	10	9	10	7	9	10	9

Aut D (1 8)(2 5)(3 10)(6 11)

Transitivity sets: {0}-(4 14 4), {2,5}-(4 14 4), {6,11}-(2 14 6), {1,8}-(0 14 8), {3,10}-(0 14 8), {4}-(0 14 8), {7}-(0 14 8), {9}-(0 14 8).

PIII-T7 N39 (8 36) (32 5 0)

6	6	7	7	7	6	7	8	9	6	8	6	8	6	7
8	8	10	9	8	9	9	10	10	10	10	7	9	9	10

Transitivity sets: {0}-(2 18 2), {2}-(2 18 2), {5}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {3}-(0 18 4), {4}-(0 18 4), {6}-(0 18 4), {7}-(0 18 4), {8}-(0 18 4), {9}-(0 18 4), {10}-(0 18 4).

PIII-T7 N43 (4 40) (24 8 1)

6	6	8	7	7	8	6	6	7	9	6	7	8	6	7
8	7	9	10	8	10	9	10	9	10	8	10	10	9	9

Aut D (0 11)(1 8)(2 7)(3 6)(4 10)(5 9)

Transitivity sets: {0,11}-(2 20 0), {1,8}-(0 20 2), {2,7}-(0 20 2), {3,6}-(0 20 2), {4,10}-(0 20 2), {5,9}-(0 20 2).

PIII-T7 N45 (4 40) (24 11 0)

6	6	8	8	7	7	6	6	7	9	6	8	7	6	7
8	7	9	10	8	10	9	9	10	10	8	10	9	10	9

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PIII-T7 N48 (4 40) (23 9 1)

6	6	8	7	7	7	8	6	6	9	6	7	7	8	6
8	7	9	10	8	9	10	10	9	10	8	10	9	10	9

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PIII-T7 N54 (4 40) (26 6 1)

6	6	8	9	7	7	7	6	6	8	6	8	7	7	6
8	7	9	10	8	9	10	10	9	10	9	10	9	10	8

Aut D (0 11)(1 8)(2 7)(3 6)(4 10)(5 9)

Transitivity sets: {0,11}-(2 20 0), {1,8}-(0 20 2), {2,7}-(0 20 2), {3,6}-(0 20 2), {4,10}-(0 20 2), {5,9}-(0 20 2).

PIII-T7 N55 (4 40) (28 4 1)

6	6	8	9	7	7	7	6	6	8	6	8	7	7	6
8	7	9	10	8	10	9	9	10	10	9	10	9	10	8

Aut D (0 11)(1 8)(2 7)(3 6)(4 10)(5 9)

Transitivity sets: {0,11}-(2 20 0), {1,8}-(0 20 2), {2,7}-(0 20 2), {3,6}-(0 20 2), {4,10}-(0 20 2), {5,9}-(0 20 2).

PIII-T7 N60 (4 40) (26 6 1)

6	6	8	8	7	7	7	6	9	6	8	6	7	7	6
8	7	10	9	8	10	9	9	10	10	10	8	9	10	9

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PIII-T7 N72 (12 32) (29 7 1)

6	7	6	8	7	7	6	6	7	9	6	8	7	8	6
8	9	9	10	8	10	8	10	9	10	9	10	10	9	7

Transitivity sets: {0}-(2 16 4), {1}-(2 16 4), {2}-(2 16 4), {3}-(2 16 4), {6}-(2 16 4), {11}-(2 16 4), {4}-(0 16 6), {5}-(0 16 6), {7}-(0 16 6), {8}-(0 16 6), {9}-(0 16 6), {10}-(0 16 6).

PIII-T7 N74 (4 40) (26 9 0)

6	7	8	6	6	6	7	8	7	9	6	8	6	7	7
8	8	9	10	7	9	10	10	9	10	8	10	9	10	9

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PIII-T7 N86 (12 32) (32 4 1)

6	7	8	6	6	7	7	8	6	9	6	8	7	7	6
8	9	10	9	7	10	8	10	9	10	10	9	10	9	8

Transitivity sets: {0}-(2 16 4), {1}-(2 16 4), {3}-(2 16 4), {4}-(2 16 4), {6}-(2 16 4), {11}-(2 16 4), {2}-(0 16 6), {5}-(0 16 6), {7}-(0 16 6), {8}-(0 16 6), {9}-(0 16 6), {10}-(0 16 6).

PIII-T7 N93 (8 36) (30 7 0)

6	7	7	6	7	8	6	6	8	9	7	6	8	7	6
8	9	10	9	8	10	7	10	9	10	10	9	10	9	8

Aut D (0 5)(1 6)(2 3)(4 9)(7 8)(10 11)

Transitivity sets: {0,5}-(2 18 2), {10,11}-(2 18 2), {1,6}-(0 18 4), {2,3}-(0 18 4), {4,9}-(0 18 4), {7,8}-(0 18 4).

PIII-T7 N97 (16 28) (36 5 0)

6	7	8	6	7	7	6	6	7	9	8	6	7	8	6
8	9	10	9	8	10	8	10	9	10	10	9	10	9	7

Aut D (0 5)(1 6)(2 3)(4 9)(7 8)(10 11)

Transitivity sets: {0,5}-(2 14 6), {1,6}-(2 14 6), {2,3}-(2 14 6), {10,11}-(2 14 6), {4,9}-(0 14 8), {7,8}-(0 14 8).

PIII-T7 N100 (4 40) (25 7 1)

6	7	7	9	6	6	7	8	8	6	7	8	6	7	6
8	8	10	10	7	9	9	10	9	10	10	10	9	9	8

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PIII-T7 N109 (8 36) (31 6 0)

6	7	8	7	7	6	6	6	7	9	8	7	6	8	6
8	9	10	10	8	9	8	10	9	10	10	10	9	9	7

Transitivity sets: {0}-(2 18 2), {1}-(2 18 2), {2}-(2 18 2), {11}-(2 18 2), {3}-(0 18 4), {4}-(0 18 4), {5}-(0 18 4), {6}-(0 18 4), {7}-(0 18 4), {8}-(0 18 4), {9}-(0 18 4), {10}-(0 18 4).

PIII-T7 N110 (8 36) (25 12 0)

6	7	8	7	7	6	6	6	7	9	8	7	6	8	6
8	10	9	9	8	9	8	10	10	10	10	9	9	10	7

Transitivity sets: {0}-(2 18 2), {1}-(2 18 2), {2}-(2 18 2), {11}-(2 18 2), {3}-(0 18 4), {4}-(0 18 4), {5}-(0 18 4), {6}-(0 18 4), {7}-(0 18 4), {8}-(0 18 4), {9}-(0 18 4), {10}-(0 18 4).

PIII-T7 N113 (12 32) (31 5 1)

7	6	6	8	7	7	6	9	8	6	6	7	8	6	7
9	8	7	9	8	10	9	10	10	10	8	9	10	9	10

Transitivity sets: {11}-(6 16 0), {1}-(4 16 2), {0}-(2 16 4), {2}-(0 16 6), {3}-(0 16 6), {4}-(0 16 6), {5}-(0 16 6), {6}-(0 16 6), {7}-(0 16 6), {8}-(0 16 6), {9}-(0 16 6), {10}-(0 16 6).

PIII-T7 N114 (4 40) (26 6 1)

7	6	6	8	7	7	6	9	8	6	6	7	8	6	7
9	8	7	10	8	9	9	10	10	10	8	10	9	9	10

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PIII-T7 N119 (4 40) (19 13 1)

7	6	6	8	7	7	8	9	6	6	6	7	8	7	6
9	7	8	10	8	9	10	10	9	10	8	10	9	10	9

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PIII-T7 N122 (4 40) (23 12 0)

7	6	6	8	7	7	8	9	6	6	6	7	8	7	6
9	7	9	10	8	9	10	10	8	10	9	10	9	10	8

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PIII-T7 N128 (20 24) (40 3 0)

7	6	7	6	7	7	6	8	9	6	8	6	8	6	7
9	8	8	9	9	10	8	10	10	10	9	7	10	9	10

Transitivity sets: {11}-(6 12 4), {1}-(4 12 6), {0}-(2 12 8), {2}-(2 12 8), {3}-(2 12 8), {5}-(2 12 8), {7}-(2 12 8), {4}-(0 12 10), {6}-(0 12 10), {8}-(0 12 10), {9}-(0 12 10), {10}-(0 12 10).

PIII-T7 N129 (24 20) (40 5 0)

7	6	7	6	7	7	6	9	8	6	8	6	8	6	7
9	8	9	8	8	10	9	10	10	10	9	7	10	9	10

Transitivity sets: {11}-(6 10 6), {1}-(4 10 8), {2}-(4 10 8), {0}-(2 10 10), {3}-(2 10 10), {4}-(2 10 10), {5}-(2 10 10), {7}-(2 10 10), {6}-(0 10 12), {8}-(0 10 12), {9}-(0 10 12), {10}-(0 10 12).

PIII-T7 N131 (8 36) (31 6 0)

7	6	7	6	7	7	6	9	8	6	8	6	8	6	7
9	8	10	8	8	10	9	10	9	10	10	7	10	9	9

Transitivity sets: {0}-(2 18 2), {2}-(2 18 2), {4}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {3}-(0 18 4), {5}-(0 18 4), {6}-(0 18 4), {7}-(0 18 4), {8}-(0 18 4), {9}-(0 18 4), {10}-(0 18 4).

PIII-T7 N134 (12 32) (29 7 1)

7	6	7	8	7	6	6	9	8	6	8	7	6	6	7
9	8	9	10	8	7	9	10	10	10	9	10	8	9	10

Transitivity sets: {0}-(2 16 4), {2}-(2 16 4), {3}-(2 16 4), {5}-(2 16 4), {7}-(2 16 4), {11}-(2 16 4), {1}-(0 16 6), {4}-(0 16 6), {6}-(0 16 6), {8}-(0 16 6), {9}-(0 16 6), {10}-(0 16 6).

PIII-T7 N146 (4 40) (25 7 1)

7 6 7 8 7 7 6 9 6 6 8 7 8 6 6
9 8 10 9 8 10 9 10 8 10 10 9 10 9 7

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PIII-T7 N155 (28 16) (45 2 0)

7 6 7 6 7 7 6 9 8 6 8 6 8 6 7
10 8 9 8 8 10 9 10 10 9 10 7 9 10 9

Aut D (1 5)(2 4)(7 10)(8 9) (0 6)(1 8)(5 9)(7 10)
(0 6)(1 9)(2 4)(5 8)

Transitivity sets: {0,6}-(8 8 6), {2,4}-(4 8 10), {11}-(4 8 10), {3}-(0 8 14), {1,5,8,9}-(0 8 14), {7,10}-(0 8 14).

PIII-T7 N167 (4 40) (20 12 1)

8 6 6 6 7 7 8 9 7 6 6 6 7 8 7
9 7 9 8 8 9 10 10 10 10 9 8 9 10 10

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PIII-T7 N169 (8 36) (20 14 1)

8 6 6 6 7 8 7 9 7 6 6 6 8 7 7
9 7 8 9 8 10 9 10 10 10 8 9 10 9 10

Transitivity sets: {0}-(2 18 2), {5}-(2 18 2), {10}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4), {3}-(0 18 4), {4}-(0 18 4), {6}-(0 18 4), {7}-(0 18 4), {8}-(0 18 4), {9}-(0 18 4).

PIII-T7 N179 (8 36) (25 12 0)

8 6 6 7 7 6 7 9 8 6 6 7 6 7 8
9 7 9 10 8 8 9 10 10 10 9 10 8 9 10

Transitivity sets: {0}-(2 18 2), {4}-(2 18 2), {9}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4), {3}-(0 18 4), {5}-(0 18 4), {6}-(0 18 4), {7}-(0 18 4), {8}-(0 18 4), {10}-(0 18 4).

PIII-T7 N180 (4 40) (16 10 3)

8 6 6 7 7 6 8 9 7 6 6 7 6 8 7
9 7 8 9 8 9 10 10 10 10 8 9 9 10 10

Aut D (0 11)(1 8)(2 7)(3 6)(4 10)(5 9)

Transitivity sets: {0,11}-(2 20 0), {1,8}-(0 20 2), {2,7}-(0 20 2), {3,6}-(0 20 2), {4,10}-(0 20 2), {5,9}-(0 20 2).

PIII-T7 N181 (4 40) (22 10 1)

8 6 6 7 7 6 8 9 7 6 6 7 6 8 7
9 7 8 10 8 9 10 10 9 10 8 10 9 10 9

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PIII-T7 N186 (4 40) (23 9 1)

8 6 6 7 7 6 8 9 7 6 6 7 6 8 7
9 7 9 10 8 8 10 10 10 9 10 9 8 10 9

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2), {2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2), {5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2), {8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PIII-T7 N189 (4 40) (22 10 1)
8 6 6 8 7 6 7 9 7 6 6 8 6 7 7
9 7 9 10 8 8 9 10 10 10 9 10 8 9 10
Aut D (0 11)(1 9)(2 10)(3 6)(4 7)(5 8)
Transitivity sets: {0,11}-(2 20 0), {1,9}-(0 20 2), {2,10}-(0 20 2),
{3,6}-(0 20 2), {4,7}-(0 20 2), {5,8}-(0 20 2).

PIII-T7 N198 (12 32) (23 16 0)
8 6 6 7 7 7 6 9 8 6 6 7 7 6 8
9 7 9 10 8 9 8 10 10 10 9 10 9 8 10
Aut D (0 11)(1 9)(2 10)(3 6)(4 7)(5 8)
Transitivity sets: {0,11}-(2 16 4), {1,9}-(2 16 4), {4,7}-(2 16 4),
{2,10}-(0 16 6), {3,6}-(0 16 6), {5,8}-(0 16 6).

PIII-T7 N200 (4 40) (20 12 1)
8 6 6 7 7 8 6 9 7 6 6 7 8 6 7
9 7 8 10 8 10 9 10 9 10 8 10 10 9 9
Aut D (0 11)(1 8)(2 7)(3 6)(4 10)(5 9)
Transitivity sets: {0,11}-(2 20 0), {1,8}-(0 20 2), {2,7}-(0 20 2),
{3,6}-(0 20 2), {4,10}-(0 20 2), {5,9}-(0 20 2).

PIII-T7 N203 (8 36) (25 12 0)
8 6 6 7 7 8 6 9 7 6 6 7 8 6 7
9 7 9 10 8 10 8 10 10 9 10 9 10 8 9
Transitivity sets: {0}-(2 18 2), {1}-(2 18 2), {7}-(2 18 2),
{11}-(2 18 2), {2}-(0 18 4), {3}-(0 18 4),
{4}-(0 18 4), {5}-(0 18 4), {6}-(0 18 4),
{8}-(0 18 4), {9}-(0 18 4), {10}-(0 18 4).

PIII-T7 N227 (4 40) (20 12 1)
8 6 7 6 7 6 8 9 6 7 6 7 6 8 7
9 7 10 9 8 8 10 10 9 10 10 9 8 10 9
Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2),
{2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2),
{5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2),
{8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PIII-T7 N234 (4 40) (24 8 1)
8 6 7 6 7 6 8 9 7 6 7 6 6 8 7
9 7 8 10 9 8 10 10 10 9 10 8 9 10 9
Aut D (0 11)(1 9)(2 10)(3 6)(4 7)(5 8)
Transitivity sets: {0,11}-(2 20 0), {1,9}-(0 20 2), {2,10}-(0 20 2),
{3,6}-(0 20 2), {4,7}-(0 20 2), {5,8}-(0 20 2).

PIII-T7 N243 (8 36) (25 12 0)
8 6 8 6 7 6 7 9 7 6 8 6 6 7 7
9 7 10 9 8 8 10 10 9 10 10 9 8 10 9
Aut D (1 8)(2 6)(3 9)(4 7)(5 10)
Transitivity sets: {0}-(2 18 2), {5,10}-(2 18 2), {11}-(2 18 2),
{1,8}-(0 18 4), {2,6}-(0 18 4), {3,9}-(0 18 4),
{4,7}-(0 18 4).

PIII-T7 N260 (8 36) (29 8 0)
8 6 8 6 7 7 6 9 7 6 8 6 7 6 7
9 7 10 8 8 10 9 10 9 10 10 8 10 9 9
Aut D (1 5)(2 4)(7 10)(8 9) (3 6)(1 7 5 10)(2 9 4 8)
(3 6)(1 10 5 7)(2 8 4 9) (0 11)(1 2 5 4)(7 9 10 8)
(0 11)(1 4 5 2)(7 8 10 9) (0 11)(1 8)(2 7)(3 6)(4 10)(5 9)
(0 11)(1 9)(2 10)(3 6)(4 7)(5 8)
Transitivity sets: {0,11}-(2 18 2), {3,6}-(2 18 2),
{1,2,4,5,7,8,9,10}-(0 18 4).

PIII-T8 N1 (4 40) (22 10 1)
6 8 8 6 7 6 7 6 7 9 6 8 6 7 7
7 9 10 8 8 9 10 9 10 10 8 10 10 9 9
Aut D (0 11)(1 8)(2 7)(3 6)(4 10)(5 9)
Transitivity sets: {0,11}-(2 20 0), {1,8}-(0 20 2), {2,7}-(0 20 2),
{3,6}-(0 20 2), {4,10}-(0 20 2), {5,9}-(0 20 2).

PIII-T8 N28 (24 20) (44 1 0)
6 7 7 6 7 6 7 8 9 6 8 6 6 8 7
8 9 10 8 8 9 10 10 10 9 10 7 10 9 9
Transitivity sets: {2}-(6 10 6), {0}-(4 10 8), {5}-(4 10 8),
{1}-(2 10 10), {6}-(2 10 10), {8}-(2 10 10),
{9}-(2 10 10), {11}-(2 10 10), {3}-(0 10 12),
{4}-(0 10 12), {7}-(0 10 12), {10}-(0 10 12).

PIII-T8 N29 (12 32) (33 6 0)
6 7 7 6 7 6 7 8 9 6 8 6 6 8 7
8 10 9 8 8 9 10 9 10 10 10 7 9 10 9
Aut D (0 11)(1 9)(2 10)(3 6)(4 7)(5 8)
Transitivity sets: {0,11}-(2 16 4), {2,10}-(2 16 4), {5,8}-(2 16 4),
{1,9}-(0 16 6), {3,6}-(0 16 6), {4,7}-(0 16 6).

PIII-T9 N1 (8 36) (20 14 1)
6 6 7 8 7 7 8 9 6 6 6 7 7 8 6
7 8 9 10 8 9 10 10 9 10 8 10 10 9 9
Aut D (1 5)(2 4)(7 10)(8 9) (0 11)(1 8)(2 7)(3 6)(4 10)(5 9)
(0 11)(1 9)(2 10)(3 6)(4 7)(5 8)
Transitivity sets: {0,11}-(2 18 2), {3,6}-(2 18 2), {1,5,8,9}-(0 18 4),
{2,4,7,10}-(0 18 4).

PIII-T9 N3 (16 28) (32 6 1)
6 6 7 9 7 7 8 8 6 6 6 7 7 8 6
7 8 9 10 9 8 10 10 9 10 9 10 10 9 8
Aut D (1 5)(2 4)(7 10)(8 9) (0 6)(1 8)(2 4)(3 11)(5 9)
(0 6)(1 9)(3 11)(5 8)(7 10)
Transitivity sets: {7,10}-(4 14 4), {0,6}-(2 14 6), {3,11}-(2 14 6),
{2,4}-(0 14 8), {1,5,8,9}-(0 14 8).

PIII-T9 N4 (8 36) (24 10 1)
6 6 7 8 7 7 8 9 6 6 6 7 7 8 6
7 9 9 10 8 9 10 10 8 10 9 10 10 9 8
Aut D (1 5)(2 4)(7 10)(8 9) (0 11)(1 8)(2 7)(3 6)(4 10)(5 9)
(0 11)(1 9)(2 10)(3 6)(4 7)(5 8)
Transitivity sets: {0,11}-(2 18 2), {3,6}-(2 18 2), {1,5,8,9}-(0 18 4),
{2,4,7,10}-(0 18 4).

PIII-T9 N13 (16 28) (30 8 1)
7 6 6 9 7 6 7 7 6 8 6 8 7 8 6
8 8 9 10 10 8 9 10 9 10 10 9 9 10 7
Aut D (0 11)(1 9)(5 8)
Transitivity sets: {0,11}-(4 14 4), {2}-(2 14 6), {4}-(2 14 6),
{6}-(2 14 6), {10}-(2 14 6), {1,9}-(0 14 8),
{3}-(0 14 8), {5,8}-(0 14 8), {7}-(0 14 8).

PIII-T9 N14 (20 24) (31 12 0)
7 6 6 8 7 6 7 7 6 9 6 8 7 8 6
8 9 9 10 10 8 9 10 8 10 10 9 9 10 7
Aut D (0 11)(1 9)(5 8) (1 5)(2 4)(7 10)(8 9)
(0 11)(1 8)(2 4)(5 9)(7 10)
Transitivity sets: {0,11}-(4 12 6), {2,4}-(4 12 6), {3}-(4 12 6),
{6}-(0 12 10), {7,10}-(0 12 10), {1,5,8,9}-(0 12 10).

<u>PIII-T10 N33</u>   (24 30)   (45 0 0)

6	7	7	9	7	6	7	6	8	6	8	7	8	6	6
8	8	10	10	9	8	9	10	10	9	10	10	9	9	7

<u>Aut D</u> (0 6)(3 9)(5 7)(8 11)   (0 8)(1 4)(3 7)(5 9)(6 11)
(0 11)(1 4)(3 5)(6 8)(7 9)   (2 10)(0 5 6 7)(3 8 9 11)
(2 10)(0 7 6 5)(3 11 9 8)   (1 4)(2 10)(0 3 6 9)(5 8 7 11)
(1 4)(2 10)(0 9 6 3)(5 11 7 8)

<u>Transitivity sets:</u> {1,4}-(2 10 10), {2,10}-(2 10 10),
{0,3,5,6,7,8,9,11}-(2 10 10).

<u>PIII-T10 N40</u>   (16 28)   (30 8 1)

7	6	6	9	7	6	7	6	8	8	6	8	7	7	6
8	8	9	10	10	7	9	9	10	10	10	9	9	10	8

<u>Aut D</u> (0 11)(1 9)(5 8)   (1 5)(2 7)(3 6)(4 10)(8 9)
(0 11)(1 8)(2 7)(3 6)(4 10)(5 9)

<u>Transitivity sets:</u> {0,11}-(4 14 4), {3,6}-(2 14 6), {4,10}-(2 14 6),
{2,7}-(0 14 8), {1,5,8,9}-(0 14 8).

<u>PIII-T10 N55</u>   (8 36)   (37 0 0)

7	7	6	9	6	7	6	7	8	6	8	7	8	6	6
8	10	8	10	8	9	9	9	10	10	10	10	9	9	7

<u>Aut D</u> (1 7)(2 10)(3 8)(4 9)(5 6)

<u>Transitivity sets:</u> {0}-(2 18 2), {2,10}-(2 18 2), {11}-(2 18 2),
{1,7}-(0 18 4), {3,8}-(0 18 4), {4,9}-(0 18 4),
{5,6}-(0 18 4).

<u>PIII-T10 N60</u>   (16 28)   (36 2 1)

7	6	6	8	6	7	7	8	9	6	6	8	7	7	6
9	8	9	10	7	10	8	10	10	9	10	9	9	10	8

<u>Aut D</u> (1 8)(3 4)(5 9)(6 10)   (0 11)(2 7)(3 10)(4 6)
(0 11)(1 8)(2 7)(3 6)(4 10)(5 9)

<u>Transitivity sets:</u> {0,11}-(4 14 4), {1,8}-(2 14 6), {5,9}-(2 14 6),
{2,7}-(0 14 8), {3,4,6,10}-(0 14 8).

<u>PIII-T10 N62</u>   (8 36)   (28 0 3)

7	6	6	8	7	6	7	6	8	9	6	8	7	7	6
9	8	9	10	10	7	8	9	10	10	10	9	9	10	8

<u>Aut D</u> (0 11)(1 9)(5 8)   (0 11)(2 7)(3 10)(4 6)
(1 8)(3 4)(5 9)(6 10)   (1 5)(2 7)(3 6)(4 10)(8 9)
(1 9)(2 7)(3 10)(4 6)(5 8)   (0 11)(1 5)(3 4)(6 10)(8 9)
(0 11)(1 8)(2 7)(3 6)(4 10)(5 9)

<u>Transitivity sets:</u> {0,11}-(4 18 0), {2,7}-(0 18 4), {3,4,6,10}-(0 18 4)
{1,5,8,9}-(0 18 4).

<u>PIII-T10 N73</u>   (24 20)   (45 0 0)

7	7	6	8	6	7	6	7	9	6	8	7	8	6	6
9	8	9	10	8	10	8	10	10	9	10	9	9	10	7

<u>Aut D</u> (0 5)(1 6)(3 11)(4 8)   (1 3)(2 9)(4 8)(6 11)(7 10)
(0 5)(1 11)(2 9)(3 6)(7 10)

<u>Transitivity sets:</u> {0,5}-(4 10 8), {1,3,6,11}-(2 10 10), {2,9}-(2 10 10),
{7,10}-(2 10 10), {4,8}-(0 10 12).

<u>PIV-T1 N4</u>   (40 4)   (53 0 0)

6	7	6	8	6	7	6	9	7	8	6	7	8	6	7
7	9	10	9	8	8	9	10	10	10	9	8	10	10	9

<u>Aut D</u> (see reducible design no.7)

<u>Transitivity sets:</u> {11}-(12 2 8), {0,5}-(6 2 14), {1,8}-(4 2 16),
{2}-(4 2 16), {6,10}-(2 2 18), {3,7}-(0 2 20),
{4}-(0 2 20), {9}-(0 2 20).

PIV-T1 N5 (36 8) (51 0 0)

6 7 6 8 6 7 6 9 7 8 6 7 8 6 7
7 9 10 10 8 8 9 10 9 10 10 8 9 9 10

Aut D (see reducible design no.5)

Transitivity sets: {0,1}-(6 4 12), {6,11}-(6 4 12), {8}-(4 4 14), {4,7}-(2 4 16), {5,10}-(2 4 16), {2,3}-(0 4 18), {9}-(0 4 18).

PIV-T1 N9 (20 24) (39 4 0)

6 7 6 8 6 7 6 8 7 8 6 7 9 6 7
7 9 9 10 9 8 10 9 10 10 8 9 10 10 8

Aut D (0 11)(1 4)(2 5)(6 8)(9 10)

Transitivity sets: {0,11}-(4 12 6), {2,5}-(2 12 8), {6,8}-(2 12 8), {9,10}-(2 12 8), {1,4}-(0 12 10), {3}-(0 12 10), {7}-(0 12 10).

PIV-T1 N11 (40 4) (53 0 0)

6 7 6 8 6 7 6 8 7 8 6 7 9 6 7
7 9 10 10 10 8 9 9 9 10 8 10 10 9 8

Aut D (see reducible design no.9)

Transitivity sets: {0,11}-(6 2 14), {6,8}-(6 2 14), {7}-(4 2 16), {9}-(4 2 16), {1,4}-(2 2 18), {2,5}-(2 2 18), {3}-(0 2 20), {10}-(0 2 20).

PIV-T1 N12 (32 12) (49 0 0)

6 7 6 9 6 7 6 8 7 8 6 7 8 6 7
7 9 10 10 9 8 9 10 8 10 10 9 9 8 10

Aut D (see reducible design no.2)

Transitivity sets: {2,3}-(6 6 10), {6,11}-(4 6 12), {0,1,8,9}-(2 6 14), {5,10}-(2 6 14), {4,7}-(0 6 16).

PIV-T1 N13 (40 0) (53 0 0)

6 7 6 9 6 7 6 8 7 8 6 7 8 6 7
7 9 10 10 10 8 9 9 8 10 9 10 10 8 9

Aut D (see reducible design no.6)

Transitivity sets: {11}-(10 2 10), {2,8}-(6 2 14), {6,9}-(4 2 16), {0,1}-(2 2 18), {3,5}-(2 2 18), {7}-(2 2 18), {4,10}-(0 2 20).

PIV-T1 N20 (36 8) (51 0 0)

6 7 6 8 7 7 6 8 6 8 7 6 9 7 6
7 9 10 9 9 8 9 10 10 10 8 9 10 10 8

Aut D (see reducible design no.3)

Transitivity sets: {0,11}-(8 4 10), {1,2}-(4 4 14), {3}-(4 4 14), {4,5,6,8}-(2 4 16), {7,9}-(0 4 18), {10}-(0 4 18).

PIV-T1 N21 (40 4) (53 0 0)

6 7 6 8 7 7 6 8 6 8 7 6 9 7 6
7 9 10 10 10 8 9 9 9 10 8 10 10 9 8

Aut D (see reducible design no.10)

Transitivity sets: {0,6,8,11}-(10 2 10), {1,2,3,4,5,7,9,10}-(0 2 20).

PIV-T1 N32 (8 36) (29 5 1)

6 6 7 9 8 6 6 7 7 7 6 8 8 6 7
9 10 8 10 9 8 10 9 10 8 9 10 10 7 9

Transitivity sets: {0}-(2 18 2), {5}-(2 18 2), {8}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4), {3}-(0 18 4), {4}-(0 18 4), {6}-(0 18 4), {7}-(0 18 4), {9}-(0 18 4), {10}-(0 18 4).

PIV-T1 N43 (12 32) (34 5 0)

6	7	6	7	6	6	7	9	7	8	8	6	6	8	7
9	8	10	9	9	8	8	10	10	10	9	7	10	10	9

Transitivity sets: {0}-(2 16 4), {4}-(2 16 4), {5}-(2 16 4), {6}-(2 16 4), {9}-(2 16 4), {11}-(2 16 4), {1}-(0 16 6), {2}-(0 16 6), {3}-(0 16 6), {7}-(0 16 6), {8}-(0 16 6), {10}-(0 16 6).

PIV-T1 N45 (16 28) (36 5 0)

6	7	6	7	6	6	7	8	7	8	8	6	6	9	7
9	9	10	8	9	8	9	10	10	10	9	7	10	10	8

Aut D (0 8)(1 7)(2 6)(3 5)(4 11)(9 10)

Transitivity sets: {0,8}-(2 14 6), {2,6}-(2 14 6), {3,5}-(2 14 6), {4,11}-(2 14 6), {1,7}-(0 14 8), {9,10}-(0 14 8).

PIV-T1 N46 (8 36) (28 6 1)

6	8	6	9	6	6	7	8	7	7	8	6	6	7	7
9	10	7	10	9	8	9	10	10	8	9	10	10	8	9

Transitivity sets: {0}-(2 18 2), {5}-(2 18 2), {6}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4), {3}-(0 18 4), {4}-(0 18 4), {7}-(0 18 4), {8}-(0 18 4), {9}-(0 18 4), {10}-(0 18 4).

PIV-T1 N47 (12 32) (30 6 1)

6	9	6	8	6	6	7	8	7	7	8	6	6	7	7
9	10	7	10	8	9	9	10	10	8	9	10	10	8	9

Transitivity sets: {0}-(4 16 2), {1}-(2 16 4), {5}-(2 16 4), {6}-(2 16 4), {11}-(2 16 4), {2}-(0 16 6), {3}-(0 16 6), {4}-(0 16 6), {7}-(0 16 6), {8}-(0 16 6), {9}-(0 16 6), {10}-(0 16 6).

PIV-T1 N49 (8 36) (31 6 0)

6	9	6	8	6	6	7	8	7	7	8	6	6	7	7
9	10	7	10	9	8	9	10	10	8	9	10	10	9	8

Transitivity sets: {0}-(2 18 2), {5}-(2 18 2), {6}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {2}-(0 18 4), {3}-(0 18 4), {4}-(0 18 4), {7}-(0 18 4), {8}-(0 18 4), {9}-(0 18 4), {10}-(0 18 4).

PIV-T1 N51 (8 36) (31 6 0)

6	7	6	9	6	8	6	7	7	8	6	7	8	6	7
9	10	7	10	9	10	10	8	8	9	10	9	10	8	9

Transitivity sets: {0}-(2 18 2), {2}-(2 18 2), {10}-(2 18 2), {11}-(2 18 2), {1}-(0 18 4), {3}-(0 18 4), {4}-(0 18 4), {5}-(0 18 4), {6}-(0 18 4), {7}-(0 18 4), {8}-(0 18 4), {9}-(0 18 4).

PIV-T1 N64 (16 28) (39 2 0)

6	7	6	7	6	7	6	7	8	8	6	9	8	6	7
9	10	10	9	9	8	10	8	10	9	7	10	10	8	9

Aut D (1 6)(2 7)(4 5)(9 10)

Transitivity sets: {0}-(4 14 4), {11}-(4 14 4), {2,7}-(2 14 6), {4,5}-(2 14 6), {1,6}-(0 14 8), {3}-(0 14 8), {8}-(0 14 8), {9,10}-(0 14 8).

PIV-T1 N69 (40 4) (53 0 0)

6	7	6	8	6	7	6	7	7	8	6	9	7	6	8
9	9	10	10	10	8	9	8	9	10	8	10	10	7	9

Aut D (see reducible design no.8)

Transitivity sets: {0,1,2,4,5,7,9,11}-(4 2 16), {3,6,8,10}-(2 2 18).

PIV-T1 N71 (8 36) (31 6 0)

6 9 6 8 6 7 6 7 7 8 6 8 7 6 7
9 10 7 10 9 9 8 10 8 10 10 9 9 10 8

Transitivity sets: {0}-(4 18 0), {1}-(2 18 2), {11}-(2 18 2), {2}-(0 18 4), {3}-(0 18 4), {4}-(0 18 4), {5}-(0 18 4), {6}-(0 18 4), {7}-(0 18 4), {8}-(0 18 4), {9}-(0 18 4), {10}-(0 18 4).

PIV-T1 N73 (12 32) (32 7 0)

6 9 6 7 6 8 6 7 7 7 6 8 8 6 7
9 10 10 8 8 9 10 9 10 8 9 10 10 7 9

Transitivity sets: {0}-(4 16 2), {1}-(2 16 4), {5}-(2 16 4), {8}-(2 16 4), {11}-(2 16 4), {2}-(0 16 6), {3}-(0 16 6), {4}-(0 16 6), {6}-(0 16 6), {7}-(0 16 6), {9}-(0 16 6), {10}-(0 16 6).

PIV-T1 N85 (8 36) (35 2 0)

6 7 6 7 7 6 6 7 8 9 6 8 8 7 6
9 8 10 9 9 8 10 8 10 10 7 9 10 10 9

Aut D (0 11)(1 2)(4 5)(8 9)

Transitivity sets: {0,11}-(4 18 0), {1,2}-(0 18 4), {3}-(0 18 4), {4,5}-(0 18 4), {6}-(0 18 4), {7}-(0 18 4), {8,9}-(0 18 4), {10}-(0 18 4).

PIV-T1 N86 (16 28) (37 4 0)

6 7 6 7 7 6 6 7 9 8 6 8 8 7 6
9 9 10 8 8 8 10 9 10 10 7 9 10 10 9

Aut D (3 10)(4 9)(5 8)(6 7) (0 11)(1 2)(4 5)(8 9)
(0 11)(1 2)(3 10)(4 8)(5 9)(6 7)

Transitivity sets: {0,11}-(6 14 2), {1,2}-(2 14 6), {3,10}-(0 14 8), {6,7}-(0 14 8), {4,5,8,9}-(0 14 8).

PIV-T1 N90 (12 32) (35 1 1)

6 7 6 7 7 6 6 7 8 8 6 8 9 7 6
9 10 10 8 9 9 10 8 10 9 7 10 10 9 8

Aut D (0 11)(3 8)(4 10)(5 9) (0 11)(1 4)(2 5)(7 8)
(0 11)(1 10)(2 9)(3 7) (1 4 10)(2 5 9)(3 7 8)
(1 10 4)(2 9 5)(3 8 7)

Transitivity sets: {0,11}-(6 16 0), {1,4,10}-(0 16 6), {2,5,9}-(0 16 6), {6}-(0 16 6), {3,7,8}-(0 16 6).

PIV-T1 N92 (12 32) (36 0 1)

6 7 6 7 7 6 6 7 8 8 6 9 8 7 6
9 10 10 9 9 8 10 8 10 9 7 10 10 8 9

Aut D (1 6)(2 7)(3 8)(4 10)(5 9)

Transitivity sets: {0}-(4 16 2), {11}-(4 16 2), {2,7}-(2 16 4), {1,6}-(0 16 6), {3,8}-(0 16 6), {4,10}-(0 16 6), {5,9}-(0 16 6).

PIV-T1 N111 (36 8) (51 0 0)

6 7 6 7 7 7 6 8 6 8 8 6 7 9 6
9 9 10 8 9 8 9 10 10 10 9 7 10 10 8

Aut D (see reducible design no.4)

Transitivity sets: {1,2}-(8 4 10), {0,11}-(4 4 14), {3}-(4 4 14), {4,5,6,8}-(2 4 16), {7,9}-(0 4 18), {10}-(0 4 18).

PIV-T1 N123 (44 0) (55 0 0)

6	7	6	7	9	7	6	7	6	8	7	6	8	8	6
9	9	10	10	10	8	9	8	8	10	9	10	10	9	7

Aut D (see reducible design no.11)

Transitivity sets: {0,1,4,10,11}-(6 0 16), {2,3,5,8,9}-(2 0 20),
{6,7}-(2 0 20).

PIV-T1 N235 (24 20) (45 0 0)

9	7	6	6	8	7	6	6	7	8	6	8	7	7	6
10	9	8	10	10	8	7	9	9	10	10	9	8	10	9

Aut D (see reducible design no.1)

Transitivity sets: {0,1,2,4,5,7,8,9,10,11}-(2 10 10), {3,6}-(2 10 10).

PIV-T3 N61 (4 40) (25 10 0)

9	6	8	6	8	7	6	7	7	6	8	6	7	6	7
10	7	10	9	9	9	10	8	10	8	10	9	9	10	8

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2),
{2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2),
{5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2),
{8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PIV-T3 N62 (8 36) (30 7 0)

9	6	8	6	8	7	6	7	7	6	8	6	7	6	7
10	7	10	10	9	10	9	8	9	8	10	10	9	9	8

Aut D (0 7)(1 4)(2 8)(3 9)(5 11)(6 10)

Transitivity sets: {0,7}-(2 18 2), {5,11}-(2 18 2), {1,4}-(0 18 4),
{2,8}-(0 18 4), {3,9}-(0 18 4), {6,10}-(0 18 4).

PIV-T3 N65 (8 36) (26 8 1)

9	6	7	6	8	7	6	7	7	6	8	6	7	6	8
10	10	8	9	9	9	10	8	10	8	10	9	9	7	10

Aut D (1 2)(3 5)(6 7)(8 10)

Transitivity sets: {0}-(2 18 2), {4}-(2 18 2), {9}-(2 18 2),
{11}-(2 18 2), {1,2}-(0 18 4), {3,5}-(0 18 4),
{6,7}-(0 18 4), {8,10}-(0 18 4).

PIV-T3 N66 (4 40) (30 5 0)

9	6	8	7	8	6	6	7	7	6	8	7	6	6	7
10	7	9	10	10	10	9	8	9	8	10	9	10	9	8

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2),
{2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2),
{5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2),
{8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PIV-T3 N67 (4 40) (25 7 1)

9	6	8	7	8	6	6	7	7	6	8	7	6	6	7
10	7	10	9	9	9	10	8	10	8	10	9	9	10	8

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2),
{2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2),
{5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2),
{8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PIV-T3 N68 (4 40) (28 4 1)

9	6	7	7	8	6	6	8	7	6	8	7	6	6	7
10	9	8	10	10	10	7	9	9	8	10	9	10	9	8

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2),
{2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2),
{5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2),
{8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PIV-T5 N23 (8 36) (29 8 0)

6 6 7 9 8 6 7 7 8 6 6 7 6 7 8
7 9 8 10 9 10 9 10 10 8 9 8 10 9 10

Aut D (1 2)(3 5)(6 7)(8 10) (0 11)(1 3 2 5)(6 8 7 10)
(0 11)(1 5 2 3)(6 10 7 8) (4 9)(1 8 2 10)(3 6 5 7)
(4 9)(1 10 2 8)(3 7 5 6) (0 11)(4 9)(1 6 2 7)(3 10 5 8)
(0 11)(4 9)(1 7 2 6)(3 8 5 10)

Transitivity sets: {0,11}-(2 18 2), {4,9}-(2 18 2),
{1,2,3,5,6,7,8,10}-(0 18 4).

PIV-T5 N28 (4 40) (25 10 0)

6 6 7 9 8 6 8 7 7 6 6 7 6 8 7
7 10 8 10 9 9 10 10 9 8 10 8 9 10 9

Aut D (1 2 3 4 5)(6 9 7 8 10) (1 3 5 2 4)(6 7 10 9 8)
(1 4 2 5 3)(6 8 9 10 7) (1 5 4 3 2)(6 10 8 7 9)
(1 7)(2 8)(3 10)(4 6)(5 9) (1 6 2 9 3 7 4 8 5 10)
(1 8 3 6 5 7 2 10 4 9) (1 9 4 10 2 7 5 6 3 8)
(1 10 5 8 4 7 3 9 2 6)

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0),
{1,2,3,4,5,6,7,8,9,10}-(0 20 2).

PIV-T5 N53 (8 36) (37 0 0)

6 9 6 7 6 7 8 7 8 6 6 7 6 7 8
7 10 8 9 9 10 9 8 10 10 9 8 10 9 10

Aut D (1 2)(3 5)(6 7)(8 10) (0 4)(3 8)(5 10)(6 7)(9 11)
(0 4)(1 2)(3 10)(5 8)(9 11) (0 9)(1 6)(2 7)(4 11)(8 10)
(0 9)(1 7)(2 6)(3 5)(4 11) (4 9)(1 8 2 10)(3 6 5 7)
(4 9)(1 10 2 8)(3 7 5 6) (0 11)(1 3 2 5)(6 8 7 10)
(0 11)(1 5 2 3)(6 10 7 8) (0 11)(4 9)(1 6 2 7)(3 10 5 8)
(0 11)(4 9)(1 7 2 6)(3 8 5 10) (0 4 11 9)(1 3 7 8 2 5 6 10)
(0 4 11 9)(1 5 7 10 2 3 6 8) (0 9 11 4)(1 8 6 3 2 10 7 5)
(0 9 11 4)(1 10 6 5 2 8 7 3)

Transitivity sets: {0,4,9,11}-(2 18 2), {1,2,3,5,6,7,8,10}-(0 18 4).

PIV-T5 N90 (44 0) (55 0 0)

6 7 6 7 9 7 6 6 8 8 6 7 6 8 7
8 9 10 10 10 8 9 7 10 9 10 8 9 10 9

Aut D (see reducible design no.12)

Transitivity sets: {0}-(22 0 0), {1,2,3,4,5,6,7,8,9,10,11}-(2 0 20).

PV-T1 N12 (4 40) (23 9 1)

6 6 8 7 9 6 7 6 8 7 7 8 6 6 7
10 7 9 8 10 8 9 9 10 10 9 10 10 9 8

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2),
{2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2),
{5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2),
{8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PV-T1 N14 (4 40) (22 13 0)

6 6 7 7 9 6 8 6 8 7 7 8 6 6 7
10 8 9 8 10 7 9 9 10 10 9 10 10 9 8

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2),
{2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2),
{5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2),
{8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PV-T2 N3 (8 36) (25 12 0)

6 6 7 8 8 6 7 6 8 7 7 6 9 6 7
7 8 9 10 9 10 9 9 10 10 8 10 10 9 8

Aut D (0 2)(1 8)(3 7)(4 5)(6 11)(9 10)

Transitivity sets: {0,2}-(2 18 2), {6,11}-(2 18 2), {1,8}-(0 18 4),
{3,7}-(0 18 4), {4,5}-(0 18 4), {9,10}-(0 18 4).

PV-T2 N30 (4 40) (23 9 1)

6 7 6 8 6 6 7 9 8 7 6 7 8 6 7

9 9 7 10 8 9 8 10 10 10 10 9 9 10 8

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2),
{2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2),
{5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2),
{8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PV-T3 N1 (4 40) (23 12 0)

6 7 6 8 6 8 6 7 7 9 6 7 6 8 7

7 8 9 10 9 10 8 9 10 10 10 8 10 9 9

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2),
{2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2),
{5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2),
{8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PV-T3 N6 (16 28) (28 10 1)

6 7 7 9 6 7 6 8 8 6 7 7 6 8 6

7 8 9 10 9 8 10 9 10 10 9 10 9 10 8

Aut D (0 11)(3 8)(4 6)

Transitivity sets: {0,11}-(4 14 4), {2}-(2 14 6), {5}-(2 14 6),
{7}-(2 14 6), {10}-(2 14 6), {1}-(0 14 8),
{3,8}-(0 14 8), {4,6}-(0 14 8), {9}-(0 14 8).

PV-T3 N8 (8 36) (29 8 0)

6 7 7 8 7 8 6 6 6 9 7 7 6 8 6

7 8 9 10 9 10 8 9 10 10 10 8 10 9 9

Aut D (0 11)(1 5)(7 10)(8 9)

Transitivity sets: {0,11}-(2 18 2), {8,9}-(2 18 2), {1,5}-(0 18 4),
{2}-(0 18 4), {3}-(0 18 4), {4}-(0 18 4),
{6}-(0 18 4), {7,10}-(0 18 4).

PV-T3 N9 (20 24) (33 10 0)

6 7 7 9 7 8 6 6 6 8 7 7 6 8 6

7 8 9 10 8 9 9 10 10 10 9 10 9 10 8

Aut D (0 11)(3 8)(4 6) (1 5)(3 6)(4 8)(7 10)
(0 11)(1 5)(3 4)(6 8)(7 10)

Transitivity sets: {0,11}-(4 12 6), {7,10}-(4 12 6), {9}-(4 12 6),
{1,5}-(0 12 10), {2}-(0 12 10), {3,4,6,8}-(0 12 10).

PV-T3 N14 (4 40) (23 9 1)

6 8 6 8 6 7 6 7 7 9 6 8 6 7 7

10 10 7 9 8 9 9 8 10 10 10 10 9 8 9

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2),
{2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2),
{5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2),
{8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PV-T3 N19 (12 32) (23 13 1)

6 8 6 7 6 8 6 7 7 9 6 8 6 7 7

10 10 8 9 7 9 9 8 10 10 10 10 9 8 9

Aut D (0 2 11)(3 8 7)(4 6 9) (0 11 2)(3 7 8)(4 9 6)

Transitivity sets: {0,2,11}-(4 16 2), {1}-(0 16 6), {3,7,8}-(0 16 6),
{5}-(0 16 6), {4,6,9}-(0 16 6), {10}-(0 16 6).

PV-T3 N20 (4 40) (25 10 0)

6 8 6 7 6 8 6 7 7 9 6 8 6 7 7

10 10 9 8 7 9 8 9 10 10 10 10 9 8 9

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2),
{2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2),
{5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2),
{8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PV-T3 N21 (8 36) (22 12 1)

6	8	6	7	6	8	6	7	7	9	6	8	6	7	7
10	10	9	8	7	9	9	8	10	10	10	10	8	9	9

Aut D (0 10)(1 11)(2 5)(3 8)(4 6)(7 9)

Transitivity sets: {0,10}-(2 18 2), {1,11}-(2 18 2), {2,5}-(0 18 4),
{3,8}-(0 18 4), {4,6}-(0 18 4), {7,9}-(0 18 4).

PV-T3 N23 (4 40) (27 8 0)

6	8	6	7	6	7	6	7	7	9	6	8	6	8	7
10	10	9	8	8	9	9	8	10	10	10	10	7	9	9

Aut D (0 11)(1 5)(6 7)(8 9)

Transitivity sets: {0,11}-(2 20 0), {1,5}-(0 20 2), {2}-(0 20 2),
{3}-(0 20 2), {4}-(0 20 2), {6,7}-(0 20 2),
{8,9}-(0 20 2), {10}-(0 20 2).

PV-T3 N28 (16 28) (30 8 1)

6	8	6	7	7	8	6	6	7	9	8	6	6	7	7
10	10	9	8	8	9	7	9	10	10	10	10	8	9	9

Aut D (3 4)(6 8)(7 9) (0 10)(1 11)(2 5)(3 6)(4 8)
(0 10)(1 11)(2 5)(3 8)(4 6)(7 9)

Transitivity sets: {7,9}-(4 14 4), {0,10}-(2 14 6), {1,11}-(2 14 6),
{2,5}-(0 14 8), {3,4,6,8}-(0 14 8).

PV-T3 N52 (8 36) (20 8 3)

8	6	6	7	6	8	6	7	7	9	8	6	6	7	7
10	10	9	8	7	9	9	8	10	10	10	10	8	9	9

Aut D (0 10)(1 11)(2 5)(3 8)(4 6)(7 9)

Transitivity sets: {0,10}-(2 18 2), {1,11}-(2 18 2), {2,5}-(0 18 4),
{3,8}-(0 18 4), {4,6}-(0 18 4), {7,9}-(0 18 4).

PV-T3 N58 (4 40) (23 9 1)

8	6	6	7	6	7	6	7	7	9	8	6	6	8	7
10	10	9	8	8	9	9	8	10	10	10	10	7	9	9

Aut D (0 11)(1 5)(6 9)(7 8)

Transitivity sets: {0,11}-(2 20 0), {1,5}-(0 20 2), {2}-(0 20 2),
{3}-(0 20 2), {4}-(0 20 2), {6,9}-(0 20 2),
{7,8}-(0 20 2), {10}-(0 20 2).

PV-T4 N14 (8 36) (24 4 3)

6	6	7	9	6	7	8	7	8	6	7	6	7	8	6
7	9	8	10	9	8	9	10	10	10	9	10	9	10	8

Aut D (0 11)(3 8)(4 6) (1 2)(3 4)(6 8)(7 9)
(0 11)(1 9)(2 7)(5 10) (1 7)(2 9)(3 6)(4 8)(5 10)
(1 9)(2 7)(3 8)(4 6)(5 10) (0 11)(1 2)(3 6)(4 8)(7 9)
(0 11)(1 7)(2 9)(3 4)(5 10)(6 8)

Transitivity sets: {0,11}-(4 18 0), {1,2,7,9}-(0 18 4), {3,4,6,8}-(0 18 4),
{5,10}-(0 18 4).

PV-T4 N30 (4 40) (26 9 0)

6	6	8	7	7	8	6	7	6	9	7	6	8	7	6
7	10	9	9	9	10	8	8	10	10	10	9	10	8	9

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2),
{2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2),
{5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2),
{8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

PV-T4 N80 (4 40) (23 9 1)

6	6	7	8	6	8	7	6	7	9	6	8	7	6	7
9	8	9	10	7	10	8	9	10	10	10	9	8	10	9

Transitivity sets: {0}-(2 20 0), {11}-(2 20 0), {1}-(0 20 2),
{2}-(0 20 2), {3}-(0 20 2), {4}-(0 20 2),
{5}-(0 20 2), {6}-(0 20 2), {7}-(0 20 2),
{8}-(0 20 2), {9}-(0 20 2), {10}-(0 20 2).

Section III

A Catalogue of the Designs Consisting Entirely of B Type Blocks

As the 3-designs of Chapter 3, Section II were all constructed using only B(1 0 9 24 9 0 1) type blocks, their embedded 2-designs must consist entirely of corresponding B(0 3 12 6 0 1) type blocks. Thus classifying these designs by the number of blocks of each block type that they contain will be waived in favour of using their q_0,q_3 and q_4 values. Using this system the representative designs retained from this earlier section are as follows:

Designs' variables q_0	q_3	q_4	# of Non-isomorphic Designs	Designs' Reference Labels (#AC #B #R) = (0 44 0).				
12	12	3	3	PI-N3	PII-N5	PIII-N30		
18	6	3	1	PIII-N29				
20	4	3	3	PI-N1	PI-N2	PIII-N27		
24	0	3	1	PI-N4				
20	10	1	5	PII-N6	PII-N8	PII-N11	PIII-N6	PIII-N16
22	8	1	3	PII-N2	PII-N4	PII-N15		
23	7	1	1	PII-N3				
24	6	1	3	PII-N1	PII-N7	PIII-N26		
26	4	1	2	PII-N12	PIII-N5			
23	10	0	3	PIV-N1	PIV-N2	PV·2-N1		
24	9	0	2	PV·2-N2	PV·2-N3			
26	7	0	1	PV·5-N2				
25	8	0	1	PV·5-N3				
27	6	0	3	PV·5-N6	PV·5-N12	PV·7-N8		
29	4	0	1	PV·7-N15				
33	0	0	1	PVII-N18				
TOTAL			34					

Each of these designs will now be presented using the following format:

NAME (#AC #B #R) (q_0 q_3 q_4)

[• • • • • •] etc.

[• • • • • •]

[• • • • • •]

[• • • • • •]

Design is specified by listing 22 blocks to be taken with their complements.

Aut D (list of designs non-trivial automorphisms)

Transitivity sets: {points in orbit}, etc.

PI-N1 (0 44 0) (20 4 3)

1 2 3 4 5 6	3 4 7 9 10 12	1 3 5 7 8 10	1 3 6 9 11 12
1 2 9 10 11 12	3 4 7 8 9 11	1 3 5 7 9 11	1 4 5 7 10 12
1 2 7 8 11 12	5 6 8 9 10 11	1 3 5 8 9 12	1 4 5 8 11 12
1 2 7 8 9 10	5 6 7 10 11 12	1 3 6 7 10 11	1 4 5 9 10 11
3 4 8 10 11 12	5 6 7 8 9 12	1 3 6 8 10 12	1 4 6 7 8 11
1 4 6 7 9 12	1 4 6 8 9 10		

Aut D (5 6)(7 12)(8 11)(9 10) (1 3)(2 4)(5 6)(7 12)(8 10)(9 11)
(1 3)(2 4)(8 9)(10 11)

Transitivity sets: {1,3}, {2,4}, {5,6}, {7,12}, {8,9,10,11}.

PI-N2 (0 44 0) (20 4 3)

1 2 3 4 5 6	3 4 7 9 10 12	1 3 5 7 8 10	1 3 6 9 10 12
1 2 9 10 11 12	3 4 7 8 9 11	1 3 5 7 9 11	1 4 5 7 10 12
1 2 7 8 11 12	5 6 8 9 10 11	1 3 5 8 9 12	1 4 5 8 10 11
1 2 7 8 9 10	5 6 7 10 11 12	1 3 6 7 10 11	1 4 5 9 11 12
3 4 8 10 11 12	5 6 7 8 9 12	1 3 6 8 11 12	1 4 6 7 8 12
1 4 6 7 9 11	1 4 6 8 9 10		

Aut D (1 2)(3 4)(5 6)(7 12)(8 11)(9 10) (1 4)(2 3)(5 6)(7 12)(8 10)(9 11)
(1 3)(2 4)(8 9)(10 11)

Transitivity sets: {5,6}, {7,12}, {1,2,3,4}, {8,9,10,11}.

PI-N3 (0 44 0) (12 12 3)

1 2 3 4 5 6	3 4 7 9 10 12	1 3 5 7 8 10	1 3 6 9 10 12
1 2 9 10 11 12	3 4 7 8 9 11	1 3 5 7 9 11	1 4 5 7 11 12
1 2 7 8 11 12	5 6 8 9 10 11	1 3 5 8 9 12	1 4 5 8 10 12
1 2 7 8 9 10	5 6 7 10 11 12	1 3 6 7 10 11	1 4 5 9 10 11
3 4 8 10 11 12	5 6 7 8 9 12	1 3 6 8 11 12	1 4 6 7 8 10
1 4 6 7 9 12	1 4 6 8 9 11		

Aut D (3 5)(4 6)(7 8)(11 12) (1 3 5)(2 4 6)(7 9 8)(10 11 12)
(1 2)(3 6)(4 5)(7 11)(8 12)(9 10) (1 4 5 2 3 6)(7 10 8 12 9 11)
(1 5)(2 6)(7 9)(10 12) (1 5 3)(2 6 4)(7 8 9)(10 12 11)
(1 6)(2 5)(3 4)(7 10)(8 11)(9 12) (1 6 3 2 5 4)(7 11 9 12 8 10)
(1 3)(2 4)(8 9)(10 11) (1 2)(3 4)(5 6)(7 12)(8 11)(9 10)
(1 4)(2 3)(5 6)(7 12)(8 10)(9 11)

Transitivity sets: {1,2,3,4,5,6}, {7,8,9,10,11,12}.

PI-N4 (0 44 0) (24 0 3)

1 2 3 4 5 6	3 4 7 9 10 12	1 3 5 7 8 10	1 3 6 9 11 12
1 2 9 10 11 12	3 4 7 8 9 11	1 3 5 7 9 11	1 4 5 7 10 11

1 2 7 8 11 12	5 6 8 9 10 11	1 3 5 8 9 12	1 4 5 8 11 12
1 2 7 8 9 10	5 6 7 10 11 12	1 3 6 7 10 12	1 4 5 9 10 12
3 4 8 10 11 12	5 6 7 8 9 12	1 3 6 8 10 11	1 4 6 7 8 12
1 4 6 7 9 11	1 4 6 8 9 10		

<u>Aut D</u> (1 2)(3 4)(5 6)(7 12)(8 11)(9 10) (1 5 3)(2 6 4)(8 9 7)(10 12 11)
(1 3 5)(2 4 6)(9 8 7)(10 11 12) (1 6 3 2 5 4)(7 11 9 12 8 10)
(1 4 5 2 3 6)(7 10 8 12 9 11)

<u>Transitivity sets:</u> {1,2,3,4,5,6}, {7,8,9,10,11,12}.

<u>PII-N1</u> (0 44 0) (24 6 1)

1 2 3 4 5 6	3 4 7 8 10 12	1 3 4 7 10 11	1 3 6 9 11 12
1 2 9 10 11 12	5 6 8 10 11 12	1 3 5 7 9 10	1 4 5 7 11 12
1 2 7 8 11 12	5 6 7 9 10 12	1 3 5 8 9 12	1 4 5 8 9 11
1 2 7 8 9 10	3 5 7 8 9 11	1 3 6 7 8 12	1 4 5 8 10 12
3 4 9 10 11 12	4 6 7 8 9 11	1 3 6 8 10 11	1 4 6 7 9 12
1 4 6 8 9 10	1 5 6 7 10 11		

<u>Aut D</u> (1 2)(3 4)(5 6)(7 8)(9 11)(10 12)

<u>Transitivity sets:</u> {1,2}, {3,4}, {5,6}, {7,8}, {9,11}, {10,12}.

<u>PII-N2</u> (0 44 0) (22 8 1)

1 2 3 4 5 6	3 4 7 8 10 12	1 3 4 7 10 11	1 3 6 8 10 12
1 2 9 10 11 12	5 6 8 10 11 12	1 3 5 7 9 10	1 4 5 7 8 12
1 2 7 8 11 12	5 6 7 9 10 12	1 3 5 8 9 12	1 4 5 8 10 11
1 2 7 8 9 10	3 5 7 8 9 11	1 3 6 7 11 12	1 4 5 9 11 12
3 4 9 10 11 12	4 6 7 8 9 11	1 3 6 8 9 11	1 4 6 7 9 12
1 4 6 8 9 10	1 5 6 7 10 11		

<u>Aut D</u> (1 2)(3 4)(5 6)(7 8)(9 11)(10 12)

<u>Transitivity sets:</u> {1,2}, {3,4}, {5,6}, {7,8}, {9,11}, {10,12}.

<u>PII-N3</u> (0 44 0) (23 7 1)

1 2 3 4 5 6	3 4 7 8 10 12	1 3 4 7 10 11	1 3 6 9 11 12
1 2 9 10 11 12	5 6 8 10 11 12	1 3 5 7 9 10	1 4 5 7 11 12
1 2 7 8 11 12	5 6 7 9 10 11	1 3 5 8 9 11	1 4 5 8 9 12
1 2 7 8 9 10	3 5 7 8 9 12	1 3 6 7 8 11	1 4 5 8 10 11
3 4 9 10 11 12	4 6 7 8 9 11	1 3 6 8 10 12	1 4 6 7 9 12
1 4 6 8 9 10	1 5 6 7 10 12		

<u>Transitivity sets:</u> {1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}, {9}, {10}, {11}, {12}.

<u>PII-N4</u> (0 44 0) (22 8 1)

1 2 3 4 5 6	3 4 7 8 10 12	1 3 4 7 10 11	1 3 6 8 10 11
1 2 9 10 11 12	5 6 8 10 11 12	1 3 5 7 9 10	1 4 5 7 8 11
1 2 7 8 11 12	5 6 7 9 10 11	1 3 5 8 9 11	1 4 5 8 10 12
1 2 7 8 9 10	3 5 7 8 9 12	1 3 6 7 11 12	1 4 5 9 11 12
3 4 9 10 11 12	4 6 7 8 9 11	1 3 6 8 9 12	1 4 6 7 9 12
1 4 6 8 9 10	1 5 6 7 10 12		

<u>Transitivity sets:</u> {1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}, {9}, {10}, {11}, {12}.

<u>PII-N5</u> (0 44 0) (12 12 3)

1 2 3 4 5 6	3 4 7 9 11 12	1 3 4 7 8 11	1 3 6 9 10 11
1 2 9 10 11 12	5 6 8 9 11 12	1 3 5 7 9 11	1 4 5 7 10 12
1 2 7 8 11 12	5 6 7 10 11 12	1 3 5 8 10 12	1 4 5 8 9 12
1 2 7 8 9 10	3 5 7 8 9 10	1 3 6 7 10 12	1 4 5 9 10 11
3 4 8 10 11 12	4 6 7 8 9 10	1 3 6 8 9 12	1 4 6 7 9 12
1 4 6 8 10 11	1 5 6 7 8 11		

<u>Aut D</u> (design is 1-transitive, see Chapter 5)

<u>Transitivity sets:</u> {1,2,3,4,5,6,7,8,9,10,11,12}.

PII-N6 (0 44 0) (20 10 1)

1 2 3 4 5 6	3 4 7 9 11 12	1 3 4 7 8 10	1 3 6 9 10 11
1 2 9 10 11 12	5 6 8 9 10 12	1 3 5 7 9 12	1 4 5 7 10 12
1 2 7 8 11 12	5 6 7 10 11 12	1 3 5 8 10 11	1 4 5 8 9 12
1 2 7 8 9 10	3 5 7 8 9 11	1 3 6 7 10 12	1 4 5 9 10 11
3 4 8 10 11 12	4 6 7 8 9 10	1 3 6 8 9 12	1 4 6 7 9 11
1 4 6 8 11 12	1 5 6 7 8 11		

Aut D (1 12)(2 11)(3 5)(4 6)(7 9)(8 10)

Transitivity sets: {1,12}, {2,11}, {3,5}, {4,6}, {7,9}, {8,10}.

PII-N7 (0 44 0) (24 6 1)

1 2 3 4 5 6	3 4 7 9 10 12	1 3 4 7 8 11	1 3 6 9 11 12
1 2 9 10 11 12	5 6 8 9 10 11	1 3 5 7 9 10	1 4 5 7 10 11
1 2 7 8 11 12	5 6 7 10 11 12	1 3 5 8 9 11	1 4 5 8 10 12
1 2 7 8 9 10	3 5 7 8 9 12	1 3 6 7 10 11	1 4 5 9 11 12
3 4 8 10 11 12	4 6 7 8 9 11	1 3 6 8 10 12	1 4 6 7 9 12
1 4 6 8 9 10	1 5 6 7 8 12		

Aut D (3 6)(4 5)(7 8)(11 12) (1 10)(2 9)(3 6)(4 5)(7 11)(8 12)
(1 10)(2 9)(7 12)(8 11)

Transitivity sets: {1,10}, {2,9}, {3,6}, {4,5}, {7,8,11,12}.

PII-N8 (0 44 0) (20 10 1)

1 2 3 4 5 6	3 4 7 9 10 12	1 3 4 7 8 11	1 3 6 9 11 12
1 2 9 10 11 12	5 6 8 9 10 11	1 3 5 7 9 11	1 4 5 7 10 12
1 2 7 8 11 12	5 6 7 10 11 12	1 3 5 8 9 10	1 4 5 8 10 11
1 2 7 8 9 10	3 5 7 8 9 12	1 3 6 7 10 11	1 4 5 9 11 12
3 4 8 10 11 12	4 6 7 8 9 11	1 3 6 8 10 12	1 4 6 7 9 10
1 4 6 8 9 12	1 5 6 7 8 12		

Aut D (3 6)(4 5)(7 8)(11 12)

Transitivity sets: {1}, {2}, {9}, {10}, {3,6}, {4,5}, {7,8}, {11,12}.

PII-N11 (0 44 0) (20 10 1)

1 2 3 4 5 6	3 4 7 9 10 12	1 3 4 7 9 11	1 3 6 9 11 12
1 2 9 10 11 12	5 6 8 9 11 12	1 3 5 7 8 9	1 4 5 7 11 12
1 2 7 8 11 12	5 6 7 9 10 11	1 3 5 9 10 12	1 4 5 8 9 11
1 2 7 8 9 10	3 5 7 8 10 11	1 3 6 7 8 12	1 4 5 8 10 12
3 4 8 10 11 12	4 6 7 8 9 12	1 3 6 8 10 11	1 4 6 7 10 11
1 4 6 8 9 10	1 5 6 7 10 12		

Aut D (1 2)(3 4)(5 6)(7 8)(9 11)(10 12)

Transitivity sets: {1,2}, {3,4}, {5,6}, {7,8}, {9,11}, {10,12}.

PII-N12 (0 44 0) (26 4 1)

1 2 3 4 5 6	3 4 7 9 10 12	1 3 4 7 9 11	1 3 6 8 10 12
1 2 9 10 11 12	5 6 8 9 11 12	1 3 5 7 8 9	1 4 5 7 8 12
1 2 7 8 11 12	5 6 7 9 10 11	1 3 5 9 10 12	1 4 5 8 10 11
1 2 7 8 9 10	3 5 7 8 10 11	1 3 6 7 11 12	1 4 5 9 11 12
3 4 8 10 11 12	4 6 7 8 9 12	1 3 6 8 9 11	1 4 6 7 10 11
1 4 6 8 9 10	1 5 6 7 10 12		

Aut D (1 2)(3 4)(5 6)(7 8)(9 11)(10 12)

Transitivity sets: {1,2}, {3,4}, {5,6}, {7,8}, {9,11}, {10,12}.

PII-N15 (0 44 0) (22 8 1)

1 2 3 4 5 6	3 4 7 9 10 12	1 3 4 7 9 11	1 3 6 9 11 12
1 2 9 10 11 12	5 6 8 9 11 12	1 3 5 7 9 12	1 4 5 7 8 12
1 2 7 8 11 12	5 6 7 9 10 11	1 3 5 8 9 10	1 4 5 8 9 11
1 2 7 8 9 10	3 5 7 8 10 11	1 3 6 7 8 11	1 4 5 10 11 12
3 4 8 10 11 12	4 6 7 8 9 12	1 3 6 8 10 12	1 4 6 7 10 11
1 4 6 8 9 10	1 5 6 7 10 12		

Aut D (1 2)(3 5)(4 6)(7 8)(9 12)(10 11)

Transitivity sets: {1,2}, {3,5}, {4,6}, {7,8}, {9,12}, {10,11}.

PIII-N5 (0 44 0) (26 4 1)

1 2 3 4 5 6	3 5 8 10 11 12	1 3 4 7 8 10	1 3 6 8 9 11
1 2 9 10 11 12	3 6 7 8 10 12	1 3 4 7 9 12	1 4 5 7 10 11
1 2 7 8 11 12	4 5 7 8 9 12	1 3 5 7 9 11	1 4 5 8 11 12
1 2 7 8 9 10	4 6 7 8 9 11	1 3 5 8 9 10	1 4 6 8 10 11
3 4 9 10 11 12	5 6 7 9 10 11	1 3 6 7 11 12	1 4 6 9 10 12
1 5 6 7 10 12	1 5 6 8 9 12		

Aut D (1 11)(2 12)(3 6)(4 5)(7 10)(8 9)

Transitivity sets: {1,11}, {2,12}, {3,6}, {4,5}, {7,10}, {8,9}.

PIII-N6 (0 44 0) (20 10 1)

1 2 3 4 5 6	3 5 8 10 11 12	1 3 4 7 8 10	1 3 6 8 11 12
1 2 9 10 11 12	3 6 7 8 9 12	1 3 4 7 9 12	1 4 5 7 11 12
1 2 7 8 11 12	4 5 7 8 10 12	1 3 5 7 9 11	1 4 5 8 9 11
1 2 7 8 9 10	4 6 7 8 9 11	1 3 5 8 9 10	1 4 6 8 10 11
3 4 9 10 11 12	5 6 7 9 10 11	1 3 6 7 10 11	1 4 6 9 10 12
1 5 6 7 10 12	1 5 6 8 9 12		

Aut D (1 11)(2 12)(3 6)(4 5)(7 10)(8 9)

Transitivity sets: {1,11}, {2,12}, {3,6}, {4,5}, {7,10}, {8,9}.

PIII-N16 (0 44 0) (20 10 1)

1 2 3 4 5 6	3 5 8 10 11 12	1 3 4 7 8 12	1 3 6 8 10 11
1 2 9 10 11 12	3 6 7 8 9 12	1 3 4 7 9 11	1 4 5 7 11 12
1 2 7 8 11 12	4 5 7 8 9 10	1 3 5 7 9 10	1 4 5 8 10 12
1 2 7 8 9 10	4 6 7 8 10 11	1 3 5 8 9 11	1 4 6 8 9 11
3 4 9 10 11 12	5 6 7 9 11 12	1 3 6 7 10 12	1 4 6 9 10 12
1 5 6 7 10 11	1 5 6 8 9 12		

Aut D (1 9)(2 10)(3 5)(4 6)(7 12)(8 11)

Transitivity sets: {1,9}, {2,10}, {3,5}, {4,6}, {7,12}, {8,11}.

PIII-N26 (0 44 0) (24 6 1)

1 2 3 4 5 6	3 5 8 9 10 12	1 3 4 7 8 12	1 3 6 8 9 12
1 2 9 10 11 12	3 6 7 8 10 11	1 3 4 7 9 11	1 4 5 7 10 12
1 2 7 8 11 12	4 5 7 8 9 11	1 3 5 7 10 11	1 4 5 8 9 10
1 2 7 8 9 10	4 6 7 9 10 12	1 3 5 9 11 12	1 4 6 8 9 11
3 4 8 10 11 12	5 6 7 9 11 12	1 3 6 7 9 10	1 4 6 10 11 12
1 5 6 7 8 12	1 5 6 8 10 11		

Aut D (1 11)(2 12)(3 4)(5 6)(7 9)(8 10)

Transitivity sets: {1,11}, {2,12}, {3,4}, {5,6}, {7,9}, {8,10}.

PIII-N27 (0 40 0) (20 4 3)

1 2 3 4 5 6	3 5 8 9 11 12	1 3 4 7 8 11	1 3 6 8 10 12
1 2 9 10 11 12	3 6 7 8 9 10	1 3 4 7 9 12	1 4 5 7 10 12
1 2 7 8 11 12	4 5 7 8 9 10	1 3 5 7 9 11	1 4 5 8 10 11
1 2 7 8 9 10	4 6 7 9 11 12	1 3 5 9 10 12	1 4 6 8 9 12
3 4 8 10 11 12	5 6 7 10 11 12	1 3 6 7 10 11	1 4 6 9 10 11
1 5 6 7 8 12	1 5 6 8 9 11		

Aut D (3 6)(4 5)(7 8)(11 12) (1 11 2 12)(3 8)(4 9 5 10)(6 7)
(1 2)(4 5)(9 10)(11 12) (1 12)(2 11)(3 7)(4 9)(5 10)(6 8)
(1 2)(3 6)(7 8)(9 10) (1 12 2 11)(3 8)(4 10 5 9)(6 7)
(1 11)(2 12)(3 7)(4 10)(5 9)(6 8)

Transitivity sets: {1,2,11,12}, {3,6,7,8}, {4,5,9,10}.

PIII-N29 (0 44 0) (18 6 3)

1 2 3 4 5 6	3 5 8 9 11 12	1 3 4 7 8 11	1 3 6 8 10 12
1 2 9 10 11 12	3 6 7 8 9 10	1 3 4 7 9 12	1 4 5 7 10 11
1 2 7 8 11 12	4 5 7 8 9 10	1 3 5 7 10 12	1 4 5 8 9 12
1 2 7 8 9 10	4 6 7 9 11 12	1 3 5 9 10 11	1 4 6 8 10 11
3 4 8 10 11 12	5 6 7 10 11 12	1 3 6 7 9 11	1 4 6 9 10 12
1 5 6 7 8 12	1 5 6 8 9 11		

Aut D (1 2)(3 8)(4 9)(5 10)(6 7)(11 12) (1 9 5)(2 10 4)(3 12 7)(6 11 8)
(1 4)(2 5)(3 11)(6 12)(7 8)(9 10) (1 10)(2 9)(3 6)(4 5)(7 11)(8 12)
(1 5 9)(2 4 10)(3 7 12)(6 8 11)

Transitivity sets: {1,2,4,5,9,10}, {3,6,7,8,11,12}.

PIII-N30 (0 44 0) (12 12 3)

1 2 3 4 5 6	3 5 8 9 11 12	1 3 4 7 8 11	1 3 6 8 9 12
1 2 9 10 11 12	3 6 7 8 9 10	1 3 4 7 10 12	1 4 5 7 9 12
1 2 7 8 11 12	4 5 7 8 9 10	1 3 5 7 10 11	1 4 5 8 9 11
1 2 7 8 9 10	4 6 7 9 11 12	1 3 5 9 10 12	1 4 6 8 10 12
3 4 8 10 11 12	5 6 7 10 11 12	1 3 6 7 9 11	1 4 6 9 10 11
1 5 6 7 8 12	1 5 6 8 10 11		

Aut D (design is 1-transitive, see Chapter 5)

Transitivity sets: {1,2,3,4,5,6,7,8,9,10,11,12}.

PIV-N1 (0 44 0) (23 10 0)

1 2 3 4 5 6	5 6 8 9 10 12	1 2 4 7 9 12	1 3 6 8 11 12
1 2 9 10 11 12	5 6 7 9 11 12	1 2 6 8 9 10	1 4 5 7 8 10
1 2 7 8 11 12	1 3 7 8 9 10	1 3 5 7 9 11	1 4 5 8 9 11
3 4 8 10 11 12	2 5 7 8 10 11	1 3 5 8 9 12	1 4 5 10 11 12
3 4 7 9 10 12	4 6 7 8 9 11	1 3 6 7 10 11	1 4 6 7 8 12
1 4 6 9 10 11	1 5 6 7 10 12		

Transitivity sets: {1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}, {9}, {10}, {11}, {12}.

PIV-N2 (0 44 0) (23 10 0)

1 2 3 4 5 6	5 6 8 9 10 11	1 2 4 7 8 10	1 3 6 8 11 12
1 2 9 10 11 12	5 6 7 10 11 12	1 2 6 8 9 10	1 4 5 7 9 11
1 2 7 8 11 12	1 3 7 9 10 11	1 3 5 7 8 9	1 4 5 8 11 12
3 4 8 9 10 12	2 5 7 8 9 12	1 3 5 8 10 12	1 4 5 9 10 12
3 4 7 8 10 11	4 6 7 8 9 12	1 3 6 7 9 12	1 4 6 7 10 12
1 4 6 8 9 11	1 5 6 7 8 10		

Aut D (1 3)(2 4)(5 6)(7 9)(8 12)(10 11)

Transitivity sets: {1,3}, {2,4}, {5,6}, {7,9}, {8,12}, {10,11}.

PV·2-N1 (0 44 0) (23 10 0)

1 2 3 4 5 6	1 5 8 9 10 12	1 2 3 7 8 9	1 3 6 8 9 12
1 2 9 10 11 12	2 6 7 8 10 11	1 2 4 7 9 10	1 3 6 8 10 11
1 2 7 8 11 12	3 5 7 8 9 11	1 3 4 7 9 11	1 4 5 7 8 12
3 4 9 10 11 12	4 6 7 8 9 10	1 3 5 7 10 11	1 4 5 8 10 11
3 4 7 8 10 12	5 6 7 9 11 12	1 3 6 7 10 12	1 4 6 7 11 12
1 4 6 8 9 11	1 5 6 7 9 10		

Aut D (1 2)(3 9)(4 10)(5 11)(6 12)(7 8)

Transitivity sets: {1,2}, {3,9}, {4,10}, {5,11}, {6,12}, {7,8}.

PV·2-N2 (0 44 0) (24 9 0)

1 2 3 4 5 6	1 5 8 9 10 12	1 2 3 7 8 9	1 3 6 7 10 11
1 2 9 10 11 12	2 6 7 8 9 10	1 2 4 7 10 11	1 3 6 8 11 12
1 2 7 8 11 12	3 5 7 8 10 11	1 3 4 8 9 11	1 4 5 7 8 12
3 4 9 10 11 12	4 6 7 8 9 11	1 3 5 7 9 10	1 4 5 7 9 11
3 4 7 8 10 12	5 6 7 9 11 12	1 3 6 7 9 12	1 4 6 7 10 12
1 4 6 8 9 10	1 5 6 8 10 11		

Transitivity sets: {1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}, {9}, {10}, {11}, {12}.

PV·2-N3 (0 44 0) (24 9 0)

1 2 3 4 5 6	1 5 8 9 10 11	1 2 3 7 8 10	1 3 6 7 10 11
1 2 9 10 11 12	2 6 7 8 9 12	1 2 4 7 9 10	1 3 6 8 9 10
1 2 7 8 11 12	3 5 7 8 9 11	1 3 4 8 9 12	1 4 5 7 8 9

3 4 9 10 11 12	4 6 7 8 10 11	1 3 5 7 11 12	1 4 5 7 10 12
3 4 7 8 10 12	5 6 7 9 10 12	1 3 6 7 9 12	1 4 6 7 9 11
1 4 6 8 11 12	1 5 6 8 10 12		

Transitivity sets: {1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}, {9}, {10}, {11}, {12}.

PV·5-N2 (0 44 0) (26 7 0)

1 2 3 4 5 6	1 5 8 9 10 12	1 2 3 7 8 9	1 3 6 8 9 12
1 2 9 10 11 12	2 6 7 9 10 11	1 2 4 7 8 10	1 3 6 9 10 11
1 2 7 8 11 12	3 5 7 8 10 11	1 3 4 7 9 11	1 4 5 7 9 10
3 4 8 10 11 12	4 6 7 8 9 12	1 3 5 7 11 12	1 4 5 8 9 11
3 4 7 9 10 12	5 6 7 8 9 11	1 3 6 7 8 10	1 4 6 7 11 12
1 4 6 8 10 11	1 5 6 7 10 12		

Transitivity sets: {1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}, {9}, {10}, {11}, {12}.

PV·5-N3 (0 44 0) (25 8 0)

1 2 3 4 5 6	1 5 8 9 10 12	1 2 3 7 8 9	1 3 6 8 9 12
1 2 9 10 11 12	2 6 7 9 10 11	1 2 4 7 8 10	1 3 6 9 10 11
1 2 7 8 11 12	3 5 7 8 9 11	1 3 4 7 11 12	1 4 5 7 9 10
3 4 8 10 11 12	4 6 7 8 9 11	1 3 5 7 10 11	1 4 5 8 9 11
3 4 7 9 10 12	5 6 7 8 10 12	1 3 6 7 8 10	1 4 6 7 9 12
1 4 6 8 10 11	1 5 6 7 11 12		

Transitivity sets: {1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}, {9}, {10}, {11}, {12}.

PV·5-N6 (0 44 0) (27 6 0)

1 2 3 4 5 6	1 5 8 9 10 12	1 2 3 7 10 11	1 3 6 8 9 10
1 2 9 10 11 12	2 6 7 8 9 10	1 2 4 7 8 9	1 3 6 8 11 12
1 2 7 8 11 12	3 5 7 8 9 11	1 3 4 7 8 10	1 4 5 7 10 11
3 4 8 10 11 12	4 6 7 9 11 12	1 3 5 7 9 12	1 4 5 8 9 11
3 4 7 9 10 12	5 6 7 8 10 11	1 3 6 7 9 11	1 4 6 7 8 12
1 4 6 9 10 11	1 5 6 7 10 12		

Transitivity sets: {1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}, {9}, {10}, {11}, {12}.

PV·5-N12 (0 44 0) (27 6 0)

1 2 3 4 5 6	1 5 8 9 10 11	1 2 3 7 10 12	1 3 6 7 9 11
1 2 9 10 11 12	2 6 7 8 9 10	1 2 4 7 8 9	1 3 6 8 9 12
1 2 7 8 11 12	3 5 7 8 9 12	1 3 4 8 9 11	1 4 5 7 8 12
3 4 8 10 11 12	4 6 7 8 10 11	1 3 5 7 10 11	1 4 5 7 9 10
3 4 7 9 10 12	5 6 7 9 11 12	1 3 6 7 8 10	1 4 6 7 11 12
1 4 6 9 10 12	1 5 6 8 10 12		

Transitivity sets: {1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}, {9}, {10}, {11}, {12}.

PV·7-N8 (0 44 0) (27 6 0)

1 2 3 4 5 6	1 5 8 9 10 11	1 2 3 7 8 10	1 3 6 8 9 11
1 2 9 10 11 12	2 6 7 8 9 12	1 2 4 7 9 10	1 3 6 10 11 12
1 2 7 8 11 12	3 5 7 9 11 12	1 3 4 7 9 12	1 4 5 7 9 11
3 4 8 9 10 12	4 6 7 10 11 12	1 3 5 7 8 12	1 4 5 8 10 12
3 4 7 8 10 11	5 6 7 8 9 10	1 3 6 7 9 10	1 4 6 7 8 11
1 4 6 8 9 12	1 5 6 7 10 12		

Aut D (2 4)(3 10)(5 8)(7 11)(9 12)

Transitivity sets: {1}, {6}, {2,4}, {3,10}, {5,8}, {7,11}, {9,12}.

PV·7-N15 (0 44 0) (29 4 0)

1 2 3 4 5 6	1 5 8 9 10 11	1 2 3 7 8 9	1 3 6 8 9 11
1 2 9 10 11 12	2 6 7 8 10 12	1 2 4 7 9 10	1 3 6 10 11 12
1 2 7 8 11 12	3 5 7 9 10 12	1 3 4 7 11 12	1 4 5 7 10 11

3 4 8 9 10 12	4 6 7 8 9 11	1 3 5 7 8 12	1 4 5 8 9 12
3 4 7 8 10 11	5 6 7 9 11 12	1 3 6 7 9 10	1 4 6 7 9 12
1 4 6 8 10 12	1 5 6 7 8 10		

<u>Aut D</u> (1 2)(3 4)(5 6)(7 9)(8 10)(11 12)

<u>Transitivity sets</u>: {1,2}, {3,4}, {5,6}, {7,9}, {8,10}, {11,12}.

<u>PVII-N18</u> (0 44 0) (33 0 0)

1 2 3 4 5 6	2 6 7 8 11 12	1 2 3 7 8 9	1 3 5 7 10 11
1 2 9 10 11 12	3 5 7 8 9 11	1 2 4 7 8 10	1 3 6 8 9 10
1 3 8 10 11 12	3 6 7 9 10 12	1 2 5 7 9 10	1 4 5 8 9 12
1 4 7 9 11 12	4 5 8 9 10 11	1 2 6 8 9 11	1 4 6 7 10 11
2 5 7 8 10 12	4 6 7 8 9 10	1 3 4 7 8 12	1 5 6 7 8 11
1 5 6 7 9 12	1 5 6 8 10 12		

<u>Aut D</u> (restriction on 5 is transitive, see Chapter 5)

(5)(2 9 3 10 8 4 6 11 7 12 1) (5)(1 8 12 10 7 3 11 9 6 2 4)
(5)(7 2 10 6 12 9 8 11 1 3 4) (5)(8 6 7 1 9 10 4 11 12 2 3)
(5)(7 10 12 8 1 4 2 6 9 11 3) (5)(8 7 9 4 12 3 6 1 10 11 2)
(5)(11 10 1 6 3 12 4 9 7 8 2) (5)(3 1 11 8 9 12 6 10 2 7 4)
(5)(10 9 1 7 6 8 3 2 12 11 4) (5)(9 2 1 12 7 11 6 4 8 10 3)

<u>Transitivity sets</u>: {5}, {1,2,3,4,6,7,8,9,10,11,12}.

SECTION IV

A Catalogue of the Designs Containing Repeated Blocks

This section presents a tabulation of the non-isomorphic designs developed in Chapter 4, followed by a key and information to provide a representative copy of each design. The first table lists the 656 non-isomorphic 2-(11,5,4) designs with repeated blocks. These designs are identified with their computer assigned name and have been categorized according to the number of blocks of each block type that they contain. A second table lists the 119 non-isomorphic 3-(12,6,4) designs with repeated blocks, and uses a similar classification system. These 3-designs are identified by the reference label assigned to each (to correspond with the order of its discovery) during the extension/restriction process.

The Non-isomorphic 2-(11,5,4) Designs With Repeated Blocks

Block Type (#A	#B	#C	#R)	Number of Designs	Designs computer assigned reference label
0	0	0	22	1	D1 1.
12	0	0	10	1	D1 2.
0	0	12	10	1	D1 3.
4	0	12	6	1	D1 4.
4	8	4	6	2	D1 5, D1 6.
0	8	8	6	4	D1 7, D2 20, D2 21, D3 27.
0	16	0	6	3	D1 8, D1 9, D4 232.
0	0	18	4	4	D2 5, D2 6, D3 18, D3 23.
0	8	10	4	8	D2 7, D2 23, D2 27, D2 29, D3 19, D3 38, D3 45, D3 51.
0	12	6	4	13	D2 8, D2 22, D2 24, D2 30, D3 22, D3 39, D3 42, D3 53, D4 179, D4 190, D4 212, D4 306, D4 311.
2	12	4	4	3	D2 11, D2 18, D3 31.
2	8	8	4	3	D2 12, D2 13, D3 29.
4	0	14	4	2	D2 14, D3 30.
4	8	6	4	4	D2 17, D3 33, D4 295, D4 297.
0	0	16	6	1	D2 19.
0	14	4	4	7	D2 25, D2 26, D2 31, D3 40, D3 47, D3 48, D3 50.
2	14	2	4	4	D2 32, D3 49, D4 191, D4 216.
4	12	2	4	2	D4 182, D4 189.

0	18	0	4	1	D4 205.
12	0	6	4	1	D4 301.
8	4	8	2	1	D5 2.
2	2	16	2	1	D5 5.
2	10	8	2	3	D5 6, D5 8, D21 45.
4	0	16	2	3	D5 13, D6 54, D6 86.
4	8	8	2	8	D5 14, D6 32, D7 68, D11 256, D20 13, D20 14, D20 15, D20 16.
4	12	4	2	13	D5 18, D5 19, D6 2, D6 3, D6 6, D6 8, D7 40, D10 19, D10 55, D10 90, D10 214, D23 203, D23 204.
0	10	10	2	12	D5 22, D5 23, D5 24, D5 25, D7 24, D7 27, D7 29, D7 36, D10 118, D10 119, D10 149, D11 131.
0	16	4	2	88	D5 26, D5 27, D5 28, D5 29, D5 30, D5 31, D5 32, D5 33, D6 26, D6 64, D6 81, D6 82, D7 30, D7 35, D9 26, D9 59, D9 81, D9 82, D10 4, D10 5, D10 12, D10 17, D10 20, D10 24, D10 30, D10 38, D10 52, D10 53, D10 63, D10 64, D10 65, D10 66, D10 69, D10 73, D10 76, D10 82, D10 92, D10 95, D10 120, D10 122, D10 126, D10 127, D10 128, D10 130, D10 134, D10 136, D10 140, D10 142, D10 185, D10 192, D10 193, D10 203, D10 204, D10 215, D10 223, D10 244, D10 248, D10 256, D10 269, D10 273, D10 289, D10 323, D10 327, D10 331, D11 114, D11 121, D20 21, D20 22, D20 23, D20 24, D20 25, D20 26, D21 62, D21 64, D22 60, D23 46, D23 47, D23 48, D23 49, D23 130, D23 131, D23 132, D23 142, D23 143, D23 217, D23 218, D23 219, D23 220.
0	4	16	2	3	D5 38, D10 25, D10 96.
0	0	20	2	4	D5 39, D6 84, D7 62, D11 260.
0	8	12	2	13	D5 40, D6 19, D6 61, D6 83, D7 63, D10 1, D10 2, D10 71, D10 225, D10 226, D10 230, D10 233, D11 235.
0	12	8	2	41	D5 41, D5 42, D6 21, D6 28, D6 65, D6 79, D6 80, D6 87, D6 89, D7 31, D7 64, D7 65, D9 21, D9 27, D9 60, D9 79, D9 80, D10 11, D10 14, D10 18, D10 23, D10 26, D10 29, D10 44, D10 45, D10 62, D10 80, D10 85, D10 87, D10 91, D10 186, D10 201, D10 202, D10 220, D11 46, D11 112, D11 118, D11 202, D11 236, D11 279, D23 40.
2	12	6	2	20	D6 9, D6 10, D6 62, D7 34, D7 41, D9 55, D10 42, D10 57, D10 123, D10 137, D10 208, D10 210, D10 212, D10 322, D11 69, D11 115, D11 122, D11 173, D11 192, D11 230.
2	16	2	2	19	D6 15, D6 16, D6 20, D6 63, D9 15, D9 16, D10 40, D10 54, D10 56, D10 138, D10 218, D11 25, D11 217, D21 67, D22 62, D23 134, D23 135, D23 144, D23 145.
0	18	2	2	50	D6 22, D6 23, D6 24, D6 25, D9 22, D9 23, D9 24, D9 25, D10 33, D10 34, D10 50, D10 51, D10 60, D10 61, D10 67, D10 116, D10 117, D10 124, D10 131, D10 143, D10 144, D10 145, D10 187, D10 188, D10 194, D10 219, D10 221, D10 226, D10 267, D10 345, D21 54, D21 55, D21 56, D21 57, D21 58, D21 59, D22 54, D22 55, D22 56, D22 57, D22 58, D22 59, D23 182, D23 183, D23 184, D23 185, D23 209, D23 210, D23 211, D23 212.
2	8	10	2	7	D6 27, D6 37, D6 46, D6 53, D6 85, D7 18, D9 86.
8	0	12	2	1	D6 31.
2	0	18	2	1	D6 36.

6	8	6	2	1	D6 45.
10	0	10	2	1	D6 50.
0	2	18	2	3	D7 23, D7 28, D10 152.
4	4	12	2	2	D10 13, D10 83.
0	14	6	2	6	D10 28, D10 181, D10 228, D10 235, D11 389, D11 395.
4	10	6	2	4	D10 305, D10 308, D21 41, D21 42.
4	2	14	2	1	D10 311.
8	10	2	2	1	D11 44.
12	2	6	2	1	D11 323.
2	14	4	2	2	D21 40, D21 44.
6	2	12	2	1	D21 43.
4	16	0	2	8	D25 22, D25 23, D26 29, 29C 2, 29E 328, 29E 329, 30B 451, 30B 523.
0	20	0	2	261	D25 29, D25 30, D25 32, D25 34, D25 35, D25 36, D25 38, D25 40, D26 39, D26 40, D26 41, D26 42, D26 43, D26 44, D26 51, D26 55, D26 59, D26 60, D26 61, D26 62, D26 63, D26 64, D26 96, D26 97, D26 98, D26 99, D26 100, D26 101, D26 103, D26 104, D26 105, D26 106, D26 107, D26 108, D26 109, D26 110, D26 111, D26 112, D26 113, D26 114, D26 115, D26 116, D26 117, D26 118, D26 119, D26 120, D28 19, D28 20, D28 24, D28 25, D28 26, D28 27, D28 64, D28 65, D28 66, D28 67, D28 69, D28 71, D28 72, D28 81, D28 82, D28 83, D28 84, D28 120, D28 127, D28 128, D28 129, D28 130, D28 150, D28 151, D28 157, D28 158, D28 177, D28 178, D28 179, D28 180, D28 181, D28 182, 29D 72, 29D 75, 29E 351, 29E 352, 30A 2, 30A 3, 30A 4, 30A 5, 30B 1, 30B 2, 30B 3, 30B 4, 30B 5, 30B 6, 30B 17, 30B 18, 30B 19, 30B 20, 30B 21, 30B 22, 30B 23, 30B 24, 30B 25, 30B 26, 30B 28, 30B 30, 30B 31, 30B 33, 30B 35, 30B 40, 30B 41, 30B 48, 30B 64, 30B 65, 30B 67, 30B 69, 30B 71, 30B 72, 30B 73, 30B 74, 30B 75, 30B 78, 30B 79, 30B 80, 30B 81, 30B 86, 30B 88, 30B 104, 30B 105, 30B 106, 30B 107, 30B 108, 30B 109, 30B 125, 30B 127, 30B 132, 30B 133, 30B 134, 30B 136, 30B 137, 30B 144, 30B 145, 30B 148, 30B 155, 30B 156, 30B 157, 30B 158, 30B 161, 30B 163, 30B 165, 30B 166, 30B 167, 30B 168, 30B 170, 30B 181, 30B 183, 30B 188, 30B 189, 30B 191, 30B 201, 30B 205, 30B 219, 30B 222, 30B 224, 30B 227, 30B 230, 30B 231, 30B 255, 30B 266, 30B 268, 30B 272, 30B 274, 30B 279, 30B 281, 30B 321, 30B 322, 30B 323, 30B 325, 30B 330, 30B 331, 30B 333, 30B 337, 30B 338, 30B 340, 30B 346, 30B 347, 30B 354, 30B 355, 30B 362, 30B 398, 30B 408, 30B 409, 30B 422, 30B 468, 30B 475, 30B 488, 30B 564, 30C 80, 30C 86, 30F 110, 30F 111, 30F 130, 30F 142, 38A 46, 38A 47, 38A 49, 38A 50, 38A 51, 38A 52, 38A 53, 38A 57, 38A 59, 38A 60, 38A 61, 38A 69, 38A 71, 38A 73, 38A 74, 38A 76, 38A 79, 38A 147, 38A 148, 38B 11, 38B 13, 38B 16, 38B 17, 38B 19, 38B 20, 38B 21, 38B 22, 38B 23, 38B 25, 38B 27, 38B 28, 38B 29, 38B 30, 38B 31, 38B 36, 38B 39, 38B 46, 38B 47, 38B 48, 38B 49, 38B 50, 38B 57, 38B 64, 38B 65, 38B 70,

2 18 0 2	10	38B 82, 38B 123, 38B 125, 38B 134, 38B 137, 38B 138, 38B 141, 38B 142, 38B 146, 38B 152, 38B 174, 38B 180, 38B 190, 38B 248, 38C 126. D26 87, D26 88, D26 89, D26 90, D26 91, D26 92, D28 78, D28 79, D28 125, D28 126.

The Non-isomorphic 3-(12,6,4) Designs

With Repeated Blocks

Block Type #AC	#B	#R	Number of Designs	Design's Reference Code
0	0	44	1	E1.
24	0	20	1	E2.
32	0	12	1	E3.
16	16	12	1	E4.
0	32	12	1	E5.
36	0	8	2	E6, E7.
20	16	8	2	E8, E11.
12	24	8	3	E9, E10, E12.
8	28	8	2	E13, E14.
0	36	8	1	E15.
40	0	4	3	E19, E33, E35.
36	4	4	2	E17, E36.
32	8	4	3	E16, E46, E57.
24	16	4	6	E20, E29, E34, E41, E42, E55.
20	20	4	2	E18, E37.
16	24	4	10	E21, E22, E26, E39, E40, E44, E47, E58, E61, E62.
12	28	4	2	E48, E60.
8	32	4	14	E23, E24, E25, E27, E28, E30, E38, E43, E45, E50, E51, E53, E54, E56.
4	36	4	5	E31, E32, E49, E52, E59.
0	40	4	57	DESIGNS E63 TO E119.

A condensed version of each of these 3-designs will now be presented. As well as listing all of the 3-designs, the previously developed non-isomorphic 2-(11,5,4) designs corresponding to their point restrictions will also be given. Each design either is accompanied by a list of all its non-trivial automorphisms, or a reference to an earlier part of the text where they have been discussed.

The 3-designs' self-complementary nature and the regularity of the skeleton for an R type block resulted in a reasonably compact means for specifying these designs. Firstly each of the twenty triples from {6,7,8,9,10,11} was assigned a reference value, namely;

Triple	6 7 8	6 7 9	6 7 10	6 7 11	6 8 9	6 8 10	6 8 11
Value	1	2	3	4	5	6	7

Triple	6 9 10	6 9 11	6 10 11	7 8 9	7 8 10	7 8 11	7 9 10
Value	8	9	10	11	12	13	14

Triple	7 9 11	7 10 11	8 9 10	8 9 11	8 10 11	9 10 11
Value	15	16	17	18	19	20

Now each 3-design was created by the extension of a 2-design, which in turn was constructed by the positioning of these triples with a fixed set of pairs from {1,2,3,4,5}. Thus an ordered list of the completing triples or their reference values can be used to specify the 2-design, and hence the 3-design via complementation.

The 3-designs will therefore be presented using the following format;

NAME (#AC #B #R) (q_0 q_3 q_4)

(twenty ordered reference values for the constituent triples)

Aut D A list of all automorphisms.

Transitivity sets: {points in orbit} - the originally created 2-(11,5,4) design to which restrictions on these points are isomorphic, ... etc.

This information is to be used in conjunction with the following key which demonstrates how design E1 is obtained.

KEY FOR THE PRODUCTION OF 3-(12,6,4) DESIGNS WITH REPEATED BLOCKS

[1 2	3 4 5		12]	[	]
[1 2	3 4 5		12]	[	]
[1 2	6 7 8	≡ (1)	12]	[	]
[1 2	6 7 8	≡ (1)	12]	[	]
[1 3	6 9 10	≡ (8)	12]	[	]
[1 3	6 9 10	≡ (8)	12]	[	]
[1 4	7 9 11	≡ (15)	12]	[	]
[1 4	7 9 11	≡ (15)	12]	[	]
[1 5	8 10 11	≡ (19)	12]	[	]
[1 5	8 10 11	≡ (19)	12]	[Complementary	]
[2 3	7 10 11	≡ (16)	12]	[Blocks	]
[2 3	7 10 11	≡ (16)	12]	[	]
[2 4	8 9 10	≡ (17)	12]	[	]
[2 4	8 9 10	≡ (17)	12]	[	]
[2 5	6 9 11	≡ (9)	12]	[	]
[2 5	6 9 11	≡ (9)	12]	[	]
[3 4	6 8 11	≡ (7)	12]	[	]
[3 4	6 8 11	≡ (7)	12]	[	]
[3 5	7 8 9	≡ (11)	12]	[	]
[3 5	7 8 9	≡ (11)	12]	[	]
[4 5	6 7 10	≡ (3)	12]	[	]
[4 5	6 7 10	≡ (3)	12]	[	]

E1 (0 0 44) (165 0 0)
(1 1 8 8 15 15 19 19 16 16 17 17 9 9 7 7 11 11 3 3)
Aut D (design is 3-transitive, see Chapter 5)
Transitivity sets: {1,2,3,4,5,6,7,8,9,10,11,12} - D1 1.

E2 (24 0 20) (105 0 0)
(1 1 8 8 15 15 19 19 16 18 10 17 9 14 7 12 4 11 3 5)
Aut D (see reducible design no 22)
Transitivity sets: {1,12} - D1 2, {2,3,4,5,6,7,8,9,10 11} - D1 3.

E3 (32 0 12) (85 0 0)
(1 1 8 8 15 16 18 19 15 18 10 17 10 14 7 12 4 12 2 5)
Aut D (see reducible design no 14)
Transitivity sets: {2,3,6,11} - D2 19, {1,4,5,7,8,9,10,12} - D1 4.

E4 (16 16 12) (73 4 0)
(1 1 8 8 15 16 18 19 15 19 9 17 10 14 7 12 4 11 3 5)
Aut D (4 5)(7 8)(9 10)
(3 6)(4 7)(5 8)
(3 6)(4 8)(5 7)(9 10)
(1 12)(9 10)
(1 12)(4 5)(7 8)
(1 12)(3 6)(4 7)(5 8)(9 10)
(1 12)(3 6)(4 8)(5 7)
Transitivity sets: {2} - D2 20, {11} - D2 21, {1,12} - D1 5,
{3,6} - D3 27, {9,10} - D1 6, {4,5,7,8} - D1 7.

E5 (0 32 12) (62 4 1)
(1 1 8 8 15 19 16 18 15 19 9 14 10 17 7 12 4 11 3 5)
Aut D (3 6)(4 7)(5 8)
(2 3)(7 10)(8 9)
(2 3 6)(4 10 7)(5 9 8)
(2 6 3)(4 7 10)(5 8 9)
(2 6)(4 10)(5 9)
(1 4)(5 12)(6 11)
(1 4 7)(3 11 6)(5 8 12)
(1 4)(2 3)(5 12)(6 11)(7 10)(8 9)
(1 4 10 7)(2 3 11 6)(5 9 8 12)
(1 4 7 10)(2 11 6 3)(5 8 9 12)
(1 4 10)(2 11 6)(5 9 12)
(1 10)(2 11)(3 6)(4 7)(5 8)(9 12)
(1 7 4)(3 6 11)(5 12 8)
(1 7)(3 11)(8 12)
(1 7 10 4)(2 6 11 3)(5 12 8 9)
(1 7)(2 6)(3 11)(4 10)(5 9)(8 12)
(1 7 10)(2 11 3)(8 9 12)
(1 7 4 10)(2 11 3 6)(5 9 12 8)
(1 10 7 4)(2 3 6 11)(5 12 9 8)
(1 10 7)(2 3 11)(8 12 9)
(1 10 4)(2 6 11)(5 12 9)
(1 10 4 7)(2 6 3 11)(5 8 12 9)
(1 10)(2 11)(9 12)
Transitivity sets: {1,4,7,10} - D1 9, {2,3,6,11} - D4 232,
{5,8,9,12} - D1 8.

E6 (36 0 8) (75 0 0)
(1 1 8 9 14 16 18 19 15 16 17 18 8 10 6 7 11 12 2 4)
Aut D (see reducible design no 19)
Transitivity sets: {1,2,12} - D2 5, {9,10,11} - D4 301,
{3,4,5,6,7,8} - D3 23.

E7 (36 0 8) (75 0 0)
(1 1 8 9 14 16 18 19 15 17 10 17 10 15 7 13 3 12 2 5)
Aut D (see reducible design no 13)
Transitivity sets: {2} - D3 18, {1,12} - D2 6, {9,10,11} - D3 30,
{3,4,5,6,7,8} - D2 14.

E8 (20 16 8) (63 4 0)
(1 1 8 9 14 16 18 19 15 17 10 18 10 14 6 13 4 12 2 5)
Aut D (4 5)(7 8)(10 11)
(1 12)(3 6)(4 7)(5 8)
(1 12)(3 6)(4 8)(5 7)(10 11)
Transitivity sets: {2} - D3 38, {9} - D4 297, {1,12} - D2 7,
{3,6} - D3 51, {10,11} - D3 33, {4,5,7,8} - D2 12.

E9 (12 24 8) (57 6 0)
(1 1 8 9 14 16 18 19 15 19 9 17 10 14 6 13 3 11 4 5)
Aut D (3 4 5)(6 7 8)(9 10 11) (1 12)(3 4)(6 7)(9 10)
(3 5 4)(6 8 7)(9 11 10) (1 12)(3 5)(6 8)(9 11)
(3 6)(4 7)(5 8) (1 12)(3 6)(4 8)(5 7)(10 11)
(3 7 5 6 4 8)(9 10 11) (1 12)(3 7)(4 6)(5 8)(9 10)
(3 8 4 6 5 7)(9 11 10) (1 12)(3 8)(4 7)(5 6)(9 11)
(1 12)(4 5)(7 8)(10 11)
Transitivity sets: {2} - D4 306, {1,12} - D2 8, {9,10,11} - D4 182,
{3,4,5,6,7,8} - D3 53.

E10 (12 24 8) (57 6 0)
(1 1 8 9 14 18 16 19 14 19 10 15 9 17 6 13 4 11 3 5)
Aut D (1 12)(7 8)(10 11)
Transitivity sets: {2} - D4 179, {3} - D2 30, {4} - D2 22, {5} - D3 39,
{6} - D4 190, {9} - D4 212, {1,12} - D2 11,
{7,8} - D3 31, {10,11} - D2 18.

E11 (20 16 8) (63 4 0)
(1 1 8 9 14 18 16 19 15 16 10 17 8 18 7 12 6 11 2 4)
Aut D (1 12)(7 8)(10 11)
Transitivity sets: {2} - D2 27, {3} - D2 23, {4} - D2 29, {5} - D3 19,
{6} - D3 45, {9} - D4 295, {1,12} - D2 13,
{7,8} - D3 29, {10,11} - D2 17.

E12 (12 24 8) (57 6 0)
(1 1 8 9 14 19 16 18 14 19 9 10 15 17 11 13 4 6 3 5)
Aut D (2 12)(3 5)(7 8)(9 10) (1 12 2)(3 4 5)(9 11 10)
(1 2)(3 4)(7 8)(9 11) (1 12)(4 5)(7 8)(10 11)
(1 2 12)(3 5 4)(9 10 11)
Transitivity sets: {6} - D4 311, {7,8} - D3 22, {1,2,12} - D2 24,
{3,4,5} - D3 42, {9,10,11} - D4 189.

E13 (8 28 8) (54 4 1)
(1 1 8 9 14 19 16 18 14 19 9 15 10 17 6 13 4 11 3 5)
Aut D (1 3)(2 4)(5 12)(6 10)(7 11)(8 9)
Transitivity sets: {1,3} - D2 26, {2,4} - D3 40, {5,12} - D2 25,
{6,10} - D4 191, {7,11} - D3 49, {8,9} - D3 47.

E14 (8 28 8) (54 4 1)
(1 1 8 9 14 19 16 18 15 19 9 14 10 17 6 13 3 11 4 5)
Aut D (3 6)(4 7)(5 8) (1 10)(2 11)(3 7)(4 6)(5 8)(9 12)
(1 10)(2 11)(3 4)(6 7)(9 12)
Transitivity sets: {1,10} - D2 32, {2,11} - D4 216, {5,8} - D3 48,
{9,12} - D2 31, {3,4,6,7} - D3 50.

E15 (0 36 8) (48 0 3)
(1 1 8 15 9 19 16 17 10 18 16 17 9 14 3 11 6 13 2 7)
Aut D (design is 1-transitive, see Chapter 5)
Transitivity sets: {1,2,3,4,5,6,7,8,9,10,11,12} - D4 205.

E16 (32 8 4) (61 0 0)
(1 2 6 8 13 15 19 20 16 18 10 17 7 14 9 12 4 11 3 5)
Aut D (see reducible design no 23)
Transitivity sets: {1,8,9,12} - D5 2, {2,3,4,5,6,7,10,11} - D5 38.

E17 (36 4 4) (63 0 0)
(1 2 6 8 13 16 18 20 15 18 10 17 10 12 7 14 4 12 2 5)
Aut D (see reducible design no 16)
Transitivity sets: {8} - D11 323, {6,11} - D10 152, {7,9,10} - D10 311,
{1,2,3,4,5,12} - D5 5.

E18 (20 20 4) (51 4 0)
(1 2 6 8 13 16 18 20 15 19 9 17 10 12 7 14 4 11 3 5)
Transitivity sets: {1} - D5 8, {2} - D5 22, {3} - D5 25, {4} - D5 23,
{5} - D5 24, {6} - D10 119, {7} - D11 131,
{8} - D11 44, {9} - D10 308, {10} - D10 305,
{11} - D10 118, {12} - D5 6.

E19 (40 0 4) (65 0 0)
(1 2 6 8 13 19 15 20 16 18 9 10 12 17 11 14 4 7 3 5)
Aut D (see reducible design no 20)
Transitivity sets: {1,12} - D5 13, {6,11} - D6 84, {2,3,4,5} - D5 39,
{7,8,9,10} - D6 31.

E20 (24 16 4) (57 0 0)
(1 2 6 8 13 19 15 20 16 18 9 17 10 12 4 14 7 11 3 5)
Aut D (2 3)(4 5)(7 10)(8 9)
(2 4 3 5)(6 11)(7 8 10 9)
(2 5 3 4)(6 11)(7 9 10 8)
(1 12)(4 5)(8 9)
(1 12)(2 3)(7 10)
(1 12)(2 4)(3 5)(6 11)(7 8)(9 10)
(1 12)(2 5)(3 4)(6 11)(7 9)(8 10)
Transitivity sets: {1,12} - D5 14, {6,11} - D6 83, {2,3,4,5} - D5 40,
{7,8,9,10} - D6 32.

E21 (16 24 4) (45 8 0)
(1 2 6 8 13 20 15 19 16 18 7 14 10 17 9 12 4 11 3 5)
Aut D (2 3)(7 10)(8 9)
(1 12)(4 5)(8 9)
(1 12)(2 3)(4 5)(7 10)
Transitivity sets: {6} - D9 79, {11} - D6 79, {1,12} - D5 18,
{2,3} - D10 62, {4,5} - D5 42, {7,10} - D6 89,
{8,9} - D6 2.

E22 (16 24 4) (45 8 0)
(1 2 6 8 13 20 15 19 16 18 9 12 10 17 7 14 4 11 3 5)
Aut D (2 3)(7 10)(8 9)
(1 12)(4 5)(8 9)
(1 12)(2 3)(4 5)(7 10)
Transitivity sets: {6} - D9 80, {11} - D6 80, {1,12} - D5 19,
{2,3} - D10 220, {4,5} - D5 41, {7,10} - D6 87,
{8,9} - D6 3.

E23 (8 32 4) (40 6 1)
(1 2 6 8 13 20 16 18 15 19 7 14 10 17 9 12 4 11 3 5)
Aut D (2 3)(7 10)(8 9)
(1 4)(5 12)(8 9)(6 11)
(1 4)(2 3)(5 12)(6 11)(7 10)
Transitivity sets: {1,4} - D5 33, {2,3} - D10 64, {5,12} - D5 26,
{6,11} - D23 218, {7,10} - D23 220, {8,9} - 30B 523.

E24 (8 32 4) (40 6 1)
(1 2 6 8 13 20 16 18 15 19 9 12 10 17 7 14 4 11 3 5)
Aut D (2 3)(7 10)(8 9)
(1 4)(5 12)(6 11)(8 9)
(1 4)(2 3)(5 12)(6 11)(7 10)
Transitivity sets: {1,4} - D5 32, {2,3} - D10 223, {5,12} - D5 27,
{6,11} - D23 217, {7,10} - D23 219, {8,9} - 30B 451.

E25 (8 32 4) (40 6 1)
(1 2 6 8 13 20 16 18 15 19 10 17 7 14 4 11 9 12 3 5)
Aut D (2 3)(7 10)(8 9)
Transitivity sets: {1} - D5 31, {4} - D5 30, {5} - D5 29, {6} - D23 142,
{11} - D23 143, {12} - D5 28, {2,3} - D10 63,
{7,10} - D23 132, {8,9} - 29C 2.

<u>E26</u> (16 24 4) (46 7 0)
(1 2 6 9 12 18 16 20 14 19 10 15 7 17 8 13 4 11 3 5)
<u>Transitivity sets:</u> {1} - D6 10, {2} - D6 21, {3} - D6 28, {4} - D9 60,
{5} - D9 55, {6} - D9 27, {7} - D6 9, {8} - D6 62,
{9} - D6 8, {10} - D6 65, {11} - D9 21, {12} - D6 6.

<u>E27</u> (8 32 4) (37 12 0)
(1 2 6 9 12 20 15 19 16 17 10 18 7 14 4 11 8 13 3 5)
<u>Aut D</u> (1 12)(4 5)(8 9)(10 11)
<u>Transitivity sets:</u> {2} - D10 269, {3} - D10 192, {6} - D10 273,
{7} - D10 65, {1,12} - D6 15, {4,5} - D9 81,
{8,9} - D9 16, {10,11} - D6 82.

<u>E28</u> (8 32 4) (39 10 0)
(1 2 6 9 12 20 15 19 16 17 10 18 8 13 4 11 7 14 3 5)
<u>Aut D</u> (1 12)(4 5)(8 9)(10 11)
<u>Transitivity sets:</u> {2} - D10 327, {3} - D10 128, {6} - D10 331,
{7} - D10 140, {1,12} - D6 16, {4,5} - D9 82,
{8,9} - D9 15, {10,11} - D6 81.

<u>E29</u> (24 16 4) (53 4 0)
(1 2 6 9 12 20 16 18 14 18 10 17 10 13 4 13 6 14 2 5)
<u>Aut D</u> (1 9)(2 7)(3 10)(4 6)(5 8)(11 12)
<u>Transitivity sets:</u> {1,9} - D6 27, {2,7} - D6 37, {3,10} - D6 61,
{4,6} - D6 46, {5,8} - D6 45, {11,12} - D6 19.

<u>E30</u> (8 32 4) (40 6 1)
(1 2 6 9 12 20 16 18 14 19 7 15 10 17 8 13 4 11 3 5)
<u>Aut D</u> (1 11)(2 7)(3 10)(4 6)(5 8)(9 12)
<u>Transitivity sets:</u> {1,11} - D6 26, {2,7} - D9 26, {3,10} - D6 64,
{4,6} - D9 59, {5,8} - D6 63, {9,12} - D6 20.

<u>E31</u> (4 36 4) (35 9 1)
(1 2 6 9 12 20 16 18 15 19 7 14 10 17 8 13 3 11 4 5)
<u>Transitivity sets:</u> {1} - D6 25, {2} - D10 67, {3} - D10 131, {4} - D9 23,
{5} - D9 24, {6} - D23 183, {7} - D23 185,
{8} - D28 126, {9} - D28 79, {10} - D23 210,
{11} - D23 212, {12} - D6 22.

<u>E32</u> (4 36 4) (35 9 1)
(1 2 6 9 12 20 16 18 15 19 8 13 10 17 7 14 3 11 4 5)
<u>Transitivity sets:</u> {1} - D6 24, {2} - D10 143, {3} - D10 194, {4} - D9 22,
{5} - D9 25, {6} - D23 182, {7} - D23 184,
{8} - D28 125, {9} - D28 78, {10} - D23 209,
{11} - D23 211, {12} - D6 23.

<u>E33</u> (40 0 4) (65 0 0)
(1 2 6 9 13 16 17 20 14 18 10 18 10 12 6 14 4 13 2 5)
<u>Aut D</u> (see reducible design no 18)
<u>Transitivity sets:</u> {4,8} - D6 50, {1,2,3,5,6,7,9,10,11,12} - D6 36.

<u>E34</u> (24 16 4) (55 2 0)
(1 2 6 9 13 19 14 20 16 17 9 17 10 13 3 15 7 11 3 5)
<u>Aut D</u> (1 12)(4 5)(8 9)(10 11)
<u>Transitivity sets:</u> {2} - D7 63, {3} - D7 68, {6} - D11 235, {7} - D11 256,
{1,12} - D6 53, {4,5} - D9 86, {8,9} - D7 18,
{10,11} - D6 85.

E35 (40 0 4) (65 0 0)
(1 2 6 9 13 19 14 20 16 17 15 17 7 10 3 9 11 13 3 5)
Aut D (see reducible design no 17)
Transitivity sets: {2} - D7 62, {6} - D11 260, {7,8,9,10,11} - D6 86,
{1,3,4,5,12} - D6 54.

E36 (36 4 4) (63 0 0)
(1 2 6 10 13 18 14 20 15 16 8 17 7 19 9 12 5 11 3 4)
Aut D (see reducible design no 21)
Transitivity sets: {1,2,5} - D7 28, {3,4,12} - D7 23,
{6,7,8,9,10,11} - D21 43.

E37 (20 20 4) (51 4 0)
(1 2 6 10 13 18 14 20 15 17 8 16 7 19 9 12 4 11 3 5)
Aut D (2 5)(6 10)(7 9)(8 11)
Transitivity sets: {1} - D7 27, {3} - D7 29, {4} - D10 149, {12} - D7 24,
{2,5} - D7 36, {6,10} - D21 45, {7,9} - D21 41,
{8,11} - D21 42.

E38 (8 32 4) (42 4 1)
(1 2 6 10 13 20 14 18 15 17 8 19 7 16 4 11 9 12 3 5)
Transitivity sets: {1} - D7 35, {2} - D10 289, {3} - D10 30,
{4} - D10 323, {5} - D10 185, {6} - D21 62,
{7} - D22 60, {8} - D26 29, {9} - D22 62,
{10} - D21 64, {11} - D21 67, {12} - D7 30.

E39 (16 24 4) (44 6 1)
(1 2 6 10 13 20 14 18 15 17 10 17 7 16 2 13 9 12 3 5)
Aut D (1 9)(2 6)(3 8)(4 11)(5 7)(10 12)
Transitivity sets: {1,9} - D7 34, {2,6} - D10 44, {3,8} - D10 55,
{4,11} - D10 212, {5,7} - D10 201, {10,12} - D7 31.

E40 (16 24 4) (46 7 0)
(1 2 6 10 13 20 15 17 14 18 8 19 7 16 4 11 9 12 3 5)
Transitivity sets: {1} - D7 41, {2} - D10 210, {3} - D10 45,
{4} - D10 202, {5} - D10 186, {6} - D7 65,
{7} - D7 64, {8} - D10 214, {9} - D10 322,
{10} - D10 29, {11} - D10 57, {12} - D7 40.

E41 (24 16 4) (49 8 0)
(1 2 6 15 7 14 19 20 10 16 17 18 8 13 9 12 5 11 3 4)
Aut D (6 7)(8 9)(10 11) (1 4)(3 12)(6 7)
(2 5)(3 12)(8 10)(9 11) (1 4)(2 5)(8 11)(9 10)
(2 5)(3 12)(6 7)(9 10)(8 11) (1 4)(2 5)(6 7)(8 10)(9 11)
(1 3)(4 12)(8 9) (1 12)(3 4)(10 11)
(1 3)(4 12)(6 7)(10 11) (1 12)(3 4)(6 7)(8 9)
(1 3 4 12)(2 5)(8 10 9 11) (1 12 4 3)(2 5)(8 11 9 10)
(1 3 4 12)(2 5)(6 7)(8 11 9 10) (1 12 4 3)(2 5)(6 7)(8 10 9 11)
(1 4)(3 12)(8 9)(10 11)
Transitivity sets: {2,5} - D10 226, {6,7} - D20 13, {1,3,4,12} - D10 1,
{8,9,10,11} - D20 16.

E42 (24 16 4) (45 12 0)
(1 2 6 15 7 14 19 20 10 16 17 18 9 12 8 13 5 11 3 4)
Aut D (6 7)(8 9)(10 11) (1 4 3)(6 9 11)(7 8 10)
(3 4 12)(6 8 10 7 9 11) (1 4 12)(6 10 8 7 11 9)
(3 4 12)(6 9 10)(7 8 11) (1 4 12)(6 11 8)(7 10 9)
(3 12 4)(6 10 9)(7 11 8) (1 4)(3 12)(8 9)
(3 12 4)(6 11 9 7 10 8) (1 4)(3 12)(6 7)(10 11)
(1 3)(4 12)(8 9)(10 11) (1 12 3)(6 10 8)(7 11 9)

(1 3)(4 12)(6 7)
(1 3 4)(6 10 9 7 11 8)
(1 3 4)(6 11 9)(7 10 8)
(1 3 12)(6 8 10)(7 9 11)
(1 3 12)(6 9 10 7 8 11)
(1 4 3)(6 8 11 7 9 10)
(1 12 3)(6 11 8 7 10 9)
(1 12 4)(6 8 11)(7 9 10)
(1 12 4)(6 9 11 7 8 10)
(1 12)(3 4)(10 11)
(1 12)(3 4)(6 7)(8 9)

Transitivity sets: {2} - D10 225, {5} - D10 233, {1,3,4,12} - D10 2, {6,7,8,9,10,11} - D20 14.

E43 (8 32 4) (36 10 1)
(1 2 6 15 7 14 19 20 10 17 16 18 8 13 9 12 4 11 3 5)
Aut D (1 3)(4 12)(8 9)
Transitivity sets: {2} - D10 244, {5} - D10 256, {6} - D20 21, {7} - D20 22, {10} - D20 24, {11} - D20 26, {1,3} - D10 5, {4,12} - D10 4, {8,9} - D25 23.

E44 (16 24 4) (49 4 0)
(1 2 6 15 7 16 17 20 8 16 17 18 10 13 9 12 7 11 2 3)
Aut D (2 5)(3 12)(8 10)(9 11)
(1 4)(3 12)(6 7)(9 11)
(1 4)(2 5)(6 7)(8 10)
Transitivity sets: {1,4} - D10 26, {2,5} - D10 80, {3,12} - D10 11, {6,7} - D23 40, {8,10} - D23 203, {9,11} - D23 204.

E45 (8 32 4) (40 6 1)
(1 2 6 15 7 16 17 20 8 19 14 18 10 13 9 12 4 11 3 5)
Transitivity sets: {1} - D10 24, {2} - D10 120, {3} - D10 20, {4} - D10 17, {5} - D10 134, {6} - D23 49, {7} - D23 47, {8} - 29E 329, {9} - D23 134, {10} - D23 130, {11} - D23 145, {12} - D10 12.

E46 (32 8 4) (53 8 0)
(1 2 6 15 7 16 17 20 10 14 17 18 10 13 9 12 5 13 2 3)
Aut D (1 4)(3 12)(6 7)(9 11)
(2 3)(5 12)(8 11)(9 10)
(1 4)(2 3 5 12)(6 7)(8 11 10 9)
(2 5)(3 12)(8 10)(9 11)
(1 4)(2 5)(6 7)(8 10)
(1 6)(2 8 5 10)(3 9 12 11)(4 7)
(1 7)(2 8 5 10)(3 11 12 9)(4 6)
(1 6)(2 9)(3 8)(4 7)(5 11)(10 12)
(1 7)(2 9 5 11)(3 10 12 8)(4 6)
(1 6)(2 10 5 8)(3 11 12 9)(4 7)
(1 7)(2 10 5 8)(3 9 12 11)(4 6)
(1 6)(2 11)(3 10)(4 7)(5 9)(8 12)
(1 7)(2 11 5 9)(3 8 12 10)(4 6)
(2 12)(3 5)(8 9)(10 11)
(1 4)(2 12 5 3)(6 7)(8 9 10 11)
Transitivity sets: {1,4,6,7} - D10 25, {2,3,5,8,9,10,11,12} - D10 13.

E47 (16 24 4) (42 8 1)
(1 2 6 15 7 16 17 20 10 17 14 18 10 13 9 12 2 13 3 5)
Aut D (1 6)(2 9)(3 8)(4 7)(5 11)(10 12)
Transitivity sets: {1,6} - D10 23, {2,9} - D10 42, {3,8} - D10 19, {4,7} - D10 18, {5,11} - D10 208, {10,12} - D10 14.

E48 (12 28 4) (45 6 0)
(1 2 6 15 7 20 14 19 10 16 12 18 9 17 8 11 5 13 3 4)
Aut D (4 5)(6 7)(8 9)(10 11)
(1 3 12)(6 8 10)(7 9 11)
(1 3 12)(4 5)(6 9 10 7 8 11)
(1 12 3)(6 10 8)(7 11 9)
(1 12 3)(4 5)(6 11 8 7 10 9)
Transitivity sets: {2} - D11 395, {4,5} - D10 235, {1,3,12} - D10 28, {6,7,8,9,10,11} - D21 44.

E49 (4 36 4) (33 11 1)
(1 2 6 15 7 20 14 19 10 18 16 17 8 13 3 11 9 12 4 5)
Transitivity sets: {1} - D10 34, {2} - D10 266, {3} - D10 51, {4} - D10 60, {5} - D10 144, {6} - D21 58, {7} - D21 59, {8} - D26 87, {9} - D26 89,

{10} - D22 57, {11} - D22 59, {12} - D10 33.

E50 (8 32 4) (41 8 0)
(1 2 6 15 7 20 16 17 8 16 13 17 10 18 9 12 7 11 2 3)
Transitivity sets: {1} - D10 69, {2} - D10 95, {3} - D10 215,
{4} - D10 92, {5} - D10 82, {6} - D23 48,
{7} - D23 46, {8} - D23 144, {9} - 29E 328,
{10} - D23 135, {11} - D23 131, {12} - D10 38.

E51 (8 32 4) (42 7 0)
(1 2 6 15 7 20 16 17 8 19 14 18 10 13 4 12 9 11 3 5)
Transitivity sets: {1} - D10 66, {2} - D11 217, {3} - D11 121,
{4} - D10 130, {5} - D10 136, {6} - D11 114,
{7} - D10 142, {8} - D10 218, {9} - D11 25,
{10} - D10 193, {11} - D10 122, {12} - D10 40.

E52 (4 36 4) (35 12 0)
(1 2 6 15 7 20 16 17 10 18 14 19 8 13 3 11 9 12 4 5)
Transitivity sets: {1} - D10 61, {2} - D10 219, {3} - D10 117,
{4} - D10 345, {5} - D10 116, {6} - D22 58,
{7} - D22 56, {8} - D26 88, {9} - D26 90,
{10} - D21 57, {11} - D21 55, {12} - D10 50.

E53 (8 32 4) (36 10 1)
(1 2 6 15 7 20 16 17 10 18 16 17 8 13 3 11 9 12 2 7)
Aut D (1 8)(2 11)(3 10)(4 7)(5 9)(6 12)
Transitivity sets: {1,8} - D10 56, {2,11} - D10 126, {3,10} - D10 127,
{4,7} - D10 204, {5,9} - D10 138, {6,12} - D10 52.

E54 (8 32 4) (32 8 3)
(1 2 6 15 7 20 16 17 10 18 16 17 9 12 3 11 8 13 2 7)
Aut D (1 5)(2 4)(3 12)(6 10)(7 11)(8 9) (1 9)(2 7)(3 6)(4 11)(5 8)(10 12)
(1 8)(2 11)(3 10)(4 7)(5 9)(6 12)
Transitivity sets: {1,5,8,9} - D10 54, {2,4,7,11} - D10 203,
{3,6,10,12} - D10 53.

E55 (24 16 4) (57 0 0)
(1 2 6 15 8 13 19 20 10 16 17 18 7 14 9 12 5 11 3 4)
Aut D (see reducible design no 24)
Transitivity sets: {2,5} - D10 230, {1,3,4,12} - D10 71,
{6,7,8,9,10,11} - D20 15.

E56 (8 32 4) (41 8 0)
(1 2 6 15 8 13 19 20 10 17 16 18 7 14 9 12 4 11 3 5)
Aut D (2 5)(3 4)(6 10)(7 11)(8 9) (1 12)(2 5)(3 4)(6 10)(7 11)
(1 12)(3 4)(8 9)
Transitivity sets: {1,12} - D10 73, {2,5} - D10 248, {3,4} - D10 76,
{6,10} - D20 23, {7,11} - D20 25, {8,9} - D25 22.

E57 (32 8 4) (61 0 0)
(1 2 6 15 8 16 18 19 10 13 17 18 10 14 9 12 5 14 1 4)
Aut D (see reducible design no 15)
Transitivity sets: {1,4,6,7} - D10 96, {2,3,5,8,9,10,11,12} - D10 83.

E58 (16 24 4) (47 6 0)
(1 2 6 15 8 16 18 19 10 17 16 18 9 12 7 11 4 14 1 8)
Aut D (1 9)(2 10)(3 11)(4 7)(5 8)(6 12)
Transitivity sets: {1,9} - D10 90, {2,10} - D10 123, {3,11} - D10 87,
{4,7} - D10 91, {5,8} - D10 137, {6,12} - D10 85.

E59 (4 36 4) (33 11 1)
(1 2 6 15 8 19 16 18 10 17 13 20 7 14 4 11 9 12 3 5)
Transitivity sets: {1} - D10 145, {2} - D10 221, {3} - D10 188,
{4} - D10 267, {5} - D10 187, {6} - D22 55,
{7} - D22 54, {8} - D26 92, {9} - D26 91,
{10} - D21 54, {11} - D21 56, {12} - D10 124.

E60 (12 28 4) (45 6 0)
(1 2 6 15 9 19 12 20 10 16 13 17 8 18 5 14 7 11 3 4)
Aut D (4 5)(6 7)(8 9)(10 11) (1 12 3)(6 10 8)(7 11 9)
(1 3 12)(6 8 10)(7 9 11) (1 12 3)(4 5)(6 11 8 7 10 9)
(1 3 12)(4 5)(6 9 10 7 8 11)
Transitivity sets: {2} - D11 389, {4,5} - D10 228, {1,3,12} - D10 181,
{6,7,8,9,10,11} - D21 40.

E61 (16 24 4) (51 2 0)
(1 2 6 16 7 20 15 17 9 16 12 18 8 19 8 11 7 11 3 4)
Aut D (1 2)(4 5)(8 9)(10 11)
Transitivity sets: {3} - D11 112, {6} - D11 279, {7} - D11 118,
{12} - D11 46, {1,2} - D11 69, {4,5} - D11 115,
{8,9} - D11 173, {10,11} - D11 122.

E62 (16 24 4) (50 0 1)
(1 2 6 16 9 17 13 20 9 17 13 19 10 14 3 15 7 11 3 5)
Aut D (2 3)(5 12)(7 9)(8 10) (1 4)(2 12)(3 5)(6 11)(7 8)(9 10)
(1 4)(2 5)(3 12)(6 11)(7 10)(8 9)
Transitivity sets: {1,4} - D11 202, {6,11} - D11 236, {2,3,5,12} - D11 192,
{7,8,9,10} - D11 230.

E63 (0 40 4) (33 12 0)
(1 8 2 6 13 20 16 18 15 19 4 17 7 14 9 12 10 11 3 5)
Aut D (1 2 4)(3 5 12)(7 8 9) (1 4 2)(3 12 5)(7 9 8)
Transitivity sets: {6} - 30F 130, {10} - 30B 41, {11} - 30F 142,
{1,2,4} - D25 35, {3,5,12} - D25 29, {7,8,9} - 30B 48.

E64 (0 40 4) (32 10 1)
(1 8 2 6 13 20 16 18 15 19 4 17 7 14 10 11 9 12 3 5)
Aut D (1 4)(5 12)(6 11)(7 10)
Transitivity sets: {2} - D25 40, {3} - D25 34, {8} - 30A 5,
{9} - 30A 3, {1,4} - D25 36, {5,12} - D25 30,
{6,11} - 30F 111, {7,10} - 30B 40.

E65 (0 40 4) (30 6 3)
(1 8 2 6 13 20 16 18 15 19 4 17 9 12 10 11 7 14 3 5)
Aut D (2 4)(3 5)(6 11)(9 10) (1 4 2)(3 12 5)(7 10 9)
(1 2)(3 12)(6 11)(7 9) (1 4)(5 12)(6 11)(7 10)
(1 2 4)(3 5 12)(7 9 10)
Transitivity sets: {8} - 30A 4, {6,11} - 30F 110, {1,2,4} - D25 38,
{3,5,12} - D25 32, {7,9,10} - 30A 2.

E66 (0 40 4) (28 8 3)
(1 8 2 7 12 20 16 18 14 19 9 13 4 17 6 15 10 11 3 5)
Aut D (1 6)(2 10)(3 7)(4 11)(5 8)(9 12)
Transitivity sets: {1,6} - D26 64, {2,10} - D26 113, {3,7} - D26 105,
{4,11} - D26 119, {5,8} - D26 98, {9,12} - D26 39.

E67 (0 40 4) (30 12 1)
(1 8 2 7 12 20 16 18 14 19 9 13 4 17 10 11 6 15 3 5)
Aut D (1 6)(2 10)(3 7)(4 11)(5 8)(9 12)

Transitivity sets: {1,6} - D26 62, {2,10} - D26 110, {3,7} - D26 106,
{4,11} - D26 117, {5,8} - D26 100, {9,12} - D26 40.

E68 (0 40 4) (32 10 1)
(1 8 2 7 12 20 16 18 14 19 9 13 6 15 4 17 10 11 3 5)
Aut D (1 6)(2 10)(3 7)(4 11)(5 8)(9 12)
Transitivity sets: {1,6} - D26 63, {2,10} - D26 114, {3,7} - D26 103,
{4,11} - D26 120, {5,8} - D26 96, {9,12} - D26 41.

E69 (0 40 4) (32 13 0)
(1 8 2 7 12 20 16 18 14 19 9 13 6 15 10 11 4 17 3 5)
Transitivity sets: {1} - D26 60, {2} - D26 112, {3} - D26 104,
{4} - D26 115, {5} - D26 101, {6} - D26 61,
{7} - D26 108, {8} - D26 97, {9} - D26 43,
{10} - D26 109, {11} - D26 118, {12} - D26 42.

E70 (0 40 4) (30 12 1)
(1 8 2 7 12 20 16 18 14 19 9 13 10 11 6 15 4 17 3 5)
Aut D (1 6)(2 10)(3 7)(4 11)(5 8)(9 12)
Transitivity sets: {1,6} - D26 59, {2,10} - D26 111, {3,7} - D26 107,
{4,11} - D26 116, {5,8} - D26 99, {9,12} - D26 44.

E71 (0 40 4) (36 0 3)
(1 8 2 7 12 20 16 18 15 19 9 13 3 17 6 14 10 11 4 5)
Aut D (3 5 12)(6 10 11)(7 8 9)
(3 12 5)(6 11 10)(7 9 8)
(1 2 4)(6 11 10)
(1 2 4)(3 5 12)(7 8 9)
(1 2 4)(3 12 5)(6 10 11)(7 9 8)
(1 4 2)(6 10 11)
(1 4 2)(3 5 12)(6 11 10)(7 8 9)
(1 4 2)(3 12 5)(7 9 8)
(1 6)(2 10)(3 7)(4 11)(5 8)(9 12)
(1 6 2 10 4 11)(3 8 12 7 5 9)
(1 6 4 11 2 10)(3 9 5 7 12 8)
(1 10)(2 11)(3 7)(4 6)(5 8)(9 12)
(1 10 2 11 4 6)(3 8 12 7 5 9)
(1 10 4 6 2 11)(3 9 5 7 12 8)
(1 11)(2 6)(3 7)(4 10)(5 8)(9 12)
(1 11 2 6 4 10)(3 8 12 7 5 9)
(1 11 4 10 2 6)(3 9 5 7 12 8)
Transitivity sets: {1,2,4,6,10,11} - D26 55, {3,5,7,8,9,12} - D26 51.

E72 (0 40 4) (45 0 0)
(1 8 2 13 6 20 15 19 10 18 11 16 7 14 9 12 3 17 4 5)
Aut D (2 3 5 4)(6 7 11 10)
(2 4 5 3)(6 10 11 7)
(2 5)(3 4)(6 11)(7 10)
(1 8)(2 6 5 11)(3 10 4 7)(9 12)
(1 8)(2 7 5 10)(3 6 4 11)(9 12)
(1 8)(2 10 5 7)(3 11 4 6)(9 12)
(1 8)(2 11 5 6)(3 7 4 10)(9 12)
(1 9)(2 6 5 11)(3 7 4 10)(8 12)
(1 9)(2 7)(3 11)(4 6)(5 10)(8 12)
(1 9)(2 10)(3 6)(4 11)(5 7)(8 12)
(1 9)(2 11 5 6)(3 10 4 7)(8 12)
(1 12)(3 4)(7 10)(8 9)
(1 12)(2 3)(4 5)(6 7)(8 9)(10 11)
(1 12)(2 4)(3 5)(6 10)(7 11)(8 9)
(1 12)(2 5)(6 11)(8 9)
Transitivity sets: {1,8,9,12} - D28 19, {2,3,4,5,6,7,10,11} - D28 178.

E73 (0 40 4) (37 8 0)
(1 8 2 13 6 20 15 19 10 18 11 16 9 12 7 14 3 17 4 5)
Aut D (1 12)(3 4)(7 10)(8 9)
(2 3 5 4)(6 7 11 10)
(1 12)(2 3)(4 5)(6 7)(8 9)(10 11)
(2 4 5 3)(6 10 11 7)
(1 12)(2 4)(3 5)(6 10)(7 11)(8 9)
(2 5)(3 4)(6 11)(7 10)
(1 12)(2 5)(6 11)(8 9)
(1 8)(2 6 5 11)(3 10 4 7)(9 12)
(1 9)(2 6 5 11)(3 7 4 10)(8 12)
(1 8)(2 7 5 10)(3 6 4 11)(9 12)
(1 9)(2 7)(3 11)(4 6)(5 10)(8 12)
(1 8)(2 10 5 7)(3 11 4 6)(9 12)
(1 9)(2 10)(3 6)(4 11)(5 7)(8 12)
(1 8)(2 11 5 6)(3 7 4 10)(9 12)
(1 9)(2 11 5 6)(3 10 4 7)(8 12)
Transitivity sets: {1,8,9,12} - D28 20, {2,3,4,5,6,7,10,11} - D28 177.

E74 (0 40 4) (39 6 0)
(1 8 2 13 6 20 16 18 9 19 12 15 7 14 10 11 3 17 4 5)
Aut D (1 10)(2 7)(3 11)(4 6)(5 9)(8 12)
Transitivity sets: {1,10} - D28 26, {2,7} - 29E 351, {3,11} - D28 129,
{4,6} - D28 72, {5,9} - D28 81, {8,12} - D28 24.

E75 (0 40 4) (34 8 1)
(1 8 2 13 6 20 16 18 9 19 12 15 10 11 7 14 3 17 4 5)
Aut D (1 10)(2 7)(3 11)(4 6)(5 9)(8 12)
Transitivity sets: {1,10} - D28 27, {2,7} - 29E 352, {3,11} - D28 130,
{4,6} - D28 71, {5,9} - D28 82, {8,12} - D28 25.

E76 (0 40 4) (33 12 0)
(1 8 2 13 7 20 14 19 10 17 11 16 4 18 6 15 9 12 3 5)
Aut D (1 12)(2 3)(4 5)(6 7)(8 9)(10 11)
Transitivity sets: {1,12} - D28 64, {2,3} - D28 179, {4,5} - D28 157,
{6,7} - D28 151, {8,9} - D28 67, {10,11} - D28 182.

E77 (0 40 4) (35 10 0)
(1 8 2 13 7 20 14 19 10 17 11 16 4 18 9 12 6 15 3 5)
Aut D (1 12)(2 3)(4 5)(6 7)(8 9)(10 11)
Transitivity sets: {1,12} - D28 65, {2,3} - D28 180, {4,5} - D28 158,
{6,7} - D28 150, {8,9} - D28 66, {10,11} - D28 181.

E78 (0 40 4) (34 8 1)
(1 8 2 13 7 20 16 17 9 19 12 15 3 18 6 14 10 11 4 5)
Aut D (1 10)(2 11)(3 7)(4 6)(5 8)(9 12) (2 3)(5 12)(7 11)(8 9)
(1 10)(2 7)(3 11)(4 6)(5 9)(8 12)
Transitivity sets: {1,10} - D28 84, {4,6} - D28 83, {2,3,7,11} - 29D 75,
{5,8,9,12} - D28 69.

E79 (0 40 4) (34 8 1)
(1 8 2 13 9 19 16 17 7 20 12 15 3 18 6 14 10 11 4 5)
Aut D (2 3)(5 12)(7 11)(8 9) (1 6)(2 11)(3 7)(4 10)(5 8)(9 12)
(1 6)(2 7)(3 11)(4 10)(5 9)(8 12)
Transitivity sets: {1,6} - D28 128, {4,10} - D28 127, {2,3,7,11} - 29D 72,
{5,8,9,12} - D28 120.

E80 (0 40 4) (25 20 0)
(1 8 2 19 4 17 13 20 6 15 16 18 7 14 9 12 10 11 3 5)
Transitivity sets: {1} - 30B 80, {2} - 30B 166, {3} - 30B 5,
{4} - 30B 268, {5} - 30B 338, {6} - 30B 333,
{7} - 30B 28, {8} - 30B 65, {9} - 30B 33,
{10} - 30B 272, {11} - 30B 188, {12} - 30B 1.

E81 (0 40 4) (24 18 1)
(1 8 2 19 4 17 13 20 6 15 16 18 7 14 10 11 9 12 3 5)
Aut D (1 7)(2 6)(3 10)(4 9)(5 11)(8 12)
Transitivity sets: {1,7} - 30B 74, {2,6} - 30B 165, {3,10} - 30B 6,
{4,9} - 30B 279, {5,11} - 30B 325, {8,12} - 30B 2.

E82 (0 40 4) (24 18 1)
(1 8 2 19 4 17 13 20 6 15 16 18 9 12 7 14 10 11 3 5)
Aut D (1 8)(2 11)(3 9)(4 10)(5 6)(7 12)
Transitivity sets: {1,8} - 30B 64, {2,11} - 30B 189, {3,9} - 30B 35,
{4,10} - 30B 266, {5,6} - 30B 340, {7,12} - 30B 3.

E83 (0 40 4) (22 14 3)
(1 8 2 19 4 17 13 20 6 15 16 18 9 12 10 11 7 14 3 5)
Aut D (1 10)(2 11)(3 7)(4 8)(5 6)(9 12)

Transitivity sets: {1,10} - 30B 69, {2,11} - 30B 132, {3,7} - 30B 20, {4,8} - 30B 281, {5,6} - 30B 323, {9,12} - 30B 4.

E84 (0 40 4) (24 18 1)
(1 8 2 19 4 17 13 20 7 14 16 18 6 15 9 12 10 11 3 5)
Transitivity sets: {1} - 30B 88, {2} - 30B 107, {3} - 30B 73, {4} - 30B 31, {5} - 30B 106, {6} - 30B 167, {7} - 30B 25, {8} - 30B 72, {9} - 30B 21, {10} - 30B 78, {11} - 30B 136, {12} - 30B 17.

E85 (0 40 4) (26 16 1)
(1 8 2 19 4 17 13 20 7 14 16 18 6 15 10 11 9 12 3 5)
Aut D (1 3)(4 12)(7 10)(8 9) (1 10)(2 6)(3 7)(4 8)(5 11)(9 12)
(1 7)(2 6)(3 10)(4 9)(5 11)(8 12)
Transitivity sets: {2,6} - 30B 170, {5,11} - 30B 137, {1,3,7,10} - 30B 81, {4,8,9,12} - 30B 18.

E86 (0 40 4) (20 16 3)
(1 8 2 19 4 17 13 20 7 14 16 18 9 12 6 15 10 11 3 5)
Aut D (1 4)(2 5)(3 12)(6 11)(7 9)(8 10) (1 10)(2 6)(3 7)(4 8)(5 11)(9 12)
(1 8)(2 11)(3 9)(4 10)(5 6)(7 12)
Transitivity sets: {1,4,8,10} - 30B 71, {2,5,6,11} - 30B 134, {3,7,9,12} - 30B 19.

E87 (0 40 4) (24 18 1)
(1 8 2 19 4 17 13 20 7 14 16 18 10 11 9 12 6 15 3 5)
Aut D (1 10)(2 6)(3 7)(4 8)(5 11)(9 12)
Transitivity sets: {1,10} - 30B 67, {2,6} - 30B 331, {3,7} - 30B 26, {4,8} - 30B 274, {5,11} - 30B 133, {9,12} - 30B 22.

E88 (0 40 4) (27 18 0)
(1 8 2 19 4 17 13 20 9 12 16 18 6 15 7 14 10 11 3 5)
Aut D (1 3)(4 12)(7 8)(9 10)
Transitivity sets: {2} - 30B 109, {5} - 30B 108, {6} - 30B 168, {11} - 30B 191, {1,3} - 30B 86, {4,12} - 30B 23, {7,8} - 30B 79, {9,10} - 30B 30.

E89 (0 40 4) (26 16 1)
(1 8 2 19 4 17 13 20 9 12 16 18 6 15 10 11 7 14 3 5)
Aut D (1 3)(4 12)(7 10)(8 9) (1 10)(2 11)(3 7)(4 8)(5 6)(9 12)
(1 7)(2 11)(3 10)(4 9)(5 6)(8 12)
Transitivity sets: {2,11} - 30B 104, {5,6} - 30B 105, {1,3,7,10} - 30B 75, {4,8,9,12} - 30B 24.

E90 (0 40 4) (20 16 3)
(1 8 2 19 4 17 16 18 7 14 13 20 9 12 6 15 10 11 3 5)
Aut D (1 4)(6 11) (1 11)(2 8)(3 9)(4 6)(5 10)(7 12)
(2 3)(5 12)(7 8)(9 10) (1 6)(2 9 5 7)(3 8 12 10)(4 11)
(1 4)(2 3)(5 12)(6 11)(7 8)(9 10) (1 11)(2 9 5 7)(3 8 12 10)(4 6)
(2 5)(3 12)(7 9)(8 10) (1 6)(2 10)(3 7)(4 11)(5 8)(9 12)
(1 4)(2 5)(3 12)(6 11)(7 9)(8 10) (1 11)(2 10)(3 7)(4 6)(5 8)(9 12)
(1 6)(2 7 5 9)(3 10 12 8)(4 11) (2 12)(3 5)(7 10)(8 9)
(1 11)(2 7 5 9)(3 10 12 8)(4 6) (1 4)(2 12)(3 5)(6 11)(7 10)(8 9)
(1 6)(2 8)(3 9)(4 11)(5 10)(7 12)
Transitivity sets: {1,4,6,11} - 30B 337, {2,3,5,7,8,9,10,12} - 30B 125.

E91 (0 40 4) (22 20 1)
(1 8 2 19 4 17 16 18 7 14 13 20 10 11 6 15 9 12 3 5)
Aut D (1 4)(6 11) (1 11 4 6)(2 7 3 8)(5 9 12 10)
(2 3)(5 12)(7 8)(9 10) (1 6 4 11)(2 8 3 7)(5 10 12 9)

(1 4)(2 3)(5 12)(6 11)(7 8)(9 10)   (1 11 4 6)(2 8 3 7)(5 10 12 9)
(1 6 4 11)(2 7 3 8)(5 9 12 10)

Transitivity sets: {1,4,6,11} - 30B 330, {2,3,7,8} - 30B 161,
{5,9,10,12} - 30B 127.

E92 (0 40 4) (26 16 1)
(1 8 2 19 4 17 16 18 7 16 11 20 6 15 9 12 8 11 3 7)
Aut D (1 9)(2 10)(3 7)(4 11)(5 8)(6 12)
Transitivity sets: {1,9} - 30B 227, {2,10} - 30B 362, {3,7} - 30B 158,
{4,11} - 30B 156, {5,8} - 30B 354, {6,12} - 30B 144.

E93 (0 40 4) (28 14 1)
(1 8 2 19 4 17 16 18 7 16 11 20 9 12 6 15 8 11 3 7)
Aut D (1 11)(2 10)(3 7)(4 6)(5 8)(9 12)
Transitivity sets: {1,11} - 30B 255, {2,10} - 30B 475, {3,7} - 30B 230,
{4,6} - 30B 155, {5,8} - 30B 408, {9,12} - 30B 145.

E94 (0 40 4) (24 12 3)
(1 8 2 19 4 17 16 18 7 16 14 18 6 15 9 12 8 11 1 10)
Aut D (3 4 12)(6 9 11)   (1 7)(2 8)(3 9)(4 6)(5 10)(11 12)
(3 12 4)(6 11 9)   (1 7)(2 8)(3 11)(4 9)(5 10)(6 12)
(1 7)(2 8)(3 6)(4 11)(5 10)(9 12)
Transitivity sets: {1,7} - 30B 231, {2,8} - 30B 422, {5,10} - 30B 488,
{3,4,6,9,11,12} - 30B 148.

E95 (0 40 4) (28 14 1)
(1 8 2 19 4 17 16 18 7 20 13 14 6 15 9 12 3 11 5 10)
Aut D (1 4)(2 5)(7 11)(8 10)   (1 11)(2 10)(3 6)(4 7)(5 8)(9 12)
(1 7)(2 8)(3 6)(4 11)(5 10)(9 12)
Transitivity sets: {3,6} - 30B 205, {9,12} - 30B 157, {1,4,7,11} - 30B 224,
{2,5,8,10} - 30B 346.

E96 (0 40 4) (20 16 3)
(1 8 2 19 4 17 16 18 9 12 13 20 10 11 6 15 7 14 3 5)
Aut D (2 12)(3 5)(7 8)(9 10)   (1 6)(2 8)(3 9)(4 11)(5 10)(7 12)
(1 4)(6 11)   (1 11)(2 7)(3 10)(4 6)(5 9)(8 12)
(1 4)(2 12)(3 5)(6 11)(7 8)(9 10)   (1 11)(2 8)(3 9)(4 6)(5 10)(7 12)
(1 6)(2 7)(3 10)(4 11)(5 9)(8 12)
Transitivity sets: {1,4,6,11} - 30B 322, {2,7,8,12} - 30B 163,
{3,5,9,10} - 30B 181.

E97 (0 40 4) (16 8 7)
(1 8 2 19 4 17 16 18 10 11 13 20 9 12 6 15 7 14 3 5)
Aut D (1 4)(6 11)   (1 11)(2 8)(3 9)(4 6)(5 10)(7 12)
(2 3)(5 12)(7 10)(8 9)   (1 6)(2 9)(3 8)(4 11)(5 7)(10 12)
(1 4)(2 3)(5 12)(6 11)(7 10)(8 9)   (1 11)(2 9)(3 8)(4 6)(5 7)(10 12)
(2 5)(3 12)(7 9)(8 10)   (1 6)(2 10)(3 7)(4 11)(5 8)(9 12)
(1 4)(2 5)(3 12)(6 11)(7 9)(8 10)   (1 11)(2 10)(3 7)(4 6)(5 8)(9 12)
(1 6)(2 7)(3 10)(4 11)(5 9)(8 12)   (2 12)(3 5)(7 8)(9 10)
(1 11)(2 7)(3 10)(4 6)(5 9)(8 12)   (1 4)(2 12)(3 5)(6 11)(7 8)(9 10)
(1 6)(2 8)(3 9)(4 11)(5 10)(7 12)
Transitivity sets: {1,4,6,11} - 30B 321, {2,3,5,7,8,9,10,12} - 30B 183.

E98 (0 40 4) (26 10 3)
(1 8 2 19 4 17 16 18 10 13 11 20 9 12 6 15 5 14 3 7)
Aut D (1 3)(2 5)(6 9)(8 10)   (1 9)(2 10)(3 6)(4 11)(5 8)(7 12)
(1 6)(2 8)(3 9)(4 11)(5 10)(7 12)
Transitivity sets: {4,11} - 30B 219, {7,12} - 30B 201, {1,3,6,9} - 30B 222,
{2,5,8,10} - 30B 355.

<u>E99</u> (0 40 4) (30 6 3)
(1 8 2 19 4 18 12 20 6 15 16 17 9 13 10 11 7 14 3 5)

<u>Aut D</u> (2 5)(3 12)(7 9)(10 11)
(1 2)(3 12)(6 11)(7 9)
(1 2 5)(6 11 10)
(1 5 2)(6 10 11)
(1 5)(3 12)(6 10)(7 9)
(1 6)(2 10)(3 7)(4 8)(5 11)(9 12)
(1 6)(2 11)(3 9)(4 8)(5 10)(7 12)
(1 10 2 6 5 11)(3 9)(4 8)(7 12)
(1 10)(2 11)(3 7)(4 8)(5 6)(9 12)
(1 11)(2 6)(3 7)(4 8)(5 10)(9 12)
(1 11 5 6 2 10)(3 9)(4 8)(7 12)

<u>Transitivity sets:</u> {4,8} - 30C 86, {3,7,9,12} - 30B 347,
{1,2,5,6,10,11} - 30B 398.

<u>E100</u> (0 40 4) (32 10 1)
(1 8 2 19 4 18 16 17 6 15 12 20 9 13 10 11 7 14 3 5)

<u>Aut D</u> (1 5)(3 12)(6 10)(7 9)
(1 6)(2 8)(3 9)(4 11)(5 10)(7 12)
(1 10)(2 8)(3 7)(4 11)(5 6)(9 12)

<u>Transitivity sets:</u> {2,8} - 30B 564, {4,11} - 30C 80, {1,5,6,10} - 30B 468,
{3,7,9,12} - 30B 409.

<u>E101</u> (0 40 4) (0 0 15)
(1 20 2 19 3 18 4 17 5 16 6 15 7 14 8 13 9 12 10 11)

<u>Aut D</u> (design is 1-transitive, see Chapter 5)

<u>Transitivity sets:</u> {1,2,3,4,5,6,7,8,9,10,11,12} - 38A 46.

<u>E102</u> (0 40 4) (8 16 7)
(1 20 2 19 3 18 4 17 5 16 6 15 7 14 8 13 10 11 9 12)

<u>Aut D</u> (5 12)(6 11)
(3 4)(9 10)
(3 4)(5 12)(6 11)(9 10)
(1 2)(7 8)
(1 2)(5 12)(6 11)(7 8)
(1 2)(3 4)(7 8)(9 10)
(1 2)(3 4)(5 12)(6 11)(7 8)(9 10)
(1 5)(2 12)(6 8)(7 11)
(1 5 2 12)(6 7 11 8)
(1 5)(2 12)(3 4)(6 8)(7 11)(9 10)
(1 5 2 12)(3 4)(6 7 11 8)(9 10)
(1 6 2 11)(3 9)(4 10)(5 7 12 8)
(1 6)(2 11)(3 9)(4 10)(5 8)(7 12)
(1 6 2 11)(3 10)(4 9)(5 7 12 8)
(1 6)(2 11)(3 10)(4 9)(5 8)(7 12)
(1 7)(2 8)(3 9)(4 10)(5 6)(11 12)
(1 7)(2 8)(3 9)(4 10)(5 11)(6 12)
(1 7)(2 8)(3 10)(4 9)(5 6)(11 12)
(1 7)(2 8)(3 10)(4 9)(5 11)(6 12)
(1 8)(2 7)(3 9)(4 10)(5 6)(11 12)
(1 8)(2 7)(3 9)(4 10)(5 11)(6 12)
(1 8)(2 7)(3 10)(4 9)(5 6)(11 12)
(1 8)(2 7)(3 10)(4 9)(5 11)(6 12)
(1 11)(2 6)(3 9)(4 10)(5 7)(8 12)
(1 11 2 6)(3 9)(4 10)(5 8 12 7)
(1 11)(2 6)(3 10)(4 9)(5 7)(8 12)
(1 11 2 6)(3 10)(4 9)(5 8 12 7)
(1 12 2 5)(6 8 11 7)
(1 12)(2 5)(6 7)(8 11)
(1 12 2 5)(3 4)(6 8 11 7)(9 10)
(1 12)(2 5)(3 4)(6 7)(8 11)(9 10)

<u>Transitivity sets:</u> {3,4,9,10} - 38A 50, {1,2,5,6,7,8,11,12} - 38A 47.

<u>E103</u> (0 40 4) (12 24 3)
(1 20 2 19 3 18 4 17 5 16 6 15 7 14 9 12 10 11 8 13)

<u>Aut D</u> (3 4 5)(9 10 11)
(3 5 4)(9 11 10)
(1 2)(7 8)
(1 2)(3 4 5)(7 8)(9 10 11)
(1 2)(3 5 4)(7 8)(9 11 10)
(1 7)(2 8)(3 9)(4 11)(5 10)(6 12)
(1 7)(2 8)(3 10)(4 9)(5 11)(6 12)
(1 7)(2 8)(3 11)(4 10)(5 9)(6 12)
(1 8)(2 7)(3 9)(4 11)(5 10)(6 12)
(1 8)(2 7)(3 10)(4 9)(5 11)(6 12)
(1 8)(2 7)(3 11)(4 10)(5 9)(6 12)

<u>Transitivity sets:</u> {6,12} - 38A 49, {1,2,7,8} - 38A 59,
{3,4,5,9,10,11} - 38B 36.

<u>E104</u> (0 40 4) (12 24 3)
(1 20 2 19 3 18 4 17 5 16 6 15 8 13 7 14 10 11 9 12)

<u>Aut D</u> (3 4)(9 10)
(1 2 12 5)(6 8 7 11)
(1 2 12 5)(3 4)(6 8 7 11)(9 10)
(1 5 12 2)(6 11 7 8)
(1 7)(2 8)(3 10)(4 9)(5 11)(6 12)
(1 8)(2 6)(3 9)(4 10)(5 7)(11 12)
(1 8)(2 6)(3 10)(4 9)(5 7)(11 12)
(1 11)(2 7)(3 9)(4 10)(5 6)(8 12)

(1 5 12 2)(3 4)(6 11 7 8)(9 10)   (1 11)(2 7)(3 10)(4 9)(5 6)(8 12)
(1 6)(2 11)(3 9)(4 10)(5 8)(7 12)   (1 12)(2 5)(6 7)(8 11)
(1 6)(2 11)(3 10)(4 9)(5 8)(7 12)   (1 12)(2 5)(3 4)(6 7)(8 11)(9 10)
(1 7)(2 8)(3 9)(4 10)(5 11)(6 12)

Transitivity sets: {3,4,9,10} - 38A 73, {1,2,5,6,7,8,11,12} - 38A 51.

E105 (0 40 4) (12 24 3)
(1 20 2 19 3 18 4 17 5 16 6 15 8 13 9 12 7 14 10 11)
Aut D (design is 1-transitive, see Chapter 5)
Transitivity sets: {1,2,3,4,5,6,7,8,9,10,11,12} - 38A 52.

E106 (0 40 4) (14 28 1)
(1 20 2 19 3 18 4 17 5 16 6 15 8 13 9 12 10 11 7 14)
Aut D (1 6)(2 11)(3 9)(4 10)(5 8)(7 12)
Transitivity sets: {1,6} - 38A 57, {2,11} - 38B 64, {3,9} - 38B 82,
{4,10} - 38B 39, {5,8} - 38B 17, {7,12} - 38A 53.

E107 (0 40 4) (14 22 3)
(1 20 2 19 3 18 4 17 5 16 6 15 9 12 10 11 7 14 8 13)
Aut D (1 6)(2 11)(3 9)(4 10)(5 8)(7 12)   (1 12)(2 5)(6 7)(8 11)
(1 7)(2 8)(3 9)(4 10)(5 11)(6 12)
Transitivity sets: {3,9} - 38B 16, {4,10} - 38A 79, {1,6,7,12} - 38A 60,
{2,5,8,11} - 38B 11.

E108 (0 40 4) (16 26 1)
(1 20 2 19 3 18 4 17 5 16 6 15 9 12 10 11 8 13 7 14)
Aut D (1 6)(2 11)(3 10)(4 9)(5 8)(7 12)   (1 12)(2 5)(6 7)(8 11)
(1 7)(2 8)(3 10)(4 9)(5 11)(6 12)
Transitivity sets: {3,10} - 38B 141, {4,9} - 38B 146, {1,6,7,12} - 38A 61,
{2,5,8,11} - 38B 70.

E109 (0 40 4) (16 26 1)
(1 20 2 19 3 18 4 17 5 16 7 14 8 13 6 15 10 11 9 12)
Aut D (1 6)(2 11)(3 10)(4 9)(5 8)(7 12)
Transitivity sets: {1,6} - 38A 71, {2,11} - 38B 123, {3,10} - 38B 29,
{4,9} - 38B 21, {5,8} - 38B 25, {7,12} - 38A 69.

E110 (0 40 4) (16 26 1)
(1 20 2 19 3 18 4 17 5 16 7 14 9 12 6 15 10 11 8 13)
Aut D (1 7)(2 8)(3 9)(4 11)(5 10)(6 12)
Transitivity sets: {1,7} - 38A 76, {2,8} - 38B 22, {3,9} - 38B 13,
{4,11} - 38B 46, {5,10} - 38B 65, {6,12} - 38A 74.

E111 (0 40 4) (18 18 3)
(1 20 2 19 3 18 4 17 6 15 5 16 9 12 10 11 7 14 8 13)
Aut D (3 12)(4 5)(7 9)(8 10)   (1 7)(2 8)(3 9)(4 10)(5 11)(6 12)
(2 4 5)(8 10 11)   (1 7 3 6 12 9)(2 10 4 8 5 11)
(2 4)(3 12)(7 9)(8 11)   (1 7)(2 10 5 8 4 11)(3 9)(6 12)
(2 5 4)(8 11 10)   (1 7 3 6 12 9)(2 11 4 10 5 8)
(2 5)(3 12)(7 9)(10 11)   (1 7)(2 11 4 8 5 10)(3 9)(6 12)
(1 3)(4 5)(6 9)(8 11)   (1 9)(2 8 4 10 5 11)(3 6)(7 12)
(1 3 12)(6 9 7)(8 10 11)   (1 9 12 6 3 7)(2 8 5 10 4 11)
(1 3)(2 4)(6 9)(10 11)   (1 9)(2 10)(3 6)(4 11)(5 8)(7 12)
(1 3 12)(2 4 5)(6 9 7)(8 11 10)   (1 9 12 6 3 7)(2 10 5 11 4 8)
(1 3)(2 5)(6 9)(8 10)   (1 9)(2 11 5 10 4 8)(3 6)(7 12)

(1 3 12)(2 5 4)(6 9 7)	(1 9 12 6 3 7)(2 11 5 8 4 10)
(1 6)(2 8 5 11 4 10)(3 7)(9 12)	(1 12 3)(6 7 9)(8 11 10)
(1 6)(2 8)(3 9)(4 11)(5 10)(7 12)	(1 12)(4 5)(6 7)(10 11)
(1 6)(2 10 4 11 5 8)(3 7)(9 12)	(1 12 3)(2 4 5)(6 7 9)
(1 6)(2 10)(3 9)(4 8)(5 11)(7 12)	(1 12)(2 4)(6 7)(8 10)
(1 6)(2 11)(3 7)(4 8)(5 10)(9 12)	(1 12 3)(2 5 4)(6 7 9)(8 10 11)
(1 6)(2 11)(3 9)(4 10)(5 8)(7 12)	(1 12)(2 5)(6 7)(8 11)
(1 7 3 6 12 9)(2 8 4 11 5 10)

Transitivity sets: {1,3,6,7,9,12} - 38A 147, {2,4,5,8,10,11} - 38B 190.

E112 (0 40 4) (20 22 1)
(1 20 2 19 3 18 4 17 6 15 5 16 9 12 10 11 8 13 7 14)
Aut D (1 6)(2 11)(3 10)(4 9)(5 8)(7 12) (1 12)(2 5)(6 7)(8 11)
(1 7)(2 8)(3 10)(4 9)(5 11)(6 12)
Transitivity sets: {3,10} - 38B 137, {4,9} - 38B 142,
{1,6,7,12} - 38A 148, {2,5,8,11} - 38B 31.

E113 (0 40 4) (17 28 0)
(1 20 2 19 3 18 5 16 4 17 6 15 8 13 9 12 10 11 7 14)
Aut D (1 11)(2 7)(3 10)(4 9)(5 6)(8 12)
Transitivity sets: {1,11} - 38B 48, {2,7} - 38B 134, {3,10} - 38B 47,
{4,9} - 38B 57, {5,6} - 38B 28, {8,12} - 38B 19.

E114 (0 40 4) (15 30 0)
(1 20 2 19 3 18 5 16 4 17 6 15 8 13 10 11 7 14 9 12)
Aut D (1 2 3 5 4)(6 10 11 7 9)	(1 7 3 6 4 11 2 9 5 10)(8 12)
(1 3 4 2 5)(6 11 9 10 7)	(1 9 4 7 5 11 3 10 2 6)(8 12)
(1 4 5 3 2)(6 9 7 11 10)	(1 10 5 9 2 11 4 6 3 7)(8 12)
(1 5 2 4 3)(6 7 10 9 11)	(1 11)(2 7)(3 9)(4 10)(5 6)(8 12)
(1 6 2 10 3 11 5 7 4 9)(8 12)
Transitivity sets: {8,12} - 38B 20, {1,2,3,4,5,6,7,9,10,11} - 38B 23.

E115 (0 40 4) (21 24 0)
(1 20 2 19 3 18 5 16 4 17 6 15 9 12 10 11 8 13 7 14)
Aut D (1 2 3)(4 5 12)(7 9 10)	(1 5 3 4 2 12)(6 8)(7 9 10)
(1 3 2)(4 12 5)(7 10 9)	(1 12 2 4 3 5)(6 8)(7 10 9)
(1 4)(2 5)(3 12)(6 8)
Transitivity sets: {11} - 38B 174, {6,8} - 38B 30, {7,9,10} - 38B 152,
{1,2,3,4,5,12} - 38B 27.

E116 (0 40 4) (21 24 0)
(1 20 2 19 3 18 5 16 4 17 7 14 9 12 10 11 8 13 6 15)
Aut D (3 12)(6 8)(7 10)(9 11)	(1 7)(2 9)(3 8)(4 10)(5 11)(6 12)
(1 2 4 5)(7 11 10 9)	(1 9 4 11)(2 10 5 7)(3 6)(8 12)
(1 2 4 5)(3 12)(6 8)(7 9 10 11)	(1 9)(2 10)(3 8)(4 11)(5 7)(6 12)
(1 4)(2 5)(7 10)(9 11)	(1 10 4 7)(2 11 5 9)(3 6)(8 12)
(1 4)(2 5)(3 12)(6 8)	(1 10)(2 11)(3 8)(4 7)(5 9)(6 12)
(1 5 4 2)(7 9 10 11)	(1 11 4 9)(2 7 5 10)(3 6)(8 12)
(1 5 4 2)(3 12)(6 8)(7 11 10 9)	(1 11)(2 7)(3 8)(4 9)(5 10)(6 12)
(1 7 4 10)(2 9 5 11)(3 6)(8 12)
Transitivity sets: {3,6,8,12} - 38B 49, {1,2,4,5,7,9,10,11} - 38B 125.

E117 (0 40 4) (15 30 0)
(1 20 2 19 3 18 5 16 4 17 7 14 10 11 6 15 8 13 9 12)
Aut D (1 3 5 12 4)(6 11 8 7 10)	(1 7)(2 9)(3 8)(4 10)(5 11)(6 12)
(1 4 12 5 3)(6 10 7 8 11)	(1 8)(2 9)(3 11)(4 7)(5 6)(10 12)
(1 5 4 3 12)(6 8 10 11 7)	(1 10)(2 9)(3 7)(4 6)(5 8)(11 12)
(1 12 3 4 5)(6 7 11 10 8)	(1 11)(2 9)(3 6)(4 8)(5 10)(7 12)
(1 6)(2 9)(3 10)(4 11)(5 7)(8 12)
Transitivity sets: {2,9} - 38B 248, {1,3,4,5,6,7,8,10,11,12} - 38B 50.

E118 (0 40 4) (29 16 0)
(1 20 2 19 3 18 5 16 6 15 4 17 10 11 9 12 7 14 8 13)

Aut D	(4 5)(6 11)(7 10)(8 9)	(1 7)(2 6)(3 10)(4 11)(5 9)(8 12)
	(2 4)(5 12)(6 8)(7 10)	(1 7)(2 8)(3 10)(4 9)(5 11)(6 12)
	(2 4 12 5)(6 11 8 9)	(1 7 3 10)(2 8 4 11 12 6 5 9)
	(2 5 12 4)(6 9 8 11)	(1 7)(2 9 12 11)(3 10)(4 6 5 8)
	(2 5)(4 12)(7 10)(9 11)	(1 7 3 10)(2 9 4 8 12 11 5 6)
	(2 12)(6 9)(7 10)(8 11)	(1 7 3 10)(2 11 4 6 12 9 5 8)
	(2 12)(4 5)(6 8)(9 11)	(1 7)(2 11 12 9)(3 10)(4 8 5 6)
	(1 3)(6 8)(7 10)(9 11)	(1 10 3 7)(2 6 5 11 12 8 4 9)
	(1 3)(4 5)(6 9)(8 11)	(1 10)(2 6 12 8)(3 7)(4 11 5 9)
	(1 3)(2 4)(5 12)(9 11)	(1 10)(2 8 12 6)(3 7)(4 9 5 11)
	(1 3)(2 4 12 5)(6 9 8 11)(7 10)	(1 10 3 7)(2 8 5 9 12 6 4 11)
	(1 3)(2 5 12 4)(6 11 8 9)(7 10)	(1 10)(2 9)(3 7)(4 6)(5 8)(11 12)
	(1 3)(2 5)(4 12)(6 8)	(1 10 3 7)(2 9 5 6 12 11 4 8)
	(1 3)(2 12)(6 11)(8 9)	(1 10 3 7)(2 11 5 8 12 9 4 6)
	(1 3)(2 12)(4 5)(7 10)	(1 10)(3 7)(2 11)(4 8)(5 6)(9 12)
	(1 7 3 10)(2 6 4 9 12 8 5 11)	

Transitivity sets: {1,3,7,10} - 38B 180, {2,4,5,6,8,9,11,12} - 38B 138.

E119 (0 40 4) (45 0 0)
(1 20 2 19 6 15 8 13 7 14 10 11 3 18 5 16 4 17 9 12)
Aut D (design is 1-transitive, see Chapter 5)
Transitivity sets: {1,2,3,4,5,6,7,8,9,10,11,12} - 38C 126.

APPENDIX 1

In Chapter 3, Section I the analysis of the six Patterns resulted in forty cases to be examined by the program. As two different data sets for any particular Pattern and Type will result in different (though isomorphic) designs, those used by the author will be presented here. More importantly, the conversion of the information provided by a partially complete skeleton into a computer digestable form will be demonstrated. Although this process deals specifically with the given case, it should be noted that the techniques used are indicative of those required when preparing the input data for the three other programs.

A fundamental requirement when formulating the input data is that the array values representing the points of any block assume an increase in numerical value with an increase in index value. For example, if the * pair for the fixed block is (1,3) then the two corresponding elements of the array ICON must be initialized as ICON(1,1) = 1, and ICON(1,2) = 3 and not vica-versa. Also, both the * and • point sets are initially represented by the numbers 1,..., 5.

As a basis for discussion, the relevant areas of the partially complete skeleton PI-Type 1 and its corresponding representation for the input file DATAREAD appear below. Note that all input information is read using free format, and numbers have been, and always will be, assigned to the letters via a = 1, b = 2, c = 3, d = 4 and e = 5.

PI-T1

	Index	T1	
	(1)	a e	
	(2)	a c	
	(3)	a d	
* pairs	(4)	a d	
with	(5)	b c	-ICON(10,2)
(0,f)	(6)	b d	

(*,0,•) triples for PI

	1	2	3	4	5
a	0	1	2	2	1
b	1	0	1	2	2
c	2	1	0	1	2
d	2	2	1	0	1
e	1	2	2	1	0

	(7)	b e		
	(8)	b e		(0,•,•) triples required for PI
	(9)	c d		12, 13, 13, 14, 14, 15, 23, 24,
	(10)	c e		24, 25, 25, 34, 35, 35, 45.

	a b	(11)	0 c d e	
* pairs	a c	(12)	0 b d e	-ICON2(5,3)
with	b d	(13)	0 a c e	
f	c e	(14)	0 a b d	
	d e	(15)	0 a b c	

DATAREAD

Assign to:	File contains:
X(5,5)	0,1,2,2,1,1,0,1,2,2,2,1,0,1,2,2,2,1,0,1,1,2,2,1,0
ISPEC(10)	12,13,14,15,23,24,0,0,0,0
IALLOW(12-23)	1,2,2,1,1
IALLOW(24-45)	2,2,1,2,1
NOFSP	6
ICON(10,2)	1,1,1,1,2,2,2,2,3,3,5,3,4,4,3,4,5,5,4,5
ICON2(5,3)	3,2,1,1,1,4,4,3,2,2,5,5,5,4,3
IREP(6)	3,7,0,0,0,2
NPERM	1
IPERM1(,),IPERM2(,)	1,10,7,8,9,6,3,4,5,2,15,14,13,12,11,10,9,8,7,6

The number of (0,*,•) triples for X are read columnwise. IALLOW stores the number of (0,•,•) triples required for balance. Here each of the ten possible • pairs are referenced as the point with the lowest numerical value multiplied by ten and added to the remaining point. The • pairs for the fixed block are also represented in ISPEC using this technique.

The only applicable permutation for PI-TI is (0)(f)(c)(b d)(a e) (3)(2 4)(1 5) so the * pair chosen for the fixed block was (a,e), however (b,d) would also have sufficed. Under this permutation,

$$\text{maps to}\quad \begin{array}{cccccc} 1,2 & 1,3 & 1,4 & 1,5 & 2,3 & 2,4 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 4,5 & 3,5 & 2,5 & 1,5 & 3,4 & 2,4 \end{array}$$

As there are six non-equivalent pairs, NOFSP = 6 and ISPEC was assigned 12,13,14,15,23,24 and the four space fillers 0. The number of permutations NPERM equals one and IPERM1 is initialized to produce the index interchanges caused by this permutation. This is done so that the columns of the resultant permuted 2 × 15 array remain in standard form, i.e.

initial index =(Data for IPERM1)	1	10	7	8	9	6	3	4	5	2	15	14	13	12	11
maps to	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
final index	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15 .

This permutation information is accompanied by the five data values for IPERM2 to effect the • point interchanges. For the case above they are interpreted as

$$\text{maps to}\quad \begin{array}{ccccc} 6 & 7 & 8 & 9 & 10 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 10 & 9 & 8 & 7 & 6\ . \end{array}$$

The data for the arrays ICON and ICON2 is reasonably self-evident and again is to be read columnwise. As there are two sets of repeated pairs, IREP(6) = 2. The indices of the first pair from each set are 3 and 7 and have been assigned to the first two elements of IREP, so the next three unused elements are assigned the space filler 0. Thus, for the program to correctly locate the repeated pairs, it is <u>essential</u> that any set have consecutive indices.

To reproduce exactly the designs examined by the author an abbreviated version of the data sets used will now be listed. As IALLOW and X are fixed for a particular Pattern they will be given once, and are to be used for each of the Types. Most of the remaining input data can be deduced from the representative Types given in Part II, Section II. The only source of variation now lies in the choice and ordering of the •

pairs for ISPEC. These will be given here along with the permutation data to be assigned to IPERM1 and IPERM2.

Pattern I

X = 0,1,2,2,1,1,0,1,2,2,2,1,0,1,2,2,2,1,0,1,1,2,2,1,0

IALLOW(12,13,14,15,23) = 1,2,2,1,1

IALLOW(24,25,34,35,45) = 2,2,1,2,1

T1	ISPEC	12,13,14,15,23,24,0,0,0,0
	IPERM1,IPERM2	1,10,7,8,9,6,3,4,5,2,15,14,13,12,11,10,9,8,7,6
T2		12,13,14,15,23,24,25,34,35,45
		1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,6,7,8,9,10
		8,6,7,5,4,2,3,1,9,10,11,13,12,15,14,7,6,10,9,8
T3		12,13,14,15,23,24,25,34,35,45
		1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,6,7,8,9,10
		7,8,6,5,4,3,1,2,10,9,11,13,12,14,15,7,6,10,9,8
T4		12,13,14,15,24,25,0,0,0,0
		(NO DESIGNS)
T5		12,13,14,15,34,35,0,0,0,0
		4,7,9,10,1,2,3,5,6,8,14,15,11,12,13,7,8,9,10,6
		10,3,6,8,4,7,9,1,2,5,12,13,14,15,11,8,9,10,6,7
		8,9,2,5,10,3,6,4,7,1,15,11,12,13,14,9,10,6,7,8
		5,6,7,1,8,9,2,10,3,4,13,14,15,11,12,10,6,7,8,9
		4,3,2,1,10,9,7,8,6,5,12,11,15,14,13,6,10,9,8,7
		10,9,7,4,8,6,3,5,2,1,15,14,13,12,11,10,9,8,7,6
		8,6,3,10,5,2,9,1,7,4,13,12,11,15,14,9,8,7,6,10
		5,2,9,8,1,7,6,4,3,10,11,15,14,13,12,8,7,6,10,9
		1,7,6,5,4,3,2,10,9,8,14,13,12,11,15,7,6,10,9,8

Pattern II

X = 1,1,2,1,1,1,0,1,2,2,2,1,0,1,2,1,2,1,1,1,1,2,2,1,0

IALLOW(12,13,14,15,23) = 1,2,2,1,1

IALLOW(24,25,34,35,45) = 2,2,1,2,1

T1	ISPEC	12,13,14,15,23,24,25,34,35,45
T2		12,13,14,15,23,24,25,34,35,45
T3		12,13,14,15,23,24,25,34,35,45
T4		12,13,14,15,23,24,25,34,35,45
T5	ISPEC	12,13,14,15,23,25,0,0,0,0
	IPERM1,IPERM2	1,6,7,10,5,2,3,9,8,4,14,12,15,11,13,9,8,7,6,10
T6		12,13,14,15,23,25,0,0,0,0
		(NO DESIGNS)
T7		12,13,14,15,23,24,25,34,35,45

T8		12,13,14,15,23,24,25,34,35,45
T9		12,13,14,15,23,25,0,0,0,0 1,8,6,10,5,3,9,2,7,4,13,12,11,15,14,9,8,7,6,10
T10		12,13,14,15,23,25,0,0,0,0 1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,6,7,8,9,10 1,2,8,5,4,9,10,3,6,7,14,15,13,11,12,9,8,7,6,10
T11		12,13,14,15,23,24,25,34,35,45 8,5,6,10,2,3,9,1,7,4,11,12,13,15,14,9,8,7,6,10

Pattern III

X = 1,1,2,1,1,1,0,1,2,2,1,1,1,2,1,1,2,1,1,1,2,2,1,0,1

IALLOW(12,13,14,15,23) = 1,2,2,1,1

IALLOW(24,25,34,35,45) = 2,2,1,2,1

T1	ISPEC IPERM1,IPERM2	12,13,23,24,25,34,0,0,0,0 1,10,7,8,9,6,3,4,5,2,15,14,13,12,11,6,10,9,8,7
T2		12,13,14,15,23,24,25,34,35,45
T3		12,13,23,24,25,34,15,14,45,35
T4		12,13,23,24,25,34,15,14,45,35
T5		12,13,23,24,25,34,0,0,0,0 1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,6,7,8,9,10 1,2,9,10,7,8,5,6,3,4,15,12,14,13,11,6,10,9,8,7
T6		12,13,23,24,25,34,15,14,45,35
T7		12,13,23,24,25,34,0,0,0,0 1,10,9,7,8,6,4,5,3,2,15,14,13,12,11,6,10,9,8,7
T8		12,13,14,15,23,24,25,34,35,45
T9		12,13,14,15,23,24,25,34,35,45 10,9,6,7,8,3,4,5,2,1,15,12,13,14,11,6,10,9,8,7
T10		12,13,14,15,23,24,25,34,35,45
T11		12,13,14,15,23,24,25,34,35,45 10,9,8,7,6,5,4,3,2,1,14,12,15,11,13,6,10,9,8,7

Pattern IV

X = 1,1,2,1,1,1,1,1,1,2,2,1,1,1,1,1,1,1,2,1,1,2,1,1,1

IALLOW(12,13,14,15,23) = 1,1,2,2,2

IALLOW(24,25,34,35,45) = 2,1,1,2,1

T1	ISPEC IPERM1,IPERM2	12,14,23,24,25,45,0,0,0,0 1,10,7,8,9,6,3,4,5,2,15,14,13,12,11,6,8,7,10,9

T2 12,14,23,24,25,45,13,15,35,34
8,6,7,5,4,2,3,1,9,10,11,13,12,15,14,7,6,10,9,8

T3 12,14,23,24,25,45,13,15,35,34
5,7,8,6,1,4,2,3,10,9,11,13,12,14,15,7,6,10,9,8

T4 12,13,14,15,24,25,0,0,0,0
1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,6,7,8,9,10
3,4,1,2,9,10,7,8,5,6,12,11,15,14,13,9,10,8,6,7
7,8,5,6,3,4,1,2,9,10,11,13,12,15,14,7,6,10,9,8
1,2,9,10,7,8,5,6,3,4,13,14,11,12,15,8,9,6,7,10
5,6,3,4,1,2,9,10,7,8,14,15,13,11,12,10,7,9,8,6
9,10,7,8,5,6,3,4,1,2,15,12,14,13,11,6,8,7,10,9
7,8,9,10,1,2,3,4,5,6,12,15,11,13,14,9,8,10,7,6
3,4,5,6,7,8,9,10,1,2,15,14,12,11,13,7,10,6,8,9
9,10,1,2,3,4,5,6,7,8,14,13,15,12,11,8,6,9,10,7
5,6,7,8,9,10,1,2,3,4,13,11,14,15,12,10,9,7,6,8

T5 12,13,14,15,34,35,0,0,0,0
1,7,6,5,4,3,2,10,9,8,14,13,12,11,15,7,6,10,9,8
5,2,9,8,1,7,6,4,3,10,11,15,14,13,12,8,9,6,7,10
8,6,3,10,5,2,9,1,7,4,13,12,11,15,14,10,7,9,8,6
10,9,7,4,8,6,3,5,2,1,15,14,13,12,11,6,8,7,10,9
4,3,2,1,10,9,7,8,6,5,12,11,15,14,13,9,10,8,6,7
4,7,9,10,1,2,3,5,6,8,14,15,11,12,13,9,8,10,7,6
10,3,6,8,4,7,9,1,2,5,12,13,14,15,11,7,10,6,8,9
8,9,2,5,10,3,6,4,7,1,15,11,12,13,14,8,6,9,10,7
5,6,7,1,8,9,2,10,3,4,13,14,15,11,12,10,9,7,6,8

Pattern V

X = 1,1,1,1,2,1,1,1,2,1,1,1,1,1,2,1,1,2,1,1,2,2,1,1,0

IALLOW(12,13,14,15,23) = 1,1,2,2,2

IALLOW(24,25,34,35,45) = 2,1,1,2,1

T1 ISPEC 12,13,14,15,23,24,25,34,35,45

T2 12,13,14,15,23,24,25,34,35,45

T3 ISPEC 12,13,14,15,23,24,25,34,35,45
IPERM1,IPERM2 2,1,3,4,7,8,5,6,10,9,12,11,13,14,15,8,9,6,7,10

T4 12,13,14,15,23,24,25,34,35,45
7,5,6,8,2,3,1,4,10,9,13,14,11,12,15,8,9,6,7,10

Pattern VI

X = 1,1,1,1,2,1,1,1,1,2,1,1,1,2,1,1,1,2,1,1,2,2,1,1,0

IALLOW(12,13,14,15,23) = 0,2,2,2,2

IALLOW(24,25,34,35,45) = 2,2,1,1,1

T1 ISPEC 13,14,15,34,35,45,0,0,0,0
IPERM1,IPERM2 1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,7,6,8,9,10

T2 13,14,15,34,35,45,0,0,0,0
1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,7,6,8,9,10

T3	13,14,15,34,35,45,0,0,0,0 1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,7,6,8,9,10 2,1,3,4,7,8,5,6,10,9,12,11,13,14,15,7,6,9,8,10 2,1,3,4,7,8,5,6,10,9,12,11,13,14,15,6,7,9,8,10
T4	13,14,15,34,35,45,0,0,0,0 1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,7,6,8,9,10 7,5,6,8,2,3,1,4,10,9,13,14,11,12,15,7,6,9,8,10 7,5,6,8,2,3,1,4,10,9,13,14,11,12,15,6,7,9,8,10

APPENDIX 2

The following gives a brief explanation and listing of the input data supplied to the programs of Chapter 3: Section II, where the designs consisting entirely of B type blocks were produced. The first of the two programs MAIN is responsible for completing the eighteen blocks of the skeleton which fulfil the $n_2 = n_4 = 9$ conditions for a design containing a B type block and requires input data, while the latter FINISH works directly off MAIN's output file 'SOLUT' and requires no further information. As with earlier programs all input data is read using free format. The method by which arrays and variables are used to store and utilise permutation information is also similar (see Appendix 1).

It is convenient to relist the input statements of MAIN and give a brief explanation of the purpose and content of each variable.

```
      COMMON/GR0/IOUT(100,5,2),LINT(2),INS(12,12,12),II(7,44)
      COMMON/GR1/ISTORE(10000,10,2),IDIS(2,10),INDEX(85,3)
      COMMON/GR2/NREP,IREP(300,10),IDIS2(3,7)
      COMMON/GR3/IPERM2(100,10),NPERM2,ICON2(100),ISTCON(100,10,6)
      DIMENSION IPTS(10)
      OPEN(12,FILE='DATA')
      OPEN(15,FILE='SOLUT')
      DATA IPTS/0,0,0,0,0,0,0,0,0,0/
      READ(12,*)(II(M1,1),M1=1,6) ---------------------- (1)
      READ(12,*)(II(M2,11),M2=1,6) --------------------- (2)
      DO 1 M3=1,2
 1    READ(12,*)(II(M3,M4),M4=2,10) -------------------- (3)
      DO 2 M5=1,4
 2    READ(12,*)(II(M5,M6),M6=12,20) ------------------- (4)
      DO 5 M11=1,10
 5    READ(12,*)IDIS(1,M11),IDIS(2,M11) ---------------- (5)
      READ(12,*)IDIS2(1,7) ----------------------------- (6)
      IF(IDIS2(1,7).EQ.0)GO TO 22
      DO 21 M21=1,IDIS2(1,7)
21    READ(12,*)(IDIS2(M22,M21),M22=1,3) --------------- (7)
22    READ(12,*)NREP ----------------------------------- (8)      Step 0
      DO 13 M13=1,NREP
13    READ(12,*)(IREP(M13,M14),M14=2,10) --------------- (9)
      READ(12,*)NPERM2 --------------------------------- (10)
      DO 6 M15=1,NPERM2
 6    READ(12,*)(IPERM2(M15,M16),M16=2,10) ------------- (11)
      READ(12,*)(ICON2(M16),M16=1,NPERM2) -------------- (12)
      DO 7 M17=1,NPERM2
      DO 8 M18=1,ICON2(M17)
 8    READ(12,*)(ISTCON(M17,M18,M19),M19=1,6) ---------- (13)
 7    CONTINUE
      READ(12,*)LINT(1),LINT(2) ------------------------ (14)
      DO 23 M21=1,LINT(1)
      DO 9 M20=1,LINT(2)
 9    READ(12,*)IOUT(M21,M20,1),IOUT(M21,M20,2) -------- (15)
23    CONTINUE
```

(1) The first block containing the * points 1,2,3,4,5,6.

(2) The other top block containing the • points 7,8,9,10,11,12.

(3) The nine * pairs read columnwise.

(4) The nine * quadruples read columnwise.

(5) All pairs of • indices against repeated * quadruples which are illegal as they would result in the occurrence of AC type blocks.

(6) The number of * points against which repeated • pairs would cause the occurrence of AC type blocks.

(7) Sets of indices for the * points of (6).

(8) Total number of index changes resulting from the occurrence of repeated * quadruples which leave the structure unchanged.

(9) The permutation of the indices for each case of (8).

(10) The total number of index changes to allow for permutations which fix the * quadruples.

(11) The actual permutation of indices for each case of (10).

(12) The number of permutations of the * points corresponding to each of the index permutations of (10), (given in the same order).

(13) Actual * point permutations corresponding to (12), (given in correct order).

(14) The number of non-equivalent • pair sets manually calculated and the number of pairs in each of these sets respectively.

(15) Each non-equivalent • pair set.

Given this brief description of the purpose of each of the variables it is a relatively simple matter to produce the input files for each of the eight possible Patterns. Below are the twenty top blocks of PIV with the standard assignment of indices. These are followed by the entire data file used to calculate the designs for this case, with each section of data labelled to correspond to its READ statement.

PIV

[1	2	3	4	5	6]	[7	8	9	10	11	12]	Index
[1	2	•	•	•	•]	[3	4	5	6	•	•]	(2)
[1	2	•	•	•	•]	[3	4	5	6	•	•]	(3)
[3	4	•	•	•	•]	[1	2	5	6	•	•]	(4)
[3	4	•	•	•	•]	[1	2	5	6	•	•]	(5)
[5	6	•	•	•	•]	[1	2	3	4	•	•]	(6)
[5	6	•	•	•	•]	[1	2	3	4	•	•]	(7)
[1	3	•	•	•	•]	[2	4	5	6	•	•]	(8)
[2	5	•	•	•	•]	[1	3	4	6	•	•]	(9)
[4	6	•	•	•	•]	[1	2	3	5	•	•]	(10)

Data for PIV

Data	
01:1,2,3,4,5,6	(1)
02:7,8,9,10,11,12	(2)
03:1,1,3,3,5,5,1,2,4 04:2,2,4,4,6,6,3,5,6	(3)
05:3,3,1,1,1,1,2,1,1 06:4,4,2,2,2,2,4,3,2 07:5,5,5,5,3,3,5,4,3 08:6,6,6,6,4,4,6,6,5	(4)
09:2,3 10:4,5 11:6,7 12:0,0 13:0,0 14:0,0 15:0,0 16:0,0 17:0,0 18:3,3	(5)
19:6	(6)
20:2,3,8 21:2,3,9 22:4,5,8 23:4,5,10 24:6,7,9 25:6,7,10	(7)
26:8	(8)
27:2,3,4,5,6,7,8,9,10 28:2,3,4,5,7,6,8,9,10 29:2,3,5,4,6,7,8,9,10 30:2,3,5,4,7,6,8,9,10 31:3,2,4,5,6,7,8,9,10 32:3,2,4,5,7,6,8,9,10 33:3,2,5,4,6,7,8,9,10 34:3,2,5,4,7,6,8,9,10	(9)
35:6	(10)
36:2,3,6,7,4,5,9,8,10 37:6,7,4,5,2,3,10,9,8 38:4,5,2,3,6,7,8,10,9 39:4,5,6,7,2,3,10,8,9 40:6,7,2,3,4,5,9,10,8 41:2,3,4,5,6,7,8,9,10	(11)
42:1,1,1,1,1,1	(12)
43:2,1,5,6,3,4 44:6,5,4,3,2,1 45:3,4,1,2,6,5 46:4,3,6,5,1,2 47:5,6,2,1,4,3 48:1,2,3,4,5,6	(13)
49:7,4	(14)
50:7,8 51:9,10 52:7,8 53:9,10 54:7,8 55:9,10 56:7,8 57:9,11 58:7,8 59:9,10 60:7,8 61:11,12 62:7,8 63:9,10 64:7,9 65:8,10 66:7,8 67:9,10 68:7,9 69:8,11	(15)

70:7,8
71:9,10
72:7,9
73:11,12 —— (15)
74:7,8
75:9,10
76:7,11
77:9,12

As the remaining seven data files are readily deduced from the representative Patterns, only parts of these will be reproduced here. In particular for each of PI to PVIII the data for statements (5),(6),(8),(10),(12),(14) and (15) will be given, so that an exact reproduction of the author's designs will be possible. Some caution must be exercised when specifying (14) and (15) as the process by which the program produces the pair sets must be taken into account.

Firstly, the algorithm is structured so that the 6! = 720 ways of arbitrarily choosing the • points from {7,8,9,10,11,12} are eliminated. Here, the important thing to remember is that 7 must have higher priority (lower index values) than 8, 8 than 9, 9 than 10, etc. Secondly, when two legitimate • pair sets are found to be equivalent, the one produced first by the algorithm is always chosen. These conditions make it apparent that the top • pair must always be (7,8) which is to be expected.

As an example, consider the following four • pair sets for PIV:

* Quadruple	Index	Case			
		C1	C2	C3	C4
[3 4 5 6 • •]	(2)	7 8	7 8	7 8	7 8
[3 4 5 6 • •]	(3)	7 9	7 9	7 9	8 9
[1 2 5 6 • •]	(4)	8 10	9 10	8 •	7 •
[1 2 5 6 • •]	(5)	10 11	10 11	• •	• •

C1 and C2 are both legitimate and equivalent but C1 is chosen over C2 as its 8's have higher priority (their 7's are equal). While C3 and C4 are equivalent, C4 is illegal as its 8's have higher priority than its 7's - a redundant variation not produced by the algorithm.

For the given statements the following data values were used.

Statement

	PI	PII	PIII	PIV	PV	PVI	PVII	PVIII
(5)	2, 3	2,3	2,3	2,3	2,3	2,3	0,0	0,0
	2, 4	2,4	2,4	4,5	4,5	0,0	0,0	0,0
	3, 4	3,4	3,4	6,7	0,0	0,0	0,0	0,0
	5, 6	5,6	0,0	0,0	0,0	0,0	0,0	0,0
	5, 7	7,8	0,0	0,0	0,0	0,0	0,0	0,0
	6, 7	0,0	0,0	0,0	0,0	0,0	0,0	0,0
	8, 9	0,0	0,0	0,0	0,0	0,0	0,0	0,0
	8,10	0,0	0,0	0,0	0,0	0,0	0,0	0,0
	9,10	0,0	0,0	0,0	0,0	0,0	0,0	0,0
	9, 9	5,5	3,3	3,3	2,2	1,1	0,0	0,0
(6)	0	4	4	6	6	6	6	6
(8)	216	24	6	8	4	2	1	1
(10)	6	4	24	6	4	4	72	12
(12)	(8,)×6	(2,)×4	(2,)×24	(1,)×6	(1,)×4	(1,)×4	(1,)×72	(1,)×12
(14)	2,4	2,4	2,4	7,4	1,4	1,4	19,4	38,4

Each of the following pair sets was used for statement (15)

PI		PII		PIII	
7, 8	7, 8	7, 8	7, 8	7, 8	7, 8
9,10	9,10	9,10	9,10	9,10	9,10
11,12	11,12	11,12	11,12	11,12	11,12
7, 8	7, 9	7, 8	7, 9	7, 8	7, 9

PIV

7, 8	7, 8	7, 8	7, 8	7, 8	7, 8	7, 8
9,10	9,10	9,10	9,10	9,10	9,10	9,10
7, 8	7, 8	7, 8	7, 9	7, 9	7, 9	7,11
9,10	9,11	11,12	8,10	8,11	11,12	9,12

PV

·1	·2	·3	·4	·5	·6	·7
7, 8	7, 8	7, 8	7, 8	7, 8	7, 8	7, 8
9,10	9,10	9,10	9,10	9,10	9,10	9,10
7, 8	7, 8	7, 8	7, 9	7, 9	7, 9	7,11
9,10	9,11	11,12	8,10	8,11	11,12	9,12

PVI

·1	·2	·3	·4	·5	·6
7, 8	7, 8	7, 8	7, 8	7, 8	7, 8
9,10	9,10	9,10	9,10	9,10	9,10
7, 9	7,11	7,11	7,11	7,11	11,12
11,12	7,12	8,12	9,12	11,12	11,12

PVII

7, 8	7, 8	7, 8	7, 8	7, 8	7, 8	7, 8	7, 8	7, 8	7, 8
7, 9	7, 9	7, 9	7, 9	7, 9	7, 9	9,10	9,10	9,10	9,10
8,10	8,10	8,10	8,10	8,10	8,10	7,11	7,11	7,11	7,11
7, 9	7,10	7,11	9,10	9,11	11,12	7, 9	7,11	7,12	8, 9

7, 8	7, 8	7, 8	7, 8	7, 8	7, 8	7, 8	7, 8	7, 8
9,10	9,10	9,10	9,10	9,10	9,10	9,10	9,10	9,10
7,11	7,11	7,11	7,11	7,11	7,11	11,12	11,12	11,12
8,11	8,12	9,10	9,11	9,12	11,12	7, 9	9,10	9,11

PVIII

7, 8	7, 8	7, 8	7, 8	7, 8	7, 8	7, 8	7, 8	7, 8	7, 8
7, 9	7, 9	7, 9	7, 9	7, 9	7, 9	7, 9	7, 9	7, 9	7, 9
8, 9	8, 9	8, 9	8, 9	8,10	8,10	8,10	8,10	8,10	8,10
7,10	8, 9	8,10	10,11	7,10	7,11	8, 9	8,10	8,11	9,10

7, 8	7, 8	7, 8	7, 8	7, 8	7, 8	7, 8	7, 8	7, 8	7, 8
7, 9	7, 9	7, 9	9,10	9,10	9,10	9,10	9,10	9,10	9,10
8,10	8,10	8,10	7,11	7,11	7,11	7,11	7,11	7,11	7,11
9,11	10,11	11,12	7, 9	7,11	7,12	8, 9	8,11	8,12	9,11

7, 8	7, 8	7, 8	7, 8	7, 8	7, 8	7, 8	7, 8	7, 8	7, 8
9,10	9,10	9,10	9,10	9,10	7, 9	7, 9	7, 9	7, 9	7, 9
7,11	7,11	11,12	11,12	11,12	10,11	10,11	10,11	10,11	10,11
9,12	11,12	7, 9	7,11	11,12	7,10	7,12	8, 9	8,10	8,12

7, 8	7, 8	7, 8	7, 8	7, 8	7, 8	7, 8	7, 8
7, 9	7, 9	9,10	9,10	9,10	9,10	9,10	9,10
10,11	10,11	7, 9	7, 9	7, 9	7, 9	7, 9	7, 9
10,11	10,12	7, 9	7,10	7,11	8,10	8,11	11,12

APPENDIX 3

The Data for the Designs with Repeated Blocks

Apart from the reference label in line one, the information is read using free format. The exact layout is best demonstrated by example, so the entire data file for D3 has been presented with accompanying explanatory comments.

Data file for D3

line	data	comment
001	D3]	Configuration label.
002	1,1,1,1,1,1,1,1,1,1,2,2,2,2,2,2,3,3,3,3,4,4	Skeleton's pairs.
003	2,2,2,2,3,3,4,4,5,5,3,3,4,4,5,5,4,4,5,5,5,5	
004	3,3,6,6	Top four blocks of the configuration read column-wise.
005	4,4,7,7	
006	5,5,8,8	
007	1,1,2,2,2,3,3,4	Pairs for the incomplete blocks.
008	4,5,3,4,5,4,5,5	
009	6,7,8	The twenty possible triples available to each incomplete block.
010	6,7,9	
011	6,7,10	
012	6,7,11	
013	6,8,9	
014	6,8,10	
015	6,8,11	
016	6,9,10	
017	6,9,11	
018	6,10,11	
019	7,8,9	
020	7,8,10	
021	7,8,11	
022	7,9,10	
023	7,9,11	
024	7,10,11	
025	8,9,10	
026	8,9,11	
027	8,10,11	
028	9,10,11	
029	6,9,10]	Triple for block (iii).
030	7,9,10]	Triple for block (iv).
031	4]	Number of triples disallowed in block (v).
032	10,15,16,19]	Pointers of disallowed triples for block (v).
033	14]	Number of triples disallowed in the first six blocks.
034	1,2,3,4,5,6,7,8,11,12,13,14,17,20]	Pointers of disallowed triples for the first six blocks.
035	7]	Number of non-trivial permutations fixing the configuration.

036 : 1,2,3,5,4,6,7,8,9,10,11
037 : 1,2,3,4,5,6,7,8,10,9,11
038 : 1,2,3,5,4,6,7,8,10,9,11
039 : 1,2,3,4,5,7,6,8,9,10,11
040 : 1,2,3,5,4,7,6,8,9,10,11
041 : 1,2,3,4,5,7,6,8,10,9,11
042 : 1,2,3,5,4,7,6,8,10,9,11

Non-trivial permutations given in the form:

$$\begin{array}{ccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 1, & 2, & 3, & 5, & 4, & 6, & 7, & 8, & 9, & 10, & 11 \end{array}$$

i.e. the first permutation just interchanges 4 and 5.

By listing the data set used for each configuration an exact reproduction of a representative set of designs can be made at any time. As most information is either constant or easily deduced from the blocks of the configurations, only lines 1,31,32,33,34, and 35 of the data files will be given. Where a configuration has been split into subcases, an integrated data file will be given as the sectionalised forms are readily derived from this.

Data List

Case ie line 1	Corresponding data for line number				
	31	32	33	34	35
D1	3	16,18,19	16	1,2,3,4,5,6,7,8,9,10, 11,12,13,14,17,20	47
D2	4	15,17,18,19	14	1,2,3,4,5,6,7,8,9,10, 11,12,13,20	15
D3	4	10,15,16,19	14	1,2,3,4,5,6,7,8,11, 12,13,14,17,20	7
D4	4	14,15,16,18	11	1,2,3,4,5,6,7,11,12, 13,20	3
D5	5	9,14,15,19,20	10	1,2,3,4,5,6,8,10,11,17	7
D6	5	8,14,15,18,20	8	1,2,3,4,5,6,9,11	3
D7	3	16,17,20	11	1,2,3,4,5,6,7,8,10, 11,19	3
D8	5	12,14,15,16,18	8	1,2,3,4,5,6,11,13	3
D9	5	11,12,14,15,20	8	1,2,3,4,5,6,11,14	3
D10	6	12,13,14,16,18,20	8	1,2,3,4,5,6,11,15	3
D11	1	0	7	1,2,3,4,5,6,11	1
D12	3	13,18,20	11	1,2,3,4,5,6,8,11,12, 17,19	3
D13	1	0	7	1,2,3,4,5,6,11	1
D14	4	13,16,18,20	12	1,2,3,4,5,6,7,10,11, 12,17,19	3
D15	1	0	7	1,2,3,4,5,6,11	1
D16	8	7,8,9,12,13,14,15,18	10	1,2,3,4,5,10,11,16, 19,20	15
D17	5	8,9,14,15,20	6	1,2,3,4,5,11	3
D18	4	9,13,15,18	12	1,2,3,4,5,6,7,10,11, 16,19,20	3
D19	7	7,8,9,12,13,14,15	12	1,2,3,4,5,10,11,16,17, 18,19,20	31
D20	3	16,18,19	16	1,2,3,4,5,6,7,8,9,10, 11,12,13,14,17,20	31

D21	4	15,17,18,19	12	1,2,3,4,5,6,7,8,9,11, 12,13	7
D22	4	10,15,16,19	14	1,2,3,4,5,6,7,8,11,12, 13,14,17,20	7
D23	4	14,15,16,18	10	1,2,3,4,5,6,7,11,12,13	3
D24	5	10,16,17,18,19	10	1,2,3,4,5,6,7,11,12,13	7
D25	11	5,7,9,10,12,14,15,17,18,19,20	4	1,2,6,8	15
D26	6	4,9,14,15,18,20	4	1,2,7,8	3
D27	6	11,13,14,15,16,20	4	1,2,8,12	3
D28	8	4,11,12,14,15,16,18,20	4	1,2,8,13	3
29A-Q	1	0	3	1,2,8	1
30A-G	10	5,7,9,10,12,14,17,18,19,20	3	1,2,8	7
31A-F	10	3,6,10,11,12,13,15,16,19,20	4	1,4,8,14	7
32A-J	6	3,6,10,12,16,19	4	1,4,8,17	3
33A-R	1	0	2	1,4	1
34A-J	6	3,6,10,12,16,19	4	1,4,8,20	3
35A-D	12	3,5,6,7,9,10,12,14,16,17,18,19	4	1,8,13,20	31
36A-E	15	5,6,7,8,9,10,12,13,14,15,16,17 18,19,20	1	0	15
38A	4	3,18,4,17	4	1,2,19,20	47
38B	4	3,18,5,16	4	1,2,19,20	3
38C	4	6,15,8,13	4	1,2,19,20	47

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